

# THE EFFECT OF MANUFACTURING ERRORS ON ROBOT ACCURACY

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## ABSTRACT

The performance of positioning and orienting the end-effector requires good knowledge of manipulator physical parameter values. However, this requirement can be difficult to meet in practice due to manufacturing errors. Kinematic parameters are perturbed to analyze positioning and orienting end-effector accuracies of the manipulator. The kinematic relationships between the links are described by utilizing the zero-reference-position method to generate the governing kinematic equations of a manipulator. Various perturbations to link dimensions and joint axis alignments are studied to monitor trajectory execution accuracies. A PUMA-type manipulator is considered as an example.

**Keywords:** Robotics kinematics, Manipulators design, Manufacturing errors, Accuracy.

## INTRODUCTION

A robot may have a real geometry equivalent to that specified in the design with ideal manufacturing and assembling processes. This, however, will never happen for an industrial robot due to manufacturing tolerances and fitting allowances. Besides, some parameters may not be considered significantly by the designer and others are too complex to model. In other words, variations always exist between the true geometry of a robot and the geometry described in the design.

Robot positioning and orienting deviations are caused by various sources of errors. Some researchers [1,2] consider the errors as the vectorial sum of the contribution from all sources. They classify them into joint errors, kinematic errors which are due to deviation of kinematic parameters from nominal values and non-kinematic errors which are due to joint compliance, backlash and link compliance.

On the other hand, other researchers [3,4] classify them into five categories: Parametric: due to variation of kinematic parameters, Computational: due to computer round-off and steady-state servo errors, Environmental: such as temperature changes, Measurement: due to resolution and non-linearity of joint position sensors and Application: due to installation errors, workpiece position and geometry errors.

Kinematics parameters are identified as dominant sources of errors. Some researchers studied robot kinematic calibration [2,5]. The calibration could be defined as a set of procedures aimed to improve robot accuracy by software, e.g. without changes in mechanical robot structure or robot control system. Some efforts in this area have been briefed in two survey papers [1,6]. The application of calibration procedure is a discrete event, contrary to control schemes where model identification is carried out continuously.

Four important steps can be distinguished in the calibration process: 1. Modeling: definition and selection of the form of a suitable kinematic relationship. 2. Measurement: physical data collection for a set of measuring configurations. 3. Identification: determination of the model parameters so that the error between the measured and the modeled poses is minimized. 4. Error compensation: implementation of the identified model in the robot positioning software.

**KINEMATIC ANALYSIS**

The inverse kinematic process is described using the zero-reference-position method. This method, which is introduced by Gupta [7], refers to the way the geometry of the robot is defined. A reference position of the robot is chosen and all the joint variables are set, for convenience, to zeros at that position. The reference position is suitably chosen, but in principle it can be arbitrary. All joint axis vectors and their locations are defined in the base coordinate system. The term initial position is used when the manipulator end-effector is at the beginning of its trajectory path. However, the initial position for manipulator is sometimes associated with its home (rest) position or with its zero reference position (when the joint values are set to be zero). This method has been used for the purpose of description and analysis of general manipulators by many researchers [2,8,10,11].

In a general six degrees-of-freedom manipulator, link *k* connects its associated joints (*k*-1) and *k*. A unit vector *u* is assigned for each joint such that *u<sub>k,0</sub>* passes through the axis of motion of joint *k* as shown Figure 1. The parameters with subscript 0 refer to the zero reference position. A body vector *b* is also assigned for each link such that *b<sub>k,0</sub>* is a vector which connects a point on joint (*k*-1) to a point on joint *k*. The magnitude of *b* is always constant throughout the movement of the manipulator and equals to its zero-reference-position value. The location of each joint vector *u* is defined by a position

vector *p* which can be obtained by summing the body vectors up to the prescribed location. Two mutually perpendicular unit vectors *u<sub>a,0</sub>* and *u<sub>t,0</sub>* are used to describe the end-effector orientation; they pass through the mid point *p* of the end-effector. The axial vector *u<sub>a,0</sub>* is along the gripper axis, while the transverse vector is *u<sub>t,0</sub>* in the traverse direction.

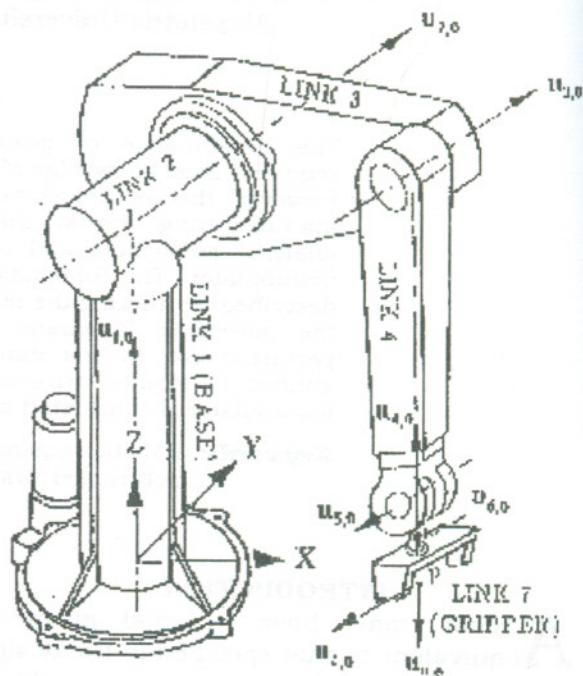


Figure PUMA-type manipulator in its Zero-Reference-position

All the above mentioned parameters are given in their zero reference position. They are converted to the current position as the manipulator moves. The current vector are derived from their zero-reference-position vectors as follows [9]

$$u_k = (2E_{0,k}^2 - 1)u_{k,0} + 2(E_{k+1} \cdot u_{k,0})E_{k+1} + 2E_{0,k+1}(E_{k+1} \times u_{k,0}) \quad (1)$$

$$b_k = (2E_{0,k}^2 - 1)b_{k,0} + 2(E_k \cdot b_{k,0})E_k + 2E_{0,k}(E_k \times b_{k,0}) \quad (2)$$

where,

$$E_{0,k} = e_{0,k-1}E_{0,k-1} - e_{k-1}E_{k-1} \quad (4)$$

$$E_k = E_{0,k-1}e_{k-1} + e_{0,k-1}E_{k-1} - e_{k-1} \times E_{k-1} \quad (5)$$

and

$$e_0 = \cos \frac{\theta}{2} \quad e_1 = u_x \sin \frac{\theta}{2} \quad e_2 = u_y \sin \frac{\theta}{2} \quad e_3 = u_z \sin \frac{\theta}{2} \quad (6)$$

These four quantities are the Euler-Rodrigues parameters.

In the inverse kinematic problem, the desired time functions of the end-effector are given. These time functions determine the position, velocity and acceleration of the end-effector. Then, the following kinematic equation could be used

$$[J] \dot{q} = \begin{Bmatrix} \omega^h \\ V^p \end{Bmatrix} \quad (7)$$

where  $[J]$  is the Jacobian matrix,  $\omega^h$  is the hand angular velocity and  $V^p$  is the linear velocity of point p. The joint variables rates ( $\dot{q}$ ) are integrated by using a predictor-corrector scheme to obtain the joint variables.

**PERTURBATIONS ANALYSIS**

Two different kinematic parameters are defined: nominal (ideal) kinematic parameters include the perfect link dimensions and joint axis alignments and actual (realistic) kinematic parameters include the actual link dimensions and joint axis misalignments. Thus, the perturbations (errors) can be split into:  $\xi_{b,k}^{\%}$  which is the percentage perturbation in  $k^{th}$  link dimensions, and  $\xi_{u,k}^{\%}$  which is the percentage perturbation in  $k^{th}$  joint axis alignment. Both satisfy the following condition

$$-1 < \xi^{\%} < 1 \quad (8)$$

The components of the  $k^{th}$  axis misalignment vector  $\vec{u}_{k,0}$  could be calculated as

$$\vec{u}_{k,0_i} = \frac{\vec{u}'_{k,0_i}}{\|\vec{u}'_{k,0}\|} \quad ; i = 1,2,3 \quad (9)$$

where,

$$\vec{u}'_{k,0_i} = \text{sign}(u_{k,0_i}) \left[ \xi_{u,k}^{\%} \|u_{k,0}\| + |u_{k,0_i}| \right] \quad (10)$$

and the components of the perturbed  $k^{th}$  link dimension  $\vec{b}_{k,0}$  could also be calculated as

$$\vec{b}_{k,0_i} = \text{sign}(b_{k,0_i}) \left[ \xi_{b,k}^{\%} \|b_{k,0}\| + |b_{k,0_i}| \right] \quad (11)$$

and their magnitudes are

$$\|\vec{u}'_{k,0}\| = \sqrt{\sum_{i=1}^3 u_{k,0_i}^2} = 1 \quad \text{and} \quad \|\vec{b}_{k,0}\| = \sqrt{\sum_{i=1}^3 b_{k,0_i}^2} \quad (12)$$

Equations (1) and (2) could be rewritten as

$$\vec{u}_k = (2\vec{E}_{0,k+1}^2 - 1) \vec{u}_{k,0} + 2(\vec{E}_{k+1} \cdot \vec{u}_{k,0}) \vec{E}_{k+1} + 2\vec{E}_{0,k+1} (\vec{E}_{k+1} \times \vec{u}_{k,0}) \quad (13)$$

$$\vec{b}_k = (2\vec{E}_{0,k}^2 - 1) \vec{b}_{k,0} + 2(\vec{E}_k \cdot \vec{b}_{k,0}) \vec{E}_k + 2\vec{E}_{0,k} (\vec{E}_k \times \vec{b}_{k,0}) \quad (14)$$

Contrary to the inverse kinematics is the direct kinematics problem. The position of the end-effector is determined uniquely with respect to a reference coordinate system for a given set of manipulator joint variables which are obtained by using the perturbed parameters.

The rotational deviations could be calculated as

$$\delta_R = \sqrt{4(1 - e_{0r}^2 + \mathbf{e}_\epsilon \cdot \mathbf{e}_\epsilon)} \quad (15)$$

where,

$$e_{0\epsilon}^2 = \cos^2 \frac{\theta_\epsilon}{2} \quad \text{and} \quad \mathbf{e}_\epsilon \cdot \mathbf{e}_\epsilon = \sin^2 \frac{\theta_\epsilon}{2} \quad (16)$$

and the positional deviations could be calculated as

$$\delta_p = \sqrt{(x_a^p - x_d^p)^2 + (y_a^p - y_d^p)^2 + (z_a^p - z_d^p)^2} \quad (17)$$

where x, y and z are components of the end-effector point. The subscripts a and d refer to the actual and desired position respectively.

**NUMERICAL EXAMPLE**

A trajectory is assigned to a PUMA-type manipulator in such a way that the end-effector draw a circle with radius of 125 mm. The manipulator in its zero reference position is shown in Figure 1 and its nominal kinematic parameters are given in Table 1.

Table 1 Nominal kinematic parameters

Index (k)	$u_{k,0}$			$b_{k+1,0}$		
	$u_x$	$u_y$	$u_z$	$b_x$	$b_y$	$b_z$
1	0	0	1	0	254	0
2	0	1	0	431.8	0	0
3	0	1	0	0	0	-431.8
4	0	0	1	0	0	0
5	0	-1	0	0	0	0
6	0	0	-1	0	0	-127

$u_{a,0} = (0, 0, -1)$  and  $u_{t,0} = (0, -1, 0)$

The deviations in position and rotation is observed for each perturbed kinematic parameter. The perturbations include 0%, ±0.02%, ±0.05%, ±0.07% and ±0.1%. Ten

parameters (four links and six joint axes) are perturbed. For the sake of simplicity, they are abbreviated as LE2, LE3, LE4 and LE7 for link errors, also JE1, JE2, JE3, JE4, JE5 and JE6 for joint axes errors. Each time, only one parameter is perturbed and the others have zero perturbations. Link dimensions and joint axes errors and their end-effector maximum positional deviations are given in Table 2. Links 5 and 6 are not included since they have zero dimensions. The joint axes misalignments and the resulted maximum rotational deviations are given in Table 3.

Table 2 Link dimensions errors and joint axes misalignments and maximum positional deviations

$\xi\%$	LE2	LE3	LE4	LE7	JE1	JE2	JE3	JE4	JE5	JE6
0	1.07E-8									
0.02	5.08E-2	8.89E-2	8.64E-2	2.54E-2	1.58E-1	2.66E-1	1.79E-1	7.05E-2	5.99E-2	7.18E-2
0.05	1.27E-1	2.22E-1	2.16E-1	6.35E-2	3.94E-1	6.65E-1	4.47E-1	1.76E-1	1.50E-1	1.80E-1
0.07	1.78E-1	3.11E-1	3.02E-1	8.89E-2	5.52E-1	9.31E-1	6.26E-1	2.47E-1	2.09E-1	2.51E-1
0.1	2.54E-1	4.44E-1	4.32E-1	1.27E-1	7.88E+0	1.33E-1	8.94E-1	3.52E-1	2.99E-1	3.59E-1

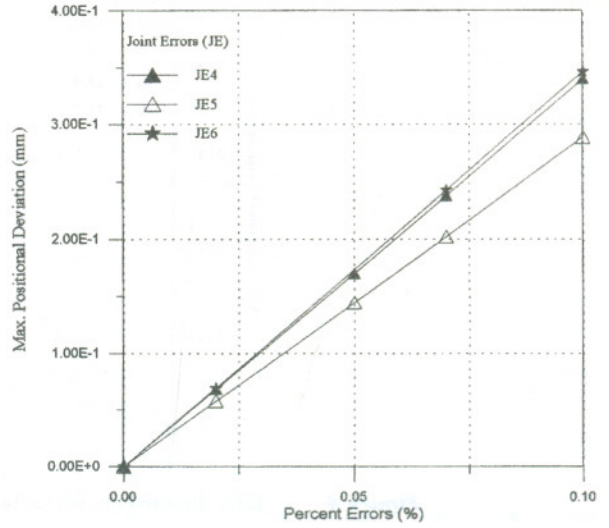
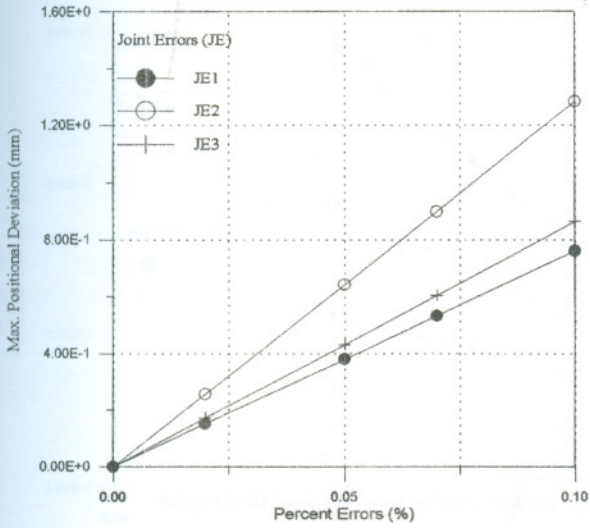
Table 3 joint axes misalignments and maximum rotational deviations

$\xi\%$	JE1	JE2	JE3	JE4	JE5	JE6
0	5.96E-8					
0.02	7.98E-4	7.65E-4	7.68E-4	8.00E-4	6.88E-4	8.00E-4
0.05	1.99E-3	1.91E-3	1.92E-3	2.00E-3	1.72E-3	2.00E-3
0.07	2.79E-3	2.68E-3	2.69E-3	2.80E-3	2.41E-3	2.80E-3
0.1	3.99E-3	3.82E-3	3.84E-3	4.00E-3	3.44E-3	4.00E-3

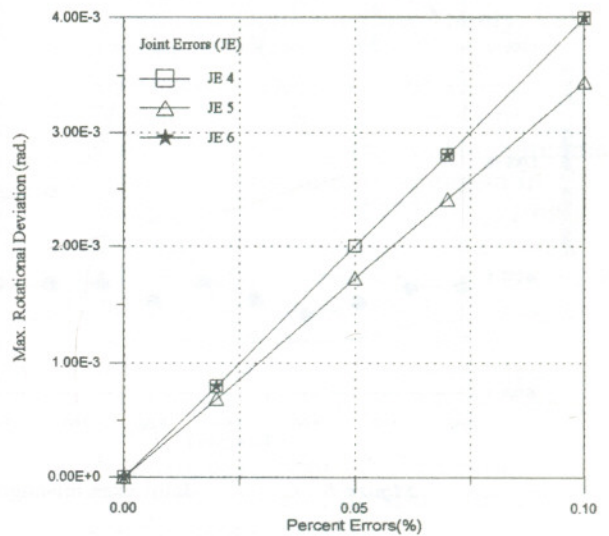
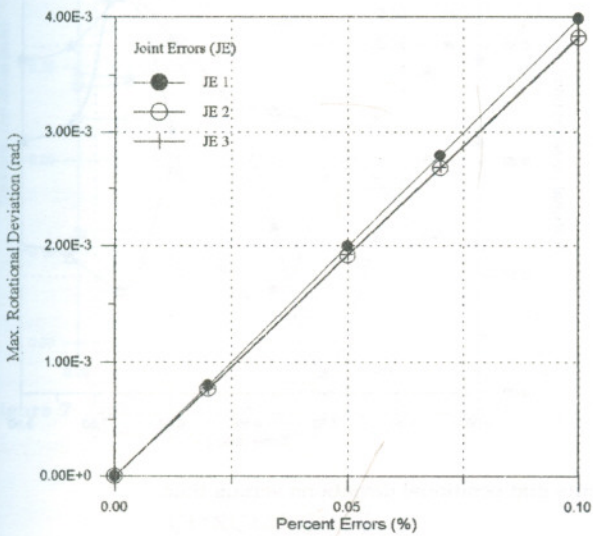
The results showed that the link dimensions errors have no effect on the maximum rotational deviations (not included). Besides, The positional deviations of the end-effector due to the joint axes misalignments are higher than those due to the link dimensions errors. Figure 2 represents the effect of joint axes misalignments on the end-effector maximum positional deviations. On the other hand, the effect of those errors to the end-effector maximum rotational deviations are shown in Figure 3 and 4 represent the effect of link

dimensions errors on the end-effector maximum positional deviations. From the shown figures, it is obvious that the adopted kinematics model has linear proportionality. Figure 5 shows the end-effector maximum positional deviations due to joint axes misalignments throughout the trajectory execution path. Whereas, Figure 6 shows the same results for the maximum rotational deviations. Moreover, Figure 7 describes the same results for the end-effector maximum positional deviations for link dimensions errors.

# The Effect of Manufacturing Errors on Robot Accuracy



**Figure 2** Joint axes misalignments versus maximum Positional deviations



**Figure 3** Joint axes misalignments versus maximum rotational deviations

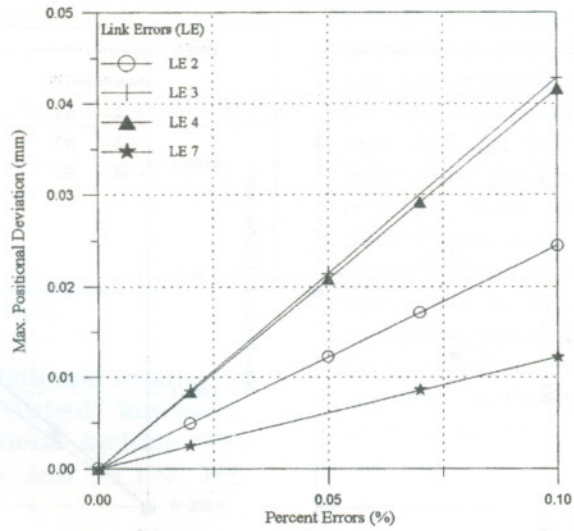


Figure 4 Link dimensions perturbations versus maximum positional deviations

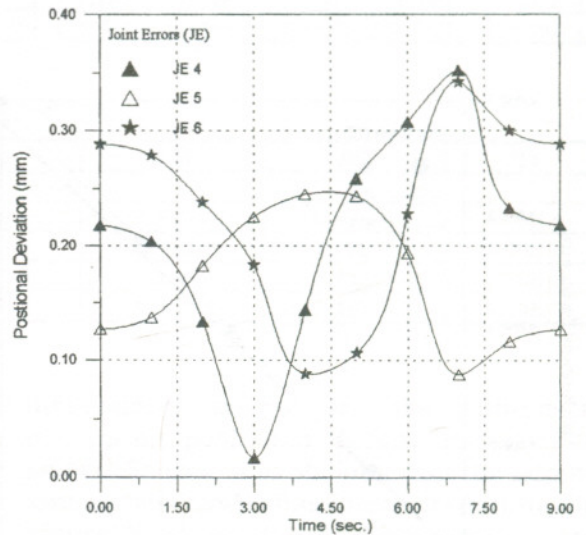
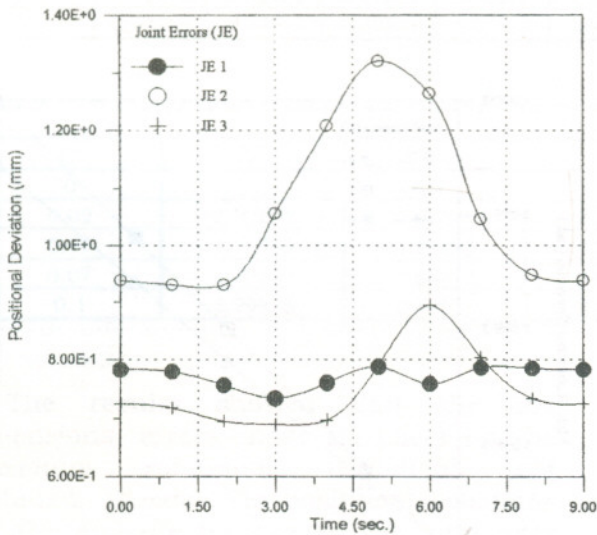
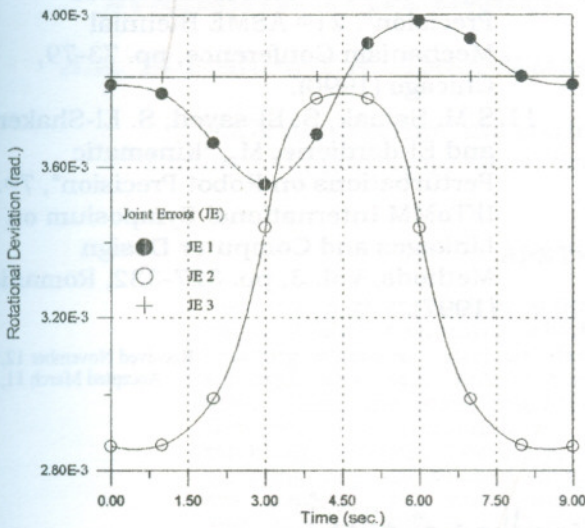
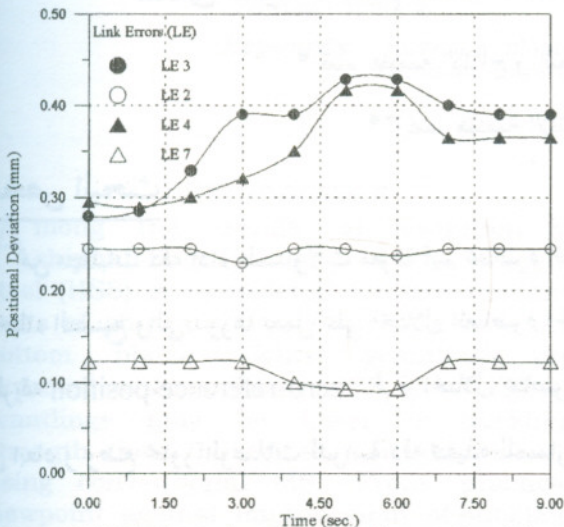
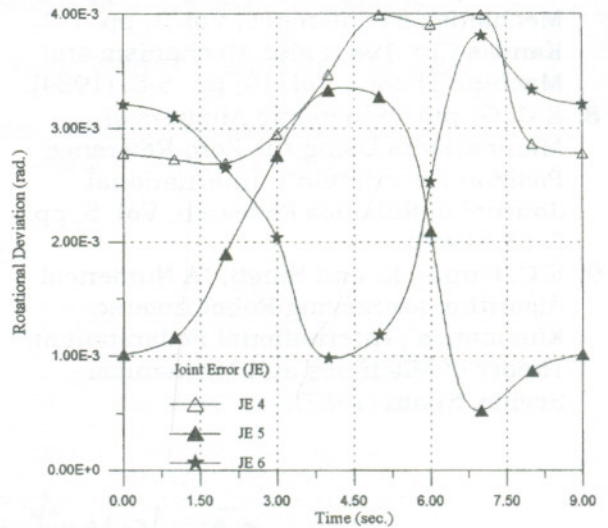


Figure 5 Joint axes misalignments and positional deviations versus time

## The Effect of Manufacturing Errors on Robot Accuracy



**Figure 6** Joint axes misalignments and rotational deviations versus time



**Figure 7** Link dimensions perturbations and positional deviations versus time

### CONCLUSIONS

Both the inverse and direct kinematics problems are developed based on the zero-reference-position method. All formulations are devoted to general manipulators. They are not limited by special manipulator configurations or dimensions. The positional deviations of the end-effector due to the joint axes misalignments are higher than those due to the link dimensions errors.

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## أخطاء تصنيع الروبوت و تأثيرها على دقة

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### ملخص البحث

من متطلبات دقة اداء المناولات معرفة قيم عناصره الطبيعية. لكن هذه المتطلبات يصعب تطبيقها فى الحياة العملية نتيجة لاختلاف التصنيع والتي بدورها تعمل على اختلال العناصر الحركية للمناول. تم تكوين المعادلات الحركية للمناول باستخدام طريقه zero-reference-position. ثم تم اختلال عناصره الحركية لدراسة دقة النهاية الطرفية حيث تم اختلال مختلف عناصره من ابعاد و وضع محاور الوصلات لدراسة دقة تنفيذه للمسار و قد أخذ المناول PUMA كمثال.