SIMULATION OF TENSILE FAILURE STRENGTH OF REINFORCED VERTICAL CUTS

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ABSTRACT

A computer code is developed to predict the failure strength of reinforced soils supporting vertical cuts under the influence of external surcharge loads. The code uses a micro-mechanical model combining the standard shear-lag equation with the chain of bundles probability model to determine the stochastical strength of the reinforced cuts. The method of analysis is Monte Carlo simulation that possesses the advantage of performing the entire simulation procedure in an automative, fast and inexpensive way. We have simulated three different soils, namely ; clay, sand, sand-gravel, reinforced with two fiber metal materials. Each soil is presented by the extreme possible conditions; the soft and the stiff for cohesive soils, and the loose and the dense states for cohesionless soils. Aluminum and stainless steel fibers are selected as reinforcing elements. In clay soils, results showed that the softer is the soil state, the higher is the effect of reinforcement on the system capacity. This conclusion does not extend to cohesionless soils, where the denser is the soil, the higher is the improvement in system capacity. Fibers with higher elastic modulus have shown more contribution to system capacity, in comparison to fibers with higher tensile strength.

For design purposes, Cumulative Distribution Functions (CDF) for each simulated condition are formed, and used as reference for comparisons. The code has potentials to extend the analysis to soils exhibiting nonlinear response, and also to fibers with anisotropic mechanical and thermal properties.

Keywords : Simulation, Reinforced Soils, Linear Elastic, Vertical Cuts, Probability.

NOMENCLATURE

CDF	Cumulative Distribution Function
	$Fs(\sigma^*)$ = Probability that the failure strength s has a value equal to or less than σ^*
k	Coefficient of lateral earth pressure
PDF	Probability Density Function
	$f\sigma(\sigma^*)$ = Probability that the failure strength σ is in the interval σ^* and $\sigma^* + d\sigma$
V_f	Volume ratio (volume of fibers per unit volume of soil mass)

INTRODUCTION

Soil reinforcement is an effective, attractive, and proved reliable technique for improving strength and stability of soils. In conventional methods of reinforced-soil construction, the inclusions of fibers, strips, fabrics, bars, grids etc. are normally oriented in a preferred direction and are introduced sequentially in alternating layers[1]. In contrast, randomly distributed discrete fiber-reinforced soil, called ply-soil [2] is similar to admixture stabilization in its preparation. The discrete fibers are simply added and mixed randomly with soil, much the same way as cement, lime, or other additives. An example of using shredded waste tires is given by Foose et al.[3]. As a means to enhance the shear strength of granular soils. Foose experimental results showed an increase in the apparent friction angle as large as 67°. Shred content and sand matrix unit weight were the most significant parameters influencing the shear strength. One of the main advantages of using randomly distributed fibers is the maintenance of strength isotropy and the absence of potential planes of weakness that may be developed parallel to the oriented reinforcement [4]. Ranjan et al. [5] conducted a series of triaxial compression test on cohesionless soils reinforced with discrete, randomly distributed fibers, both synthetic and natural. The study aimed at investigating the influence of fiber and soil characteristics, along with the applied confining stress on shear strength of cohesionless reinforced soils, having a uniformity coefficient ($D_{60\%}/D_{10\%}$) ranges between 2.28 and 2.38. Test results indicate the existence of curvilinear failure envelopes, with a transition taking place at certain confining stress termed as " critical confining stress ", below which the fiber tends to slip. The value of the critical confining stress is highly influenced by the fiber aspect ratio. Fiber inclusion is proven to increase significantly the soil shear strength, which is greatly inspired by fiber weight fraction, aspect ratio, and soil grain size distribution.

The analysis of vertically faced structures could be classified into internal (microscopic level), and external (macroscopic level), [6]. In the first level, studies such as; stresses within the structure, arrangement and back-fill properties are considered. In design terms, internal analysis is associated with adhesion and tension failure mechanisms. The external analysis covers the basic stability of the earth reinforced structure as a unit. This includes sliding, tilt/bearing failure, and slip within the surrounding sub-soil, or slips passing through the reinforced earth structure. In addition, stresses imposed upon the reinforced earth structure due to particular external conditions such as creep of the sub-soil have to be considered.

In the current study, we have focused on the microscopic analysis of the vertically faced structure, under the influence of external surcharge loads which exist at the top of the structure. In particular, we have investigated the reinforced system capacity subject to the condition that, tension failure controls. Our analysis explores the statistical nature of the capacity of a given vertically faced structure, originated from the randomness born in the failure strength of the supporting fibers. The term "capacity " means the maximum possible external load that causes structure's failure.

STATISTICAL FAILURE MODELS

It is possible to categorize the failure models developed on statistical bases in relation to fibers/sheets reinforced soil system into two categories; the weakest link model, and the fracture models. The weakest link model, also known as the series model is based on the assumption that the whole fails if the weakest link fails. The first application to study the material strength by this model is due to Weibull [7]. The series model does apply to reinforced soils, since the unbroken fibers continue to carry loads after the weakest fibers break. In the fracture models, two major models are presented in the literature; the cumulative fracture model, and the fracture propagation model. In the first modeling, the continuum (soil) is assumed not to contribute directly to the system strength, but it provides means to transfer the load in shear to the fibers. The system is divided into layers (bundles) of a length known as ineffective length. When the system is loaded, the fibers are assumed to be stressed uniformly, and as the load increases, the fibers in each bundle start to break randomly, whilst stresses are redistributed uniformly among the unbroken fibers in each bundle. When a sufficient number of fibers fail in a bundle, the system fails. Since, the stress redistribution is assumed to be uniform along all the unbroken fibers, no stress concentration factors were employed. It is clear that, the cumulative fracture model or the parallel model does not account for the stresses developed between fibers. This disadvantage is overcome by the fracture propagation model. The fracture propagation (series-parallel) model is an alternative to the weakest link model, the cumulative fracture model. In this model the fibers-soil system is modeled as a chain of n links in series, and each link is a bundle of m fibers in parallel. The use of a series-parallel model is an attempt to account for the interaction between the fibers when the system is loaded to failure.

The Model

The model is based on two basic assumptions; the fiber strength is described statistically by two parameters. Weibull distribution, and shear lag equation govern the fibers displacement field, whereas the soil does not contribute directly to the system, but it provides a means to transfer the loads in shear to the fibers.

The first assumption states that, if $F(\sigma^*)$ is the probability that the fiber strength is less than or equal to σ^*, then:

$$F(\sigma^*) = 1 - \exp[-\left(\frac{\sigma^*}{\beta}\right)^{\alpha} \Delta x] \qquad (1)$$

α and β are Weibull shape and scale parameters respectively. Δx is the fiber segment size. The second assumption considers the unidirectional reinforced soil as thin sheet consists of m number of fibers spaced uniformly parallel to x axis, see Figure 1.

If the reinforced soil is loaded in the x direction, the force equilibrium equation or the shear lag equation in a non-dimensional form is,

$$\frac{\partial u_i^2}{\partial \rho^2} + (u_{i+1} - 2u_i + u_{i-1}) = 0 \qquad (2)$$

where $\rho = \left(\frac{E_f A_f S}{G_m h_m}\right)^{1/2} X$

u_i is the displacement of the i^{th} fiber, $E_f A_f$ is the fiber tensile stiffness, $G_m h_m$ is the soil shear stiffness, and S is the spacing between fibers. S is assumed to be constant and uniform. G_m depends on the applied stress in the non-linear elastic soil constitutive response. For soil-reinforced system pulled out in simple tension, the boundary conditions are,

$$u_i(0) = 0, \qquad \frac{\partial u_i}{\partial \rho} = \frac{\sigma_c}{E_f} \qquad (3)$$

where σ_c (Pk) is the applied stress on the system, see Figure 1. Equation 2 guarantees the system equilibrium, whereas Equation 3 ensures the equilibrium of each fiber by itself. Equation 2 takes a slightly different form if it is applied to the first, or to the last fiber in the bundle.

Solution Scheme

The reinforced system is divided into m by n mesh points, where m is the number of fibers, and n is the total number of mesh points in the x direction (i.e the number of bundles).

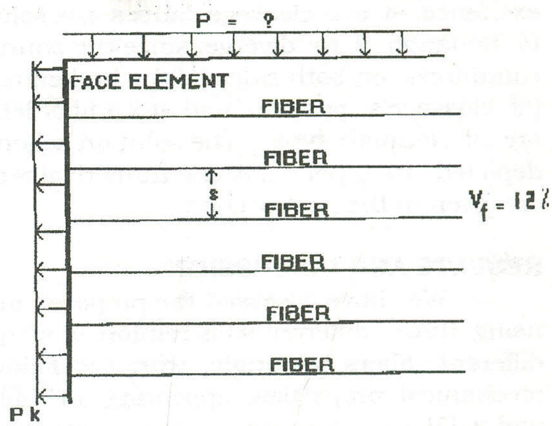

Figure1 Problem definition

The segment Δx (fiber length/n) is also known as the ineffective length and depends on the mechanical properties of the fiber and the soil, the size of the system, and its geometry. Fariborz [8,9] suggested a conservative value of 6 to 10 fiber diameter. For m by n mesh points, random number (σ^*) corresponds to each segment strength is generated using the distribution defined in Equation 1, Greco [10] and Song [11]. The load is applied in increments $\Delta\sigma$, and Equation 2 is solved with the boundary conditions in Equation 3, using the successive over-relaxation method.

Once the solution of Equation 2 is known at each load increment, the stress in each fiber segment ($\sigma = E_p\, du_{i,j}/d\rho$, i = 1, 2, ..., m, j = 1, 2, ..., n) is checked against its strength (σ^*). Two cases exist, either $\sigma^* \geq \sigma$ or $\sigma^* < \sigma$. The first case indicates that this segment is in the state of constant stress, and the displacement is a single valued function. On the other hand, the second case implies that the fiber has been broken at this segment, and a multi-valued displacement function exists in this segment. As long the first case prevails, Equation 2 holds. If the first case is violated, and the second case exists, Equation 2 has to be modified to allow the displacement to assume multi-values. Appendix A presents the solution to Equation 2 in a finite difference form for the two mentioned cases. For each increment of loading, the broken fiber and breaks locations are defined. The system fails, if one cleavage of breaks is formed. The existence of one cleavage causes the solution to Equation 2 to diverge, since the boundary conditions on both sides of the broken region (at cleavage's position, and at x = fiber length) are of Neuman type. The solution scheme is depicted in Appendix A and numerical results are given in the next section.

RESULTS AND DISCUSSION

We have assessed the proposed model using three different soils reinforced with two different fibers materials, with the following mechanical properties, appearing in Tables 1 and 2, [1].

Table 1 Soil mechanical properties (E = elastic modulus, ν = Possion's ratio, G = shear modulus, Gr=Gvavel)

SOIL TYPE	E(MPa)	ν	G(MPa)
Soft Clay	3.0	0.40	1.07
Stiff Clay	14.0	0.25	5.60
Loose Sand	15.0	0.20	6.25
Dense Sand	80.0	0.30	30.77
Loose Sand/Gr.	100.0	0.20	41.67
Dense Sand/Gr.	150.0	0.30	57.69

Table 2 Properties of the reinforced materials, t (Thickness),E (elastic modulus),G (Shear Modulus), β (Weibull's scale factor), α (Weibull's shape factor)

Reinforcing Material	t(mm)	E(MPa)	G(MPa)	β(Mpa)	α
Aluminum Alloy	6.0	150.0	72.0	120.0	4.0
Stainless Steel	6.0	330.0	132.0	220.0	4.0

Figure 1 shows the system arrangement of a vertical cut reinforced with a uniformly distributed fibers attached to a face element. We define the problem as follows; for a given reinforced cut (with a specified number of reinforced fibers, volume ratio, and fiber properties), find the capacity (the maximum surcharge load) of the cut,...which is the upper bound value of the surcharge load that causes the cut to fail!

Figures 2 to 4 illustrate the PDF for a cut strength reinforced with aluminum fibers. These fibers are embedded into three different soils, namely; Clay, Sand and Sand-Gravel mixture. It is interesting to note that the effect of reinforcement in all soils is the same. The weaker is the soil state, the higher is the effect of reinforcement.

In Figure 2, the reinforcement has improved the system capacity up to 45 MPa in soft clay, and up to 38.5 MPa in hard clay. However, the lower values for such improvement are 33.5 MPa, and 31.5 MPa for the soft and the hard clays, respectively.

This insinuated higher scatter in the results is expected in softer soil conditions. This conclusion does extend to cohesionless soils, where the looser is the soil, the higher is the improvement in system capacity, and scatter values. The following tables are formed based on the system average capacity values to clarify further this conclusion. The conclusion with respect to the scatter in the results, is slightly violated in gravel-sand mixture reinforced with aluminum fibers. Columns number 4, and 5 in Tables 3 and 4 show the average system capacity and standard deviation evaluated at the ground level G. L.

In view of Equation 2, the influence of soft/loose soil conditions on the redistribution of stresses between unbroken fiber segments, as a consequence of failure of some fiber segments is quite clear. The soil shear modulus G_m does strongly exist in the shear lag equation. It is obvious that, having a small soil shear modulus shall release less stresses from broken fiber segments to unbroken ones. As such, the increase in load level on unbroken fibers as a result of any failures takes place is proportional to the soil shear modulus. This shall lead to more failure breaks, and shall expedite the overall failure of the system

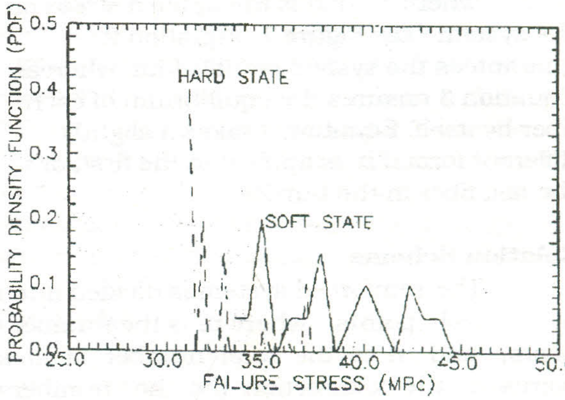

Figure 2 Probability density function for failure stress (aluminum fibers in clay soils)

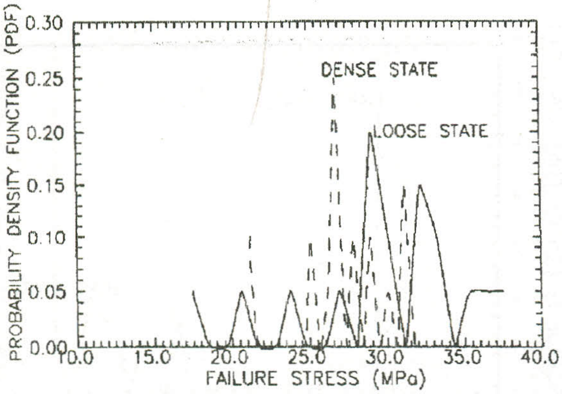

Figure 3 Probability density function for failure stress (aluminum fibers in sand soils)

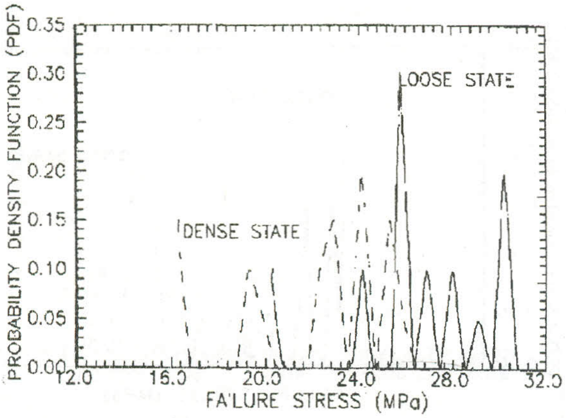

Figure 4 Probability density function for failure stress (aluminum fibers in sand/gravel soils)

Table 3 Average values and standard deviations for system capacity for soils reinforced with aluminum fibers, (A.C. = average capacity, S. D. = standard Deviation).

Soil Type	A.C. (MPa)	S.D.	A.C./k at G.L. (MPa)	S.D./k at G.L. (MPa)	E_f/G_m
Soft Clay	38.65	3.77	38.65	3.77	140.2
Hard Clay	32.90	2.30	32.90	2.30	26.80
Loose Sand	30.55	5.30	81.47	14.13	24.00
Dense Sand	27.70	3.07	127.65	14.15	4.88
Loose Sand-Gravel	26.70	2.98	71.20	7.95	3.60
Dense Sand-Gravel	22.20	3.37	102.30	15.53	2.60

Table 4 Average values and standard deviations for system capacity for soils Reinforced with stainless Steel fibers, (A.C. = average capacity, S. D. = standard deviation).

Soil Type	A.C. (MPa)	S.D.	A.C./k at G.L. (MPa)	S.D./k at G.L	E_f/G_m
Soft Clay	90.25	10.59	90.25	10.59	308.4
Hard Clay	43.35	2.24	43.35	2.24	58.93
Loose Sand	53.65	7.70	143.06	20.53	52.80
Dense Sand	38.00	4.89	175.12	22.53	10.72
Loose Sand-Gravel	52.30	6.16	139.47	16.40	7.92
Dense Sand-Gravel	50.55	5.88	232.95	27.10	5.72

For design purposes, Figures 5 to 7 are created. From the definition of the Cumulative Distribution Function (CDF), one could calculate the permissible system capacity that guarantees a certain degree of confidence. A note is in order. The failure stress presented by the x axis in the mentioned figures demonstrate the uniformly distributed forces pulling the system out at the facing element. These forces are related to the maximum surcharge load at the ground level through the appropriate coefficient of lateral earth pressure. As such, the comparison between the listed figures is just qualitative, rather than quantitative.

Obviously, dividing this axis by the appropriate coefficient of lateral earth pressure shall yield the required quantitative comparison, which we have done on average value bases as shown in Tables 3, and 4. It is assumed here that, k follows the simple formula given by Rankine.

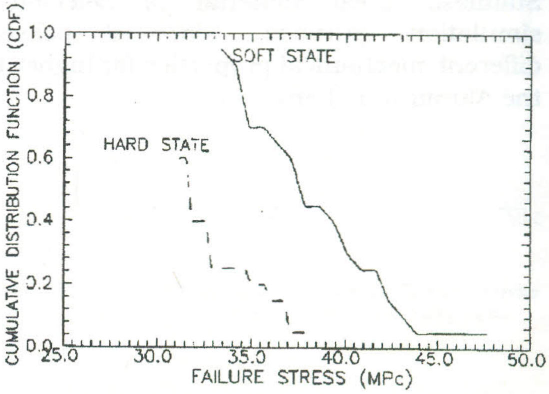

Figure 5 Cumulative distribution function for failure stress (aluminum fibers in clay soils)

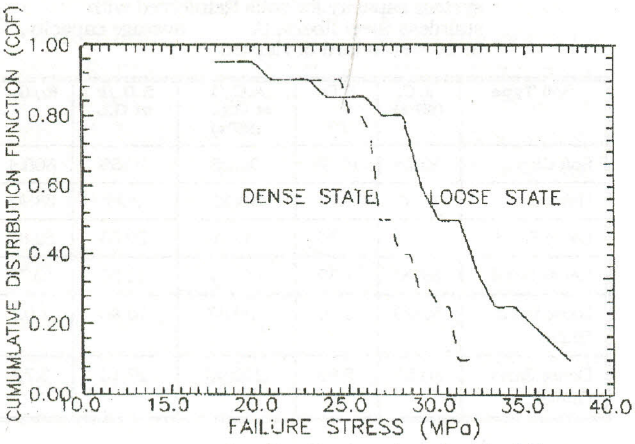

Figure 6 Cumulative distribution function for failure stress (aluminum fibers in sand soils)

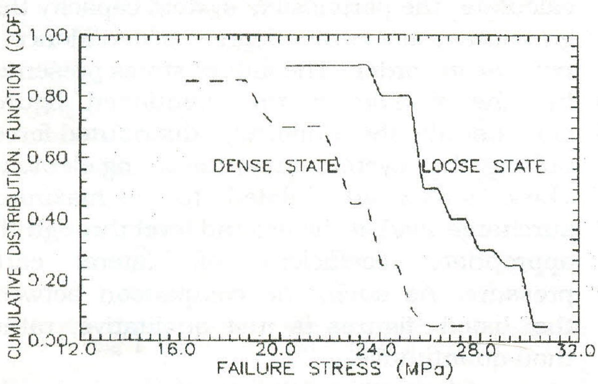

Figure 7 Cumulative distribution function for failure stress (aluminum fibers in sand/gravel soils)

To substantiate the effect of fiber's material on probability distributions of cut capacity, Figures 8 to 13 are constructed. Stainless steel material is selected for simulation purpose, since it possesses different mechanical properties far higher than the Aluminum fibers.

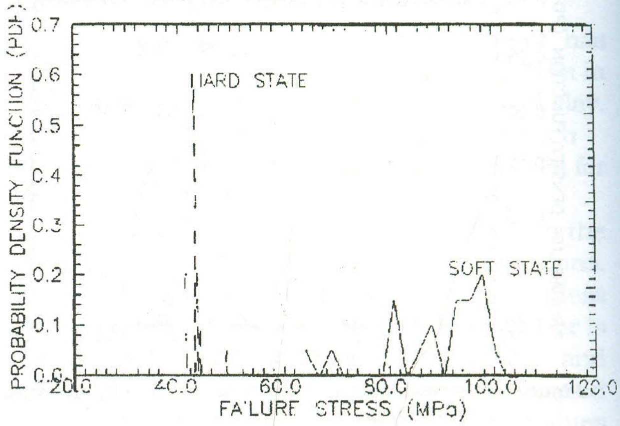

Figure 8 Probability density function for failure stress (Stainless Steel fibers in clay soils)

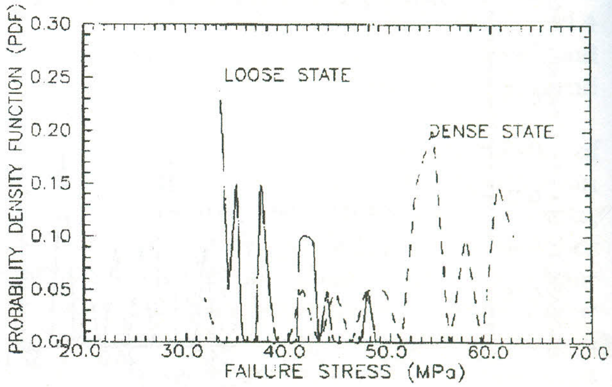

Figure 9 Probability density function for failure stress (Stainless steel fibers in sand soils)

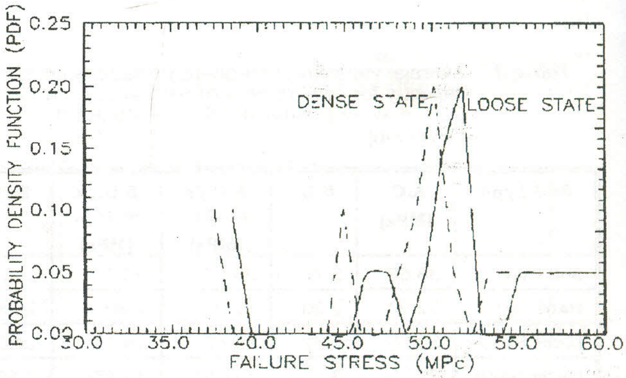

Figure 10 Probability density function for failure stress (Stainless steel fibers in sand/gravel soils)

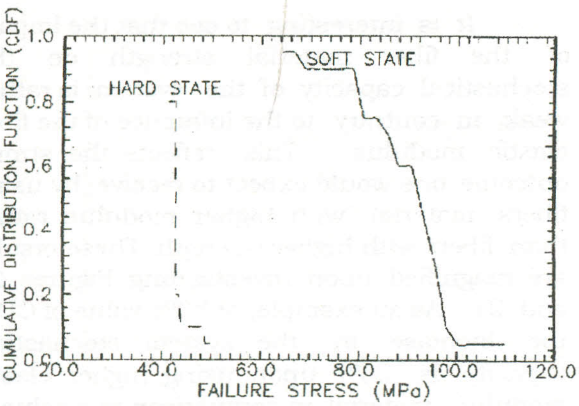

Figure 11 Cumulative distribution function for failure stress (stainless steel fibers in clay soils)

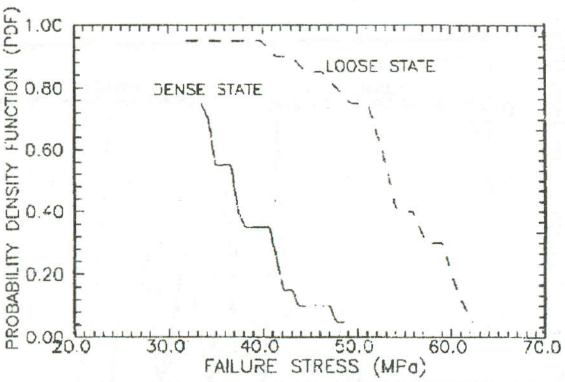

Figure 12 Cumulative distribution function for failure stress (Stainless steel Fibers in sand soils)

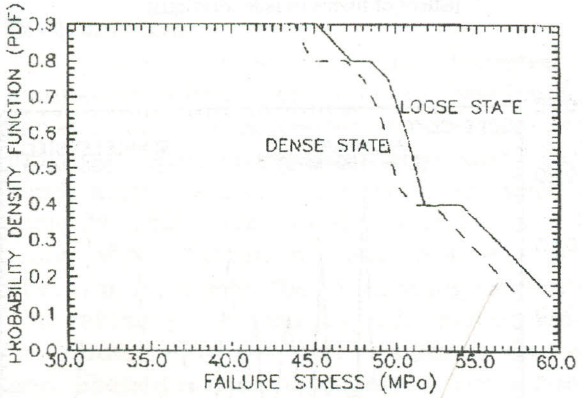

Figure 13 Cumulative distribution function for failure stress (Stainless steel fibers in sand/gravel soils)

With the aid of Tables 3 and 4, we could summarize the previous conclusion derived for soils reinforced with Aluminum fibers, and with stainless steel fibers as follows:

1. In clay soils, the result is quite clear; the softer the soil state, the higher is the improved system capacity, and the more is the scatter in the results.
2. In sand/Sand-Gravel soils, and in the spirit of the data shown in columns 4 and 5 in the tables, the denser the soil, the higher is the improvement in the system capacity, and the scatter in the results.

To further assess the effect of the reinforcement fibers mechanical properties, a comparison between the two materials reinforcing soft and hard clays is depicted in Figures 14 to 17. In view of the data included in Table 2, the influence of the fiber high strength, and high elastic modulus is quite acknowledged.

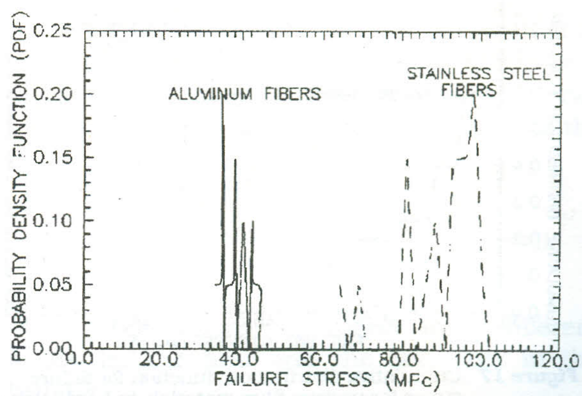

Figure 14 Probability density function for failure stress for various fiber materials in soft clay soils

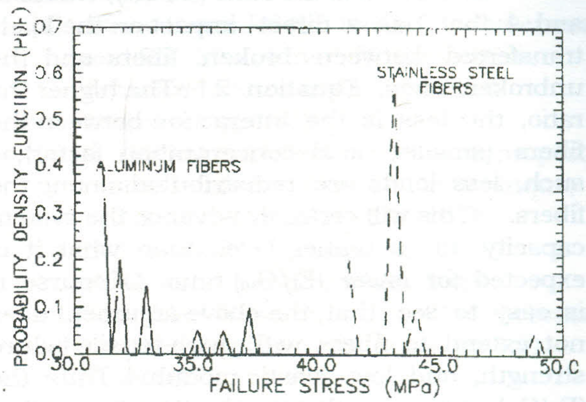

Figure 15 Probability density function for failure stress for various fiber materials in hard clay soils

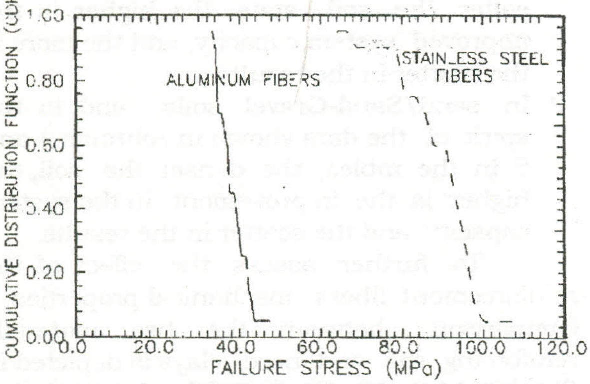

Figure 16 Cumulative distribution function for failure stress for various fiber materials in soft clay soils

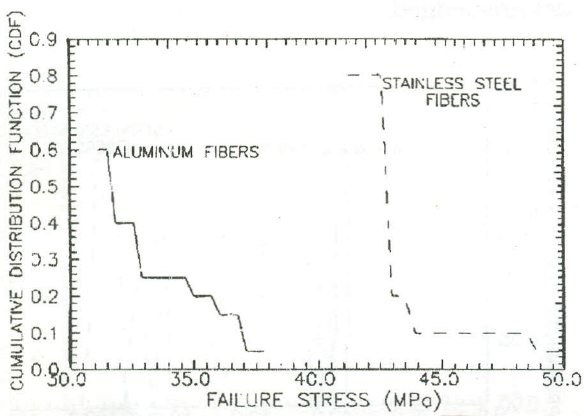

Figure 17 Cumulative distribution function for failure stress for various fiber materials in hard clay soils

In fact, it is the ratio (E_f/G_m), Tables 3, and 4 that has a direct impact on the loads transferred between broken fibers and the unbroken ones, Equation 2. The higher the ratio, the less is the interaction between the fibers (smaller load concentration factor),as such, less loads are redistributed among the fibers. This will certainly advance the system capacity to a higher level, than what it is expected for lower (E_f/G_m) ratio. Of course, it is easy to see that, the above argument does not extend to fibers with high tensile failure strength, and low elastic modulus. Thus, the (E_f/G_m) ratio is the dominating factor that controls the overall system capacity.

Figures 18 and 19 demonstrate the Probability Density Function PDF for the effect of fiber strength and fiber elastic modulus, respectively.

It is interesting to see that the impact of the fiber material strength on the stochastical capacity of the system, is rather weak, in contrary to the influence of the fiber elastic modulus. This reflects the strong outcome one would expect to receive, by using fibers material with higher modulus rather than fibers with higher strength. These results are magnified upon investigating Figures 20 and 21. As an example, at 80% value of CDF, the increase in the system stochastical capacity is 26% upon using higher elastic modulus material, in comparison to a value of 6.5%, when working with material of higher strength.

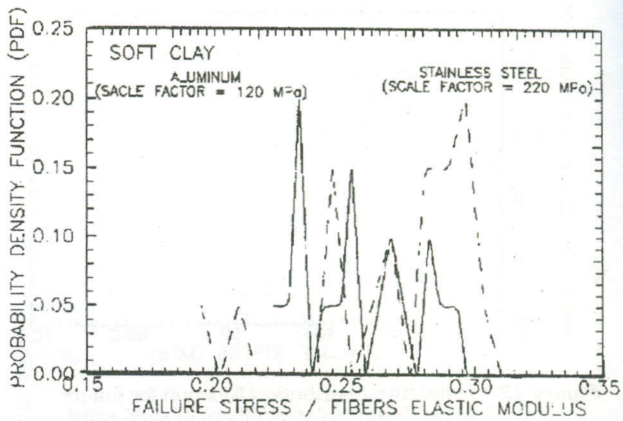

Figure 18 Probability density function for failure stress (Effect of fibers tensile strength)

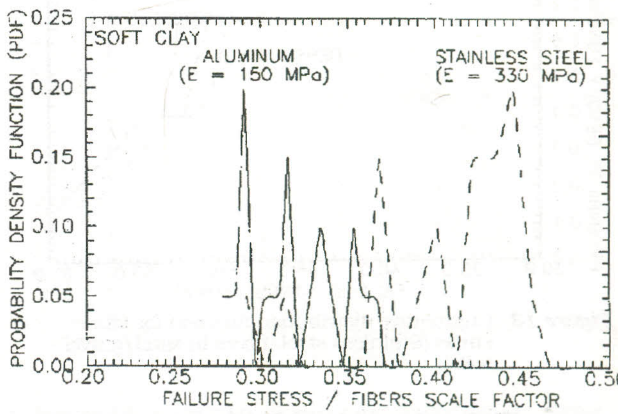

Figure 19 Probability density function for failure stress (Effect of fibers elastic modulus)

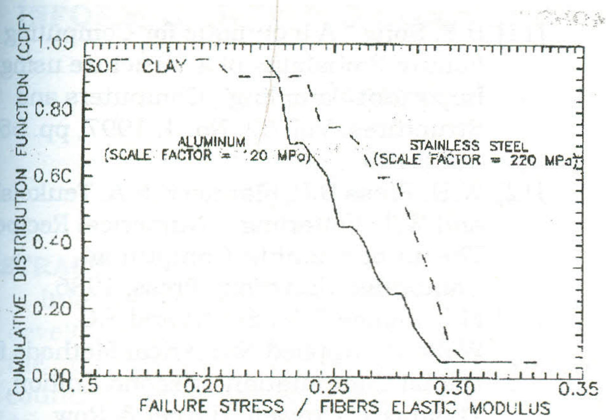

Figure 20 Cumulative distribution function for failure stress (Effect of fibers tensile strength)

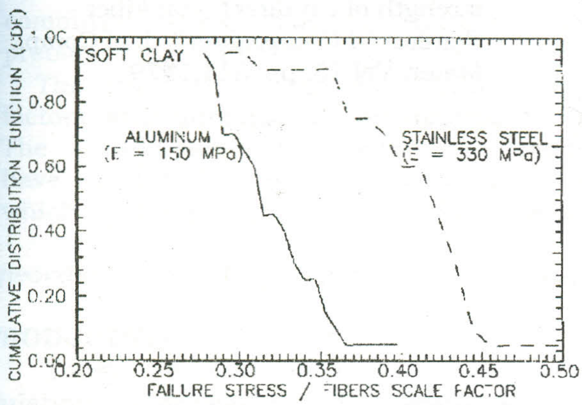

Figure 21 Cumulative distribution function for failure stress (effect of fibers elastic modulus)

CONCLUSION

The article has demonstrated a simulation scheme to predict the stochastical capacity of a reinforced vertical cut under the influence of excessive surcharge load. Clay, sand, sand-gravel mixture soils are used in their extreme states in the simulation code, along with Aluminum and Stainless steel fibers to represent the reinforcing elements. The reinforced system capacity has shown to be influenced by soil state conditions, and soil and fibers mechanical properties. The softer is the cohesive soil, the higher is the improvement in system capacity, and the scatter in the results. This conclusion is reversed in cohesionless soils, where the improvement in system capacity increases with the increase in soil density. Also, smaller scatter in the results are demonstrated to exist for denser soils. Fibers elastic modulus has shown higher influence in the system capacity in comparison to fibers tensile failure strength. In fact, the ratio between the fiber elastic modulus and the soil shear stress is the dominating parameter in over all improvement in system capacity.

APPENDIX A

Equation 1 in a finite difference form using the successive over relaxation method SOR, Press et al. [12] and James et al. [13], is

$$u(i,j)=\omega\Big[\frac{(u(i-1,j)+u(i+1,j))\,\Delta\rho+u(i,j-1)+u(i,j+1)}{2(1+\Delta\rho^2)}+(1-\omega)u(i,j) \qquad \text{(A-1)}$$

where w is the relaxation factor of a value between 1 , and 2.

The use of Christopherson method as demonstrated by Oh [14] allows Equation A-1 for a broken fiber to have the following two forms; if it is evaluated to the right, or to the left of the break, respectively,

$$u(i,j)=\omega\Big[\frac{3\Delta\rho^2(u(i-1,j)+u(i+1,j))+4u(i,j+1)}{2(3\Delta\rho^2+2)}+(1-\omega)u(i,j) \qquad \text{(A.2)}$$

The above equations take different forms for the first and the last fiber in the bundle.

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