# ON THE STABILITY OF COMPRESSIBLE BOUNDARY LAYERS WITH HIGH MACH NUMBERS: PART II- HEAT TRANSFER

***Ayman A. AL-Maaitah and Monther A. AL-Farah Nimri***

Department of Mechanical Engineering Mu'tah University P.O. Box 7
Mu' tah, Al-Karak - Jordan

**ABSTRACT**

Controlling the supersonic laminar boundary layer stability by wall cooling is investigated. As commonly known, cooling stabilizes first- mode waves whereas it destabiliizes second-mode waves. The present calculations demonstrate that cooling does not only increase the growth rates of the second-mode waves, but also creates an undesired disturbance structure. Effect of cooling on basic flow characteristics and its influence on the stability problem is further investigated. Various frequencies, wave angles, Mach numbers and Reynolds numbers are considered.

***Keywords:*** Supersonic boundary layer, Stability, Heat transfer, Free stream velocity, Laminar flow control.

## INTRODUCTION

The linear instability theory gained a big boost when it proved to give excellent accuracy in predicting the transition location. Smith [1] and Van Inggen [2] independently put forward a criterion for transition in two-dimensional boundary layers. According to this criterion, transition takes place when T-S waves in the boundary layer have amplified by a factor of $e^N$. By comparison with low-speed experimental transition data, Smith [1] found that the exponent N has a value of about nine for two-dimensional boundary layers with and without pressure gradient. Later, for three-dimensional boundary layers on a swept-swing leading edge, Sorkowski and Orzsag [3] as well as Hefner and Bushnell [4] found that the exponent N, hereafter referred to as N factor, is in the range of 7-11. Moreover, Malik et al. [5] found that the N factor correspoding to traansition over a rotating disk is around 11. Smith [6] suggested that N is about 10 for the Görtler vortices. More recently Nayfeh et al. [7] and Al-Maaitah et al. [8] found that the N factor corresponding to transition for flows over a rough element is around 9. All of the previous work suggests that when the amplification of the linear wave reaches a certain level a breakdown to turbulence takes place. For engineering and design purposes the $e^N$-method is the most widely used in predicting transition location. Experiments show that similar methods can be used for compressible boundary layers.

The most important difference between the stability of supersonic and subsonic boundary layers is demonstrated by extensive work of Mack [9] who showed by numerical computations that there exists an infinite sequence of modes of instability for supersonic boundary layers. The response of one mode to a laminar flow technique can be quite different from other modes. Moreover, more than one instability mechanism coexist in supersonic flows, the viscous and inviscid mechanisms. Furthermore, theoretical and experimental investigations of supersonic boundary-layer stability is a rather difficult task. Consequently, information about LFC of supersonic flows is considerably less than for incompressible flow.

Effect of wall cooling on the inviscid instability waves was first investigated by Mack [10]. Using the inviscid theory, he found that while cooling stabilizes first-mode waves it destabilizes the second mode waves. For moderate Mach numbers, enough cooling can be applied to remove the generalized inflection point, which is the source of the first-mode instability. The source of the second-mode instability, however, cannot be removed by cooling. Stetson et al. [11] experimentally verified these results. They performed stability experiments on 7-degree half cone angle at fiee stream Mach ngmber of eight. They found that the transition Reynolds number was reduced from $4.8\times10^{6}$ to $3.2\times10^{6}$ due to cooling. Theoretically, Gasperas [12] investigated two temperature distributions, one with a constant wall temperature across the surface, and the other with ramp decrease in wall temperature. His calculations showed that near the ramp, the second mode is stabilized by cooling. These results, however, are questionable due to his inaccurate calculations of the mean profile. In solving the non-similar conventional boundary layer equations, Gasperas [12] did not account for the upstream influence and the viscous-inviscid interaction that results from the abrupt change in wall temperature.

Like cooling, the effect of suction on supersonic flow stability was first investigated by Mack [10]. Using the inviscid stability theory, Mack showed that suction stabilizes both of the first and second-mode waves. More recently, Malik [13] conducted a primary study on the effect of suction on the stability of supersonic flows at a Mach number of 4.0 and Reynolds number of 1500. He used a suction distribution which results in a self-similar velocity profile. He concluded that suction significantly stabilizes the boundary-layer. However, the more detailed and comprehensive work of Al-Maaitah et al. [14] and Massad et al. [15] demonstrated that this was not true for high Mach numbers. The effect of suction stabilizing the flow diminishes significantly as the free steam Mach number increases above5.5.

In Part I of this work we presented an investigation of the of pressure gradient on the control of laminar flow transition. in the present paper, however, we present a detailed and comprehensive investigation of the effect of heat transfer on the stability of supersonic boundary layers.

## PROBLEM FORMULATION

similar to the analysis of Part I-pressure gradient, the mean flow fow field is still governed by the boundary layer equations. Nevertheless, Equation 7 of the boundary condition in Part I, which is $\{\delta T/\delta y\}=0$ at y = 0, is not valid for the case of Instead:

$$T = T_w \text{ at } y = 0 \qquad (1)$$

Moreover, for flows over a flat plate $T_e = M_e = U_e = 1$. Hence $\beta_o = 0$ and the self- similar solution for the following equations (10 and 11 of Part I) is valid, provided that $T_w$ is constant

$$(cf_{\eta\eta})_\eta + ff_{\eta\eta} + \beta_o[\rho_e/\rho]f_\eta)^2] = 2\xi(ff_{\eta\eta}f_\xi - ff_{\eta\eta}f_\xi) \qquad (2\text{-a})$$

$$(a_1Q_\eta + a_2f_\eta f_{\eta\eta})_\eta + fQ_\eta = 2\xi(f_\eta Q_\xi\ Q_\eta f_\xi) \qquad (2\text{-b})$$

The boundary conditions are now

$$f = f_\eta = 0, \text{and } Q = Q_w \text{ at } y = 0 \qquad (2\text{-c})$$

$$Q \to 1 \text{ and } f_\eta \to 1 \text{ as } \eta \to \infty \qquad (3)$$

The stability analysis is also the same as described in Part I -effect of free-stream velocity. Hence for certain mean flow profile and certain R, $\alpha$ can be found for specific F and $\beta$.

## RESULTS AND DISCUSSION

After calculating the mean velocity profile for the flow conditions described in the previous section, the stability characteristics of the flow is then analyzed as will be described in this section. Many physical aspects of the stability problem, however, can be understood by investigating the effect of flow control techniques on some mean flow characteristics.

One of the mean flow features that is important to the compressible stability problem is the Generalized Inflection Point (GIP). It is defined as the point where

$$\delta/\delta y\,\{\rho(\delta u/\delta y)\} = 0 \qquad (4)$$

The existence of the GIP is responsible for the inviscid instability mechanism. For flow over adiabatic flat plate with no suction there is one generalized inflection point inside the boundary layer. As the wall temperature decreases, another GIP is created in the boundary layer as shown by

Figure 1. By further cooling the wall, the two points become closer to each other until they merge and disappear. For Mach numbers up to 7.5, the minimum wall temperatures needed to remove the GIP are plotted in Figure 2. For Mach numbers of 7.5 and higher the generalized inflection point cannot be removed by cooling.

Although the wall temperature needed to get rid of the generalized inflection point at $M_\infty$ = 2. 1 is 0 .866 $T_r$, Figure 3-a shows the first-mode waves at F= 13.55x$10^{-6}$ are stable when $T_w \leq 0.851T_r$. At this Mach number, however, viscosity is known to destabilize first-mode waves [4], which may explain why they are still unstable even when the source of the inviscid instability is removed. On the other hand, viscosity is known to be a stabilizing mechanism for $M_\infty$>3.6 [21]. Figure 3-b shows the variation of the growth rates with wall temperature at M=4.5 As expected, first-mode waves are stable when $T_w/T_r$ is higher than that needed to remove the generalized inflection point as seen in Figure 2.

Next we investigate the effect of cooling on the most amplified first-mode waves as the wall is cooled. Figure 4 shows that growth rates are reduced, the frequency corresponding to the most amplified wave shifts slightly to the left and the range of frequencies receiving amplification is decreased by cooling .

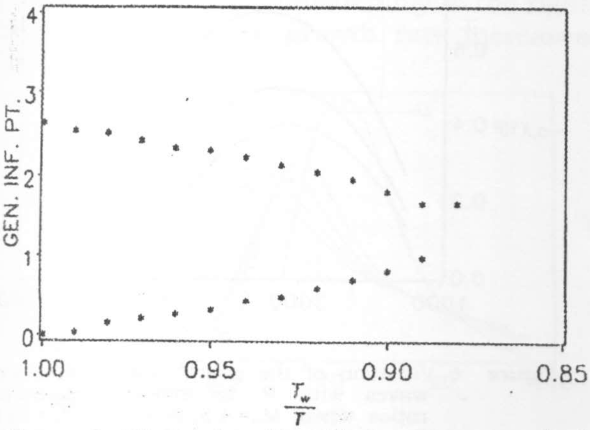


**Figure 1** Effect of wall cooling on the generalized inflection points: $M_\infty$=2.0, $T_\infty$ = 266 K and Pr = 0.72.

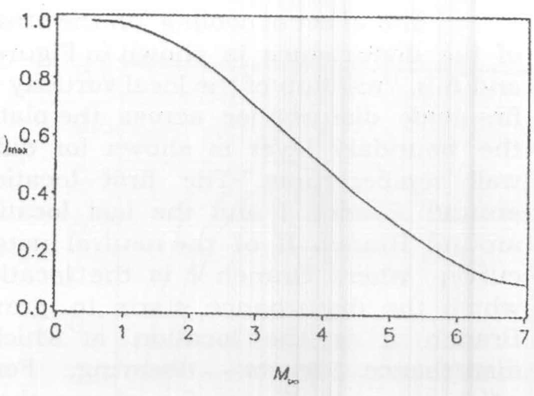


**Figure 2** The maximum temperature ratio needed to remove the generalized inflection point: Wind-tunnel temperature and Pr=0.72

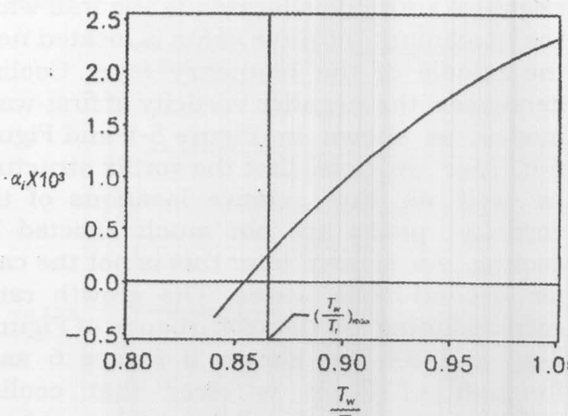


**Figure 3-a** Variation of the growth rate of first-mode waves with wall temperature when $M_\infty$=2.1, $T_\infty^*$=165 K, Pr = 0.72, F =13.55x$10^{-6}$, R=1500 and ψ=60°.

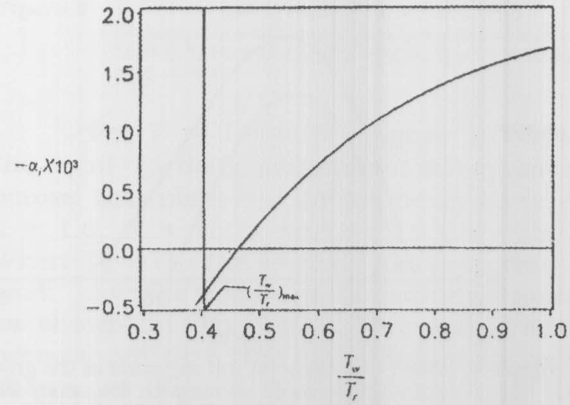


**Figure 3-b** Variation of the growth rate of first-mode waves with wall temperature when $M_\infty$ = 4.5, $T^*_\infty$ = 62 K, Pr = 0.72, F = 13.55x$10^{-6}$ R=1500 and ψ=60°.

The effect of cooling on the structure of the disturbance is shown in Figures 5-a and 5-b. Variation of the local vorticity of the firs-mode disturbance across the plate and the boundary layer is shown for different wall temperatures. The first location is around Branch I and the last location is around Branch II of the neutral instability curve; where Branch I is the location at which the disturbance starts to grow and Branch II is the location at which the disturbance starts decaying. For an adiabatic wall, Figure 5-a shows that the vorticity has two peaks at x= 1.0; one positive and one negative. At x = 6.0, however, the vorticity has two positive peaks and one negative peak. The maximum negative vorticity is located at the wall while the maximum positive value is located near the middle of the boundary layer. Cooling decreases the negative vorticity of first-wave modes, as shown in Figure 5-b and Figure 5-c. Also we note that the vortex structure as well as the relative locations of the vorticity peaks are not much affected by cooling. As shown later this is not the case for second-mode waves. The growth rates corresponding to the disturbance of Figures 5-a and 5-b are shown if Figure 6 as a function of R. It is clear that cooling stabilizes the flow for all Reynolds numbers. Furthermore, it more effective at larger values of R.

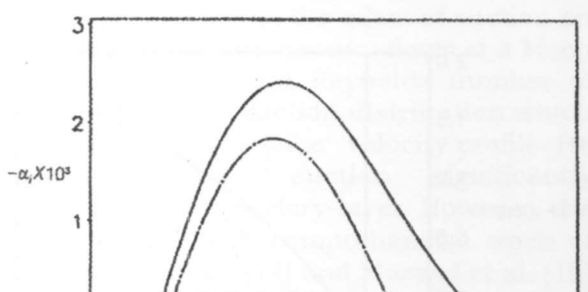


***Figure 4*** Effect of cooling on the variation of the growth rate of first-mode waves with frequency when $M_\infty = 2.1$, $T^*_\infty = 165$ K, Pr = 0.72, R = 1500, and $\psi = 60°$: ____ Tw = Tr, and ___ _ ___ Tw = 0.9Tr.

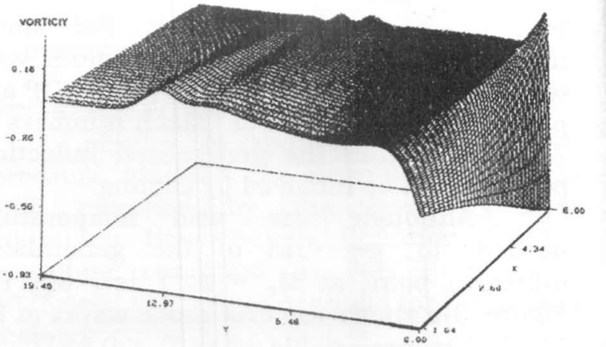


***Figure 5-a*** The local vorticity of the first-mode disturbance across the plate and the boundary-layer: $M_\infty = 4.5$, $T^*_\infty = 62$ K, Tw = Tr, F = $20.89 \times 10^{-6}$ and $\psi = 0°$

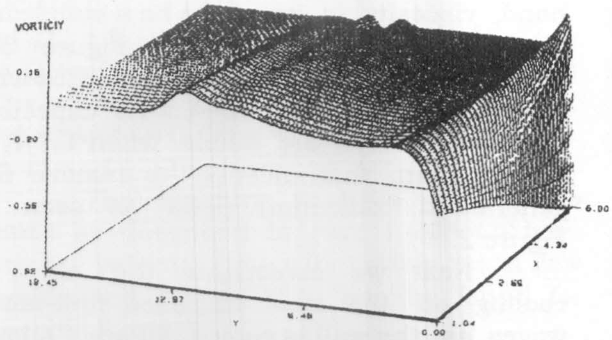


***Figure 5-b*** The local vorticity of the first-mode disturbance across the plate and the boundary-layer: $M_\infty = 4.5$, $T^*_\infty = 62°$, $T_w = 0.9T_r$, F = $20.86 \times 10^{-6}$ and $\psi = 0°$.

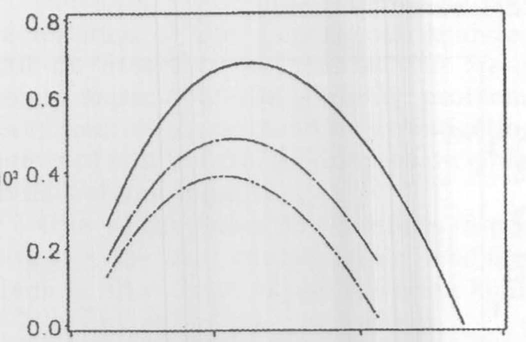


***Figure 6*** Variation of the growth rate of first-mode waves with R for different temperature ratios when $M_\infty = 4.5$, Pr = 0.72, $T^*_\infty = 62$ K, F = $20.89 \times 10^{-6}$ and $\psi = 0°$: ____ $T_w = T_r$, _ _ _ $T_w = 0.9T_r$ and ---- - --- $T_w = 0.85T_r$.

Although the instability of first-mode waves is dependent on the location of the generalized inflection points, it is not the case for second-modes. Hence, wall cooling does not stabilize second-mode waves. In fact, it destabilizes it. This is known to be for viscous and inviscid second-mode waves. Although cooling stabilizes second-mode waves for certain frequencies as shown in Figure 7, the maximum growth rate increases with cooling Moreover, the frequency corresponding to the growing waves shifts considerably to right as the wall temperature decreases. In the case of heating, the situation is reversed. The maximum growth rate decreases and the frequency band corresponding to the growing waves shifts to the left as the wall is heated. For $T_w = 0.5T_r$ the maximum growth rate is nearly 1.5 that of adiabatic case Figure 7.

The effect of cooling on the maximum growth rates of the second-mode waves is shown in Figure 8 for various $M_\infty$. Although cooling is most effective around $M_\infty = 5.0$, it's effect does not diminish for high Mach numbers, as in the case of suction. Moreover, the critical Mach number at which the second mode starts to appear, decreases as $T_w / T_r$ decreases.

As the waves propagate downstream, Figure 9 shows the variation of the growth rates of the second-mode with R when $T_w = 0.5T_r$, Branches I and II of the neutral stability curve shift considerably to the right and the maximum growth rate increases greatly.

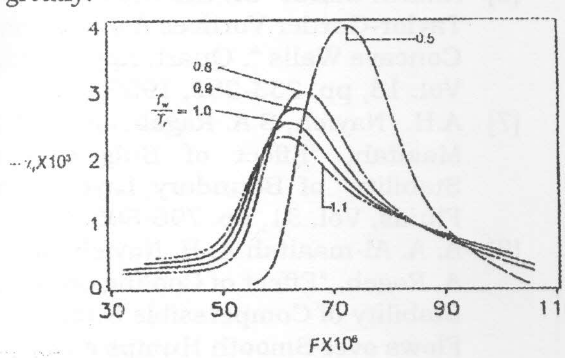


**Figure7** Variation of the growth rate of second-mode waves with F for different temperature ratios when $M_\infty$ = 7, Pr = 0.72, $T^*_\infty$ = 50 K, R = 1500 and $\psi$=0°.

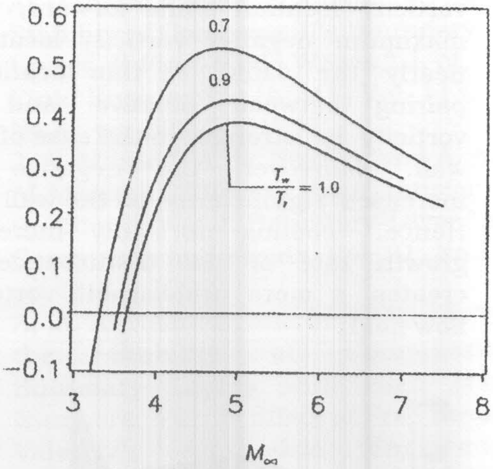


**Figure 8** Variation of the growth rate of second-mode waves with $M_\infty$ for different temperature ratios: Pr=0.72, wind tunnel temperature, R = 1500 and $\psi$ = 0°.

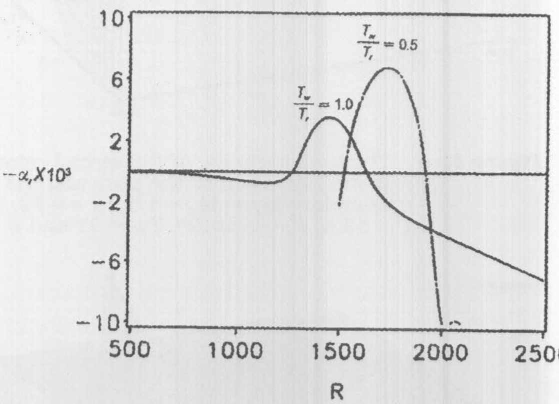


**Figure 9** Effect of cooling on the variation of the growth rates of second-mode waves with R: $M_\infty$= 4.5, Pr = 0.72, $T^*_\infty$ = 62 K, F = 140x10$^{-6}$, and $\psi$ = 0°.

For F = 140x10$^{-6}$, Figure 10 shows the local vorticity variation of disturbances across the plate and the boundary layer. At x = 1.0, R = 1500 and x = 2.93, R = 2500. When $T_w = T_r$ the vorticity has one positive peak near the middle of the boundary layer, as shown in Figure 10-a. This peak is more pronounced than that of the first-mode wave shown in Figure 5-a. Near the wall the vorticity is negative.

The x-location of the minimum positive vorticity corresponds to that of the maximum negative vorticity and vice versa.

When $T_w = 0.5T$, the minimum positive vorticity location shifts forward, while the maximum negative vorticity location stays nearly the same. At this location vortex pairing between positive and negative vorticity is stronger in the case of adiabatic wall. Moreover, the negative vorticity increases significantly as the wall is cooled. Hence, cooling not only increases the growth rate of the disturbance, it also creates a more undesirable vortex in the flow field.

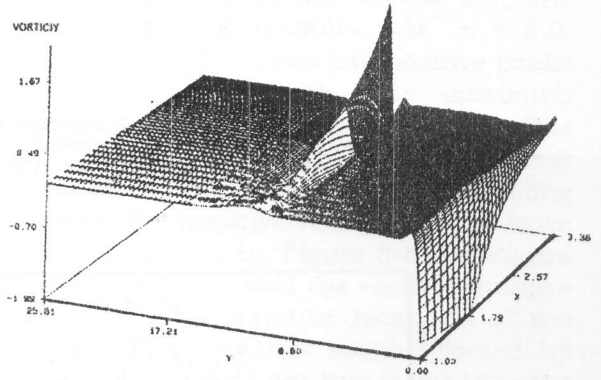

***Figure 10-a*** The local vorticity of the second- mode disturbance across the plate and the boundary layer: $M_\infty = 4.5$, $Pr = 0.72$, $T^*_\infty = 62$ K, $F = 145\times10^{-6}$, Tw = Tr and $\psi = 0°$

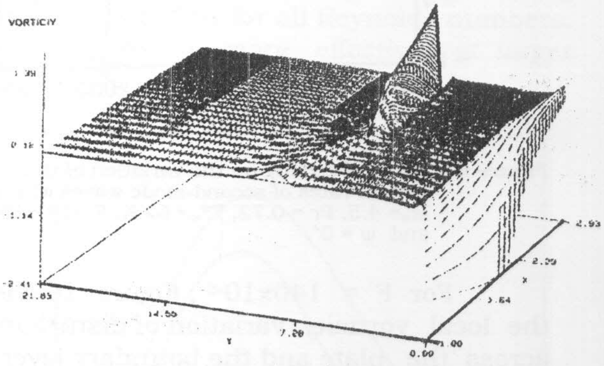

***Figure 10-b*** The local vorticity of the second-mode disturbance across the plate and the boundary layer: $M_\infty = 4.5$, $Pr = 0.72$, $T^*_\infty = 62$ K, $F = 145\times10^{-6}$, $T_w = T_r$ and $\psi = 0°$

## CONCLUSIONS

After performing the analysis with variable frequency and various flow parameters the following conclusions have been deduced:

- Cooling increases the growth rate of the second mode of instability.
- Inspite of cooling, the second mode still has the highest growth rate of instability.
- The most dangerous frequency slightly changes due to cooling.

## ACKNOWLEDGMENT

The authors would like to thank Prof. A. H. Nayfeh for his help and for facilitating the use of the computer facility at and SU, VA, U.S.A.

## REFERENCES

[1] A.M.O. Smith, “Transition, Pressure Gradient and Stability Theory,” IX International Congress for Applied Mechanic, Brussels, 1996.

[2] J.L., Van Ingen, “A Suggested Semi-Empirical Method for the Calculation of the Boundary Layer Transition Region,” University of technology, Department of Aerie Engineering Report UTH-1956.

[3] A.J. Srokowski, and S.A. Orszag, “Mass Flow Requirements for LFC Wing Design,“ AIAA Paper No. 77-1222,August 1977.

[4] J.N. Hefner, and D.M. Bushnell, “Application of Stability Theory to Laminar Flow Control, “ AIAA Paper No. 79-1493, 1979.

[5] M.R. Malik, S.P. Wilkinson, and A.A. Orszag, “Instability and Transition in Rotating Disk Flow,” AIAAJ, Vol. 19, pp. 1131-1138. 1981.

[6] A.M.O. Smith “On the Growth of Taylor-Gortler Vortices Along Highly Concave Walls “, Quart. Appl. Math, Vol. 13, pp. 233-262, 1955.

[7] A.H. Nayfeh, S.A. Ragab, and A. A. Al-Maaitah, “Effect of Bulges on the Stability of Boundary Layers” Phys. Fluids, Vol. 31, pp. 796-806, 1988.

[8] A. A. Al-maaitah A. H. Nayfeh, and S. A. Rageb, “Effect of Cooling on the Stability of Comperssible Subsonic Flows over Smooth Humps and Backward-Facing Steps,” Phys. Fluids A, Vol. No.3, pp. 381-389, 1990.

[9] L. M. Mack, "Linear Stability Theory and the Problem of Supersonic Boundary-Layer Transition," AIAAJ, 13, pp. 290-299, 1975.

[10] L. M. Mack, "Remarks on Disputed Numerical Results in Compressible Boundary Layers Stability Theory," Phys. Fluids, Vol. 27, pp.347, 1984.

[11] K.F. Stetson, E.R. Thompson, J.C. Donaldson, and L.G. Siler, "Laminar Boundary-Layer Stability Experiments on a Cone at Mach 8, Part 5: Tests with a Cooled Model, " AIAA paper No. 89-1895, 1989.

[12] G. Gasperas, "Effect of Wall Temperature Distribution on the Stability of Compressible Boundary-Layer," AIAA Paper No. 89-1894, 1989.

[13] R. M. Malik, "Prediction and Control of Transition in Hypersonic Boundary-Layers," AIAA Paper No. 87- 1414, 1987.

[14] A.A. Al-Maaitah A.H. Nayfeh, an J. A Massad, "Effect of Suction on the Stability of Compressible Boundary-Layers, I.Second-Mode Waves," J. of Fluid Engineering ( ASME) Vol. 113, No. 4, pp. 596, 1991.

[15] J.A. Massad, A. H. Nayfeh, and A. A. Al-Maaitah, "Effect of suction on the Stability of Compressible Boundary-Layers, II First-Mode Waves," J. of Fluid Engineering (ASME) Vol. 113, No.4, pp. 597-6021991.

[16] J. A. Massad, A. H. Nayfeh, and A. A. Al-Maaitah, "Effect of Heat Transfer on the Stability of J. Boundary-Layer," of Computers and Fluids, Vol. 21, No.1, pp.43-61, 1992.

[17] A. A. Al-Naaitah, and M. A. Nimri, "On the Stability of Compressible Boundary Layers with High Mach Numbers: Part I- Effect of Free Stream Velocity", Alexandria Engineering journal, Vol. 36, No.6, pp A263-271, 1997.