ACCURACY IN POSITIONING OF NEW STATIONS IN HORIZONTAL CONTROL NETWORKS - A SURVEYING COMPUTER LIBRARY

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ABSTRACT

This paper is the third of a series forming the documentation of a computer library for surveying. It will answer the questions of accuracy in positioning of new stations by different methods in horizontal control networks, namely, triangulation, trilateration, hybrid, intersection and resection using the adjustment technique by variation of coordinates in matrix notation with precision analysis. A comprehensive general program in "FORTRAN" which performs both the adjustment and analysis of precision of any network is introduced.

Keywords: Triangulation, Trilateration, Hybrid, Intersection, Resection, Error Ellipse.

BACKGROUND REVIEW

Adjustment by Variation of Coordinates

The method of variation of coordinates involves the use of observations and parameters. The number of condition equations is exactly the same as the number of observations and each condition equation contains only one observation with a unit coefficient.

The general equation for the adjustment in horizontal control networks can be formulated in matrix notation as follows:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \cdots\cdots & b_{1u} \\ b_{21} & b_{22} \cdots\cdots & b_{2u} \\ \vdots & & \\ b_{n1} & b_{n2} \cdots\cdots & b_{nu} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_u \end{bmatrix} - \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

And in more concise form becomes:

$V_{n,1}=B_{n,u} \cdot \Delta_{u1}-f_{n,1}$

The most probable corrections Δx, Δy and the most probable coordinates $\hat{x},\hat{y}$ may be calculated:

$N=B^T.W.B$

$t =B^T.W.f$

$\Delta= N^{-1}.t$

$$\hat{x} = x_o + \Delta x$$

$$\hat{y} = y_o + \Delta y$$

Where:

$V_1,V_2,\ldots\ldots,V_n$ are the residuals for the "n" observations; $b_{11},b_{12},\ldots\ldots,b_{nu}$ are the numerical coefficients of the unknown parameters; Δ_1, $\Delta_2,\ldots\ldots$, Δ_u are the "u" unknown parameters (corrections to provisional coordinates); f_1, $f_2,\ldots\ldots$, f_n : are the numerical constant terms; B^T is a transpose matrix of "B".W is a weight matrix; x_o,y_o is a vector of provisional coordinates.

The mathematical model of observation equations for different methods of observation will be given in the following sections:

Triangulation

Referring to Figure 1, the observation equation for a measured angle can be written [1]:

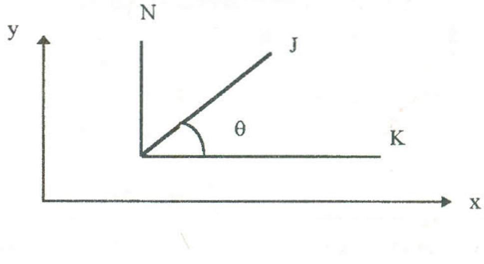


Figure 1 An observed angle

$$v_{jik} + \rho\left[\left(\frac{y_k^0 - y_i^0}{(s_{ik}^0)^2} - \frac{y_j^0 - y_i^0}{(s_{ij}^0)^2}\right)\Delta x_i + \left(\frac{x_j^0 - x_i^0}{(s_{ij}^0)^2} - \frac{x_k^0 - x_i^0}{(s_{jk}^0)^2}\right)\Delta y_i + \left(\frac{y_j^0 - y_i^0}{(s_{ij}^0)^2}\right)\Delta x_j + \left(\frac{x_i^0 - x_j^0}{(s_{ij}^0)^2}\right)\Delta y_j + \left(\frac{y_i^0 - y_k^0}{(s_{ik}^0)^2}\right)\Delta x_k + \left(\frac{x_k^0 - x_i^0}{(s_{ik}^0)^2}\right)\Delta y_k\right] = f_{jik} \quad (1)$$

where:

$$f_{jik} = \overset{0}{\theta}_{jik} - \theta_{jik}$$

$$\overset{0}{\theta}_{jik} = \arctan\frac{x_k^0 - x_i^0}{y_k^0 - y_i^0} - \arctan\frac{x_j^0 - x_i^0}{y_j^0 - y_i^0}$$

where θ_{jik} is the observed value of the angle

and v_{jik} is the corresponding residuals.

Trilateration

Referring to Figure 2 the observation equation for measured distances is:

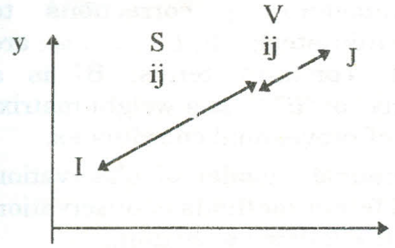


Figure 2 A measured Distance

$$v_{ij} + \frac{x_j^0 - x_i^0}{s_{ij}^0}\Delta x_i + \frac{y_j^0 - y_i^0}{s_{ij}^0}\Delta y_i + \frac{x_i^0 - x_j^0}{s_{ij}^0}\Delta x_j + \frac{y_i^0 - y_j^0}{s_{ij}^0}\Delta y_j = f_{ij} \quad (2)$$

where:

$$f_{ij} = \overset{0}{s}_{ij} - s_{ij}$$

$$\overset{0}{s}_{ij} = \sqrt{(x_{j^0} - x_{i^0})^2 + (y_{j^0} - y_{i^0})^2}$$

where s_{ij} is the measured distance and v_{ij} are the corresponding residuals.

Hybrid System

A hybrid system is a combination of triangulation and trilateration, in which both angles and distances are observed. Homogeneity of dimension and weight can be achieved by giving each observation a weight equal to the reciprocal of its variance σ_i^2. This can be effected by dividing each observation equation by the corresponding standard deviation(σ_i) and using the following to form the normal equations[2]:

$$[B^T . B].[\Delta] = [B^T].[f]$$
$$[\Delta] = [B^T . B]^{-1}.[B^T].[f] \quad (3)$$

Intersection

Observation equations for angular measurement, Figure 3 [3] :

$$V_{\theta_{uab}} + \rho\left[\left(\frac{y_{uo} - y_a}{S^2_{au_0}}\right)\Delta x_u + \left(\frac{x_a - x_{u_0}}{S^2_{au_0}}\right)\Delta y_u\right] = f_{\theta_{uab}} \quad (4)$$

where:

$$f_{\theta_{uab}} = \overset{0}{\theta}_{uab} - \theta_{uab}$$

For redundancy, at least three observations are required

Observation equations for distance measurement, Figure 3 [3]:

$$v_{au} + \frac{x_a - x_{u_0}}{S_{au_0}}\Delta x_u + \frac{y_a - y_{u_0}}{S_{au_0}}\Delta y_u = S_{au_0} - S_{au} \quad (5)$$

where:

$$S_{au_0} = \sqrt{(x_{u_0} - x_a)^2 + (y_{u_0} - y_a)^2}$$

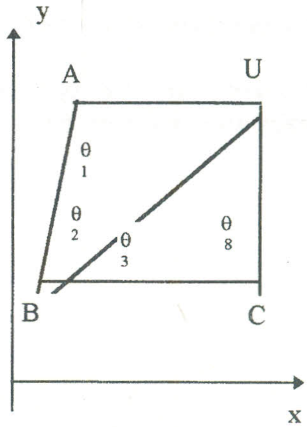


Figure 3 Angle intersection

Again, at least three observations are required.

Resection

Linear observation equations for observed angle, Figure 4, [3]:

$$v_{\theta_{aub}} + \rho\left[\left(\frac{y_b^o - y_u^o}{(s_{bu}^o)^2} - \frac{y_a^o - y_u^o}{(s_{au}^o)^2}\right)\Delta x_u + \left(\frac{x_a^o - x_u^o}{(s_{au}^o)^2} - \frac{x_b^o - x_u^o}{(s_{bu}^o)^2}\right)\Delta y_u\right] = f_{\theta_{aub}} \qquad (6)$$

(the same for θ_{bcu} *and* θ_{aud})

Linear observation equations for measured distance, Figure 4, [3]:

$$v_{ua} + \frac{x_a - x_u^o}{S_{au_o}}\Delta x_u + \frac{y_a - y_u^o}{S_{au_o}}\Delta y_u = S_{au_o} - S_{au} \qquad (7)$$

(the same for 'UB', 'UC' and 'UD')

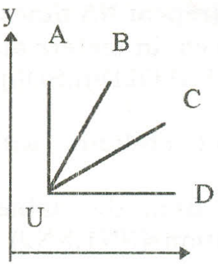


Figure 4 Resection of distance

Precision of Unknowns

The combined effect of both linear and angular random error causes an area of uncertainty around the point to be determined. The general shape of this area is an ellipse which is called the error ellipse. To determine the error ellipse three parameters are required, namely: semi- major and semi- minor axes, and the angle of orientation"θ" of the semi-major axis, Figure 5 Two types of error ellipses are used: absolute and relative.

Absolute Error Ellipse

It indicates the maximum and minimum error in position relative to the fixed station. The procedure of calculating the angle 'θ', maximum and minimum axes are presented in the following:

$$Q_{xx} = Q_{\Delta\Delta} = N^{-1} = (B^T.W.B)^{-1}$$

Q_{xx} is the cofactor matrix of the parameter estimates.

If the reference variance σ_o^2 is :

$$\sigma_o = \sqrt{\frac{v^T v}{n-u}}$$

where:

n: is the number of observations.

u: the number of unknowns

then, we calculate $\sum_{xx}$ as:

$$\sigma_{x_i} = \sigma_o\sqrt{Q_{xx_{ii}}} = \sqrt{\Sigma_{xx}}$$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_y^2 - \sigma_x^2} \qquad (8)$$

$$\sigma_u^2 = \frac{1}{2}(\sigma_y^2 + \sigma_x^2 + k) \qquad (9)$$

$$\sigma_v^2 = \frac{1}{2}(\sigma_y^2 + \sigma_x^2 - k) \qquad (10)$$

$$k = [(\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy})^2]^{\frac{1}{2}}$$

Equation 8 gives the angle which the semi-major axis makes with 'y' axis. Equation 9 gives the variance corresponding to semi-major axis. the square root of this variance is the value of the semi-major axis for the standard error ellipse. Equation 10 gives the variance corresponding to semi-major axis. The square root of this variance gives the minimum axis of the standard error ellipse.

Relative Error Ellipse

It represents the uncertainty in length and azimuth of a line joining two stations. It is computed by considering the components of the line joining the two stations:

$$\sigma_{\Delta x}^2 = \sigma_{x2}^2 + \sigma_{x1}^2 - 2\,\sigma_{x1x2}$$

$\Delta x = x2 - x1$

$\Delta y = y2 - y1$

$$\sigma_{\Delta y}^2 = \sigma_{y2}^2 + \sigma_{y1}^2 - 2\,\sigma_{y1y2}$$

$$\sigma_{\Delta x\,\Delta y} = \sigma_{x2y2} - \sigma_{x2y1} - \sigma_{x1y2} + \sigma_{x1y1}$$

the calculations of relative error ellipse are [4]:

$$a^2 = \frac{1}{2}\left(\sigma^2_{\Delta y} + \sigma^2_{\Delta x} + \sqrt{(\sigma^2_{\Delta y} - \sigma^2_{\Delta x})^2 + (\sigma_{\Delta x\,\Delta y})^2}\right)$$

$$b^2 = \frac{1}{2}\left(\sigma^2_{\Delta y} + \sigma^2_{\Delta x} - \sqrt{(\sigma^2_{\Delta y} - \sigma^2_{\Delta x})^2 + (\sigma_{\Delta x\,\Delta y})^2}\right)$$

$$\tan 2\theta = \frac{2\,\sigma_{\Delta x\,\Delta y}}{\sigma^2_{\Delta y} - \sigma^2_{\Delta x}}$$

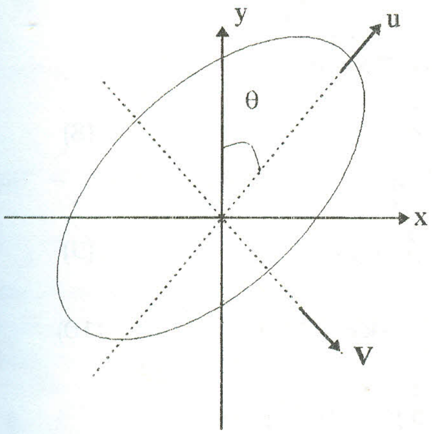


Figure 5 Enor ellipse

The standard error has approximately 35 percent probability that the adjusted point lies within or on it. It can be modified depending on the number of degrees of freedom in adjustment problem with any confidence interval. In surveying, the 95% probability is used and the 'F' statistic modifiers for that are [5]:

degrees of freedom	'F' statistic
1	200
2	19.0
3	9.55
4	6.94
5	5.79
10	4.10
15	3.68
20	3.49
25	3.39
50	3.19

COMPUTER PROGRAM

General

A comprehensive computer program was written. The basis of the program was obtained from [2] . The original program was developed extensively to involve evaluation of precisions of the observations and parameters after adjustment including determination of error ellipse dimensions (both absolute and relative)[6]

Description of the program

Input

- No. of stations 'N'/No. of fixed stations 'NF'/ No. of observed angles 'NA'/ No. of measured distances' ND'/JSQRT1: index to define system, =1 for triangulation, 2 for trilateration, 3 for hybrid.
- Names of fixed stations 'NNF'.
- Provisional coordinates of network stations(distinguishing fixed and new stations)
  (X(I), Y(I), repeat (N) times).
- Observed angles in degrees, minutes, seconds and the prior standard deviation of observed angle (NNA(I,J),J=1,3) AD,AM,AS,SA(I) (repeat NA times)
- Measured distances in meters and standard error(NND(I,J), J=1,2) OLD(I),SD(I)) (repeat ND times)
  Serial number of stations without fixed stations (NN).
- Statistic modifier from the table number of two connected stations(NN1,NN2)

Computations Steps

- Compute: No. of observations : NR=NA+ND,
  No. ofparameters:NCC=2(N- NF)
  No. of freedom degree: NDF=NR-NCC
- Convert observed angles into radians (OLA(I)),(function DEG)
- Compute the bearings and distances for sides of observed angles or measured distances ,or both them (subroutine BB)
- Form observation equations A(NR,NCC)
- Form normal equations:(C(I,J), J=1, NCC+1).
- Solve simultaneous equations using Gauss Jordan algorithm. (C(I,NCC+1)).
- Compute corrected coordinates X(I), Y(I).
- Compute adjusted observations after computing corrections to observed angles and measured distances 'v' by back substitution in the original observation equations v =Ax-L.
- Compute a posterior standard deviations σ_0 .

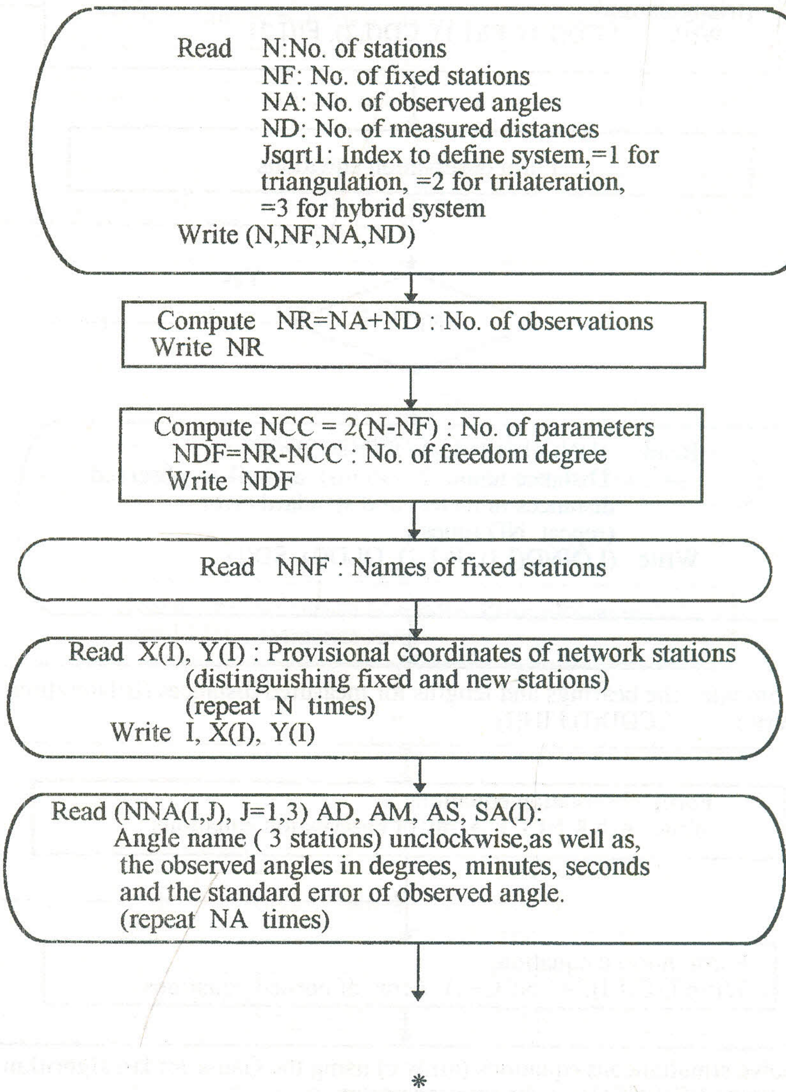
Form inverse of normal equations AN1(I,J),(subroutine minv).

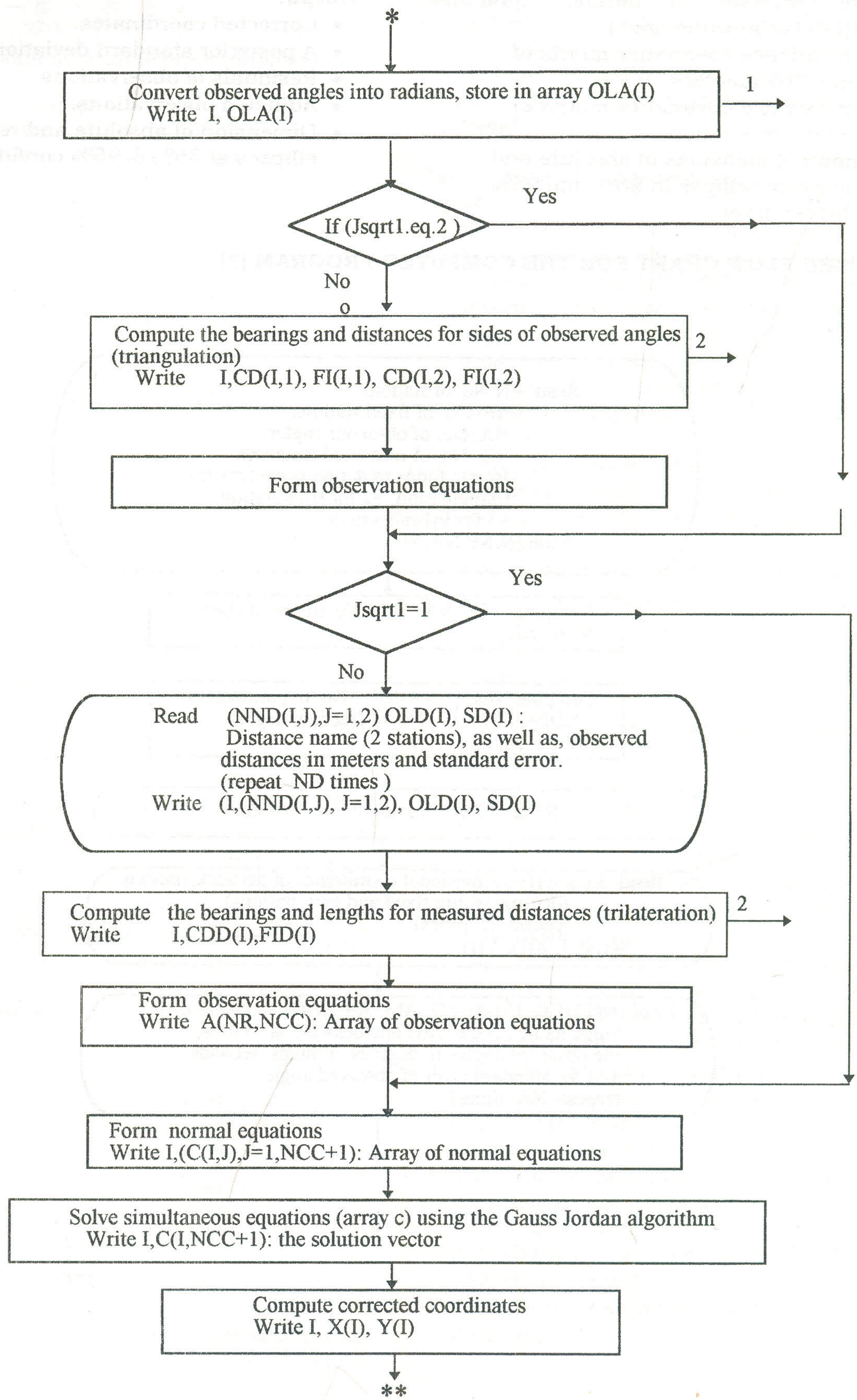
- Form variance-covariance matrix of adjusted parameters
- Form variance-covariance matrix of adjusted observation.
- Compute dimensions of absolute and relative error ellipse in 35% and 95% confidence level.

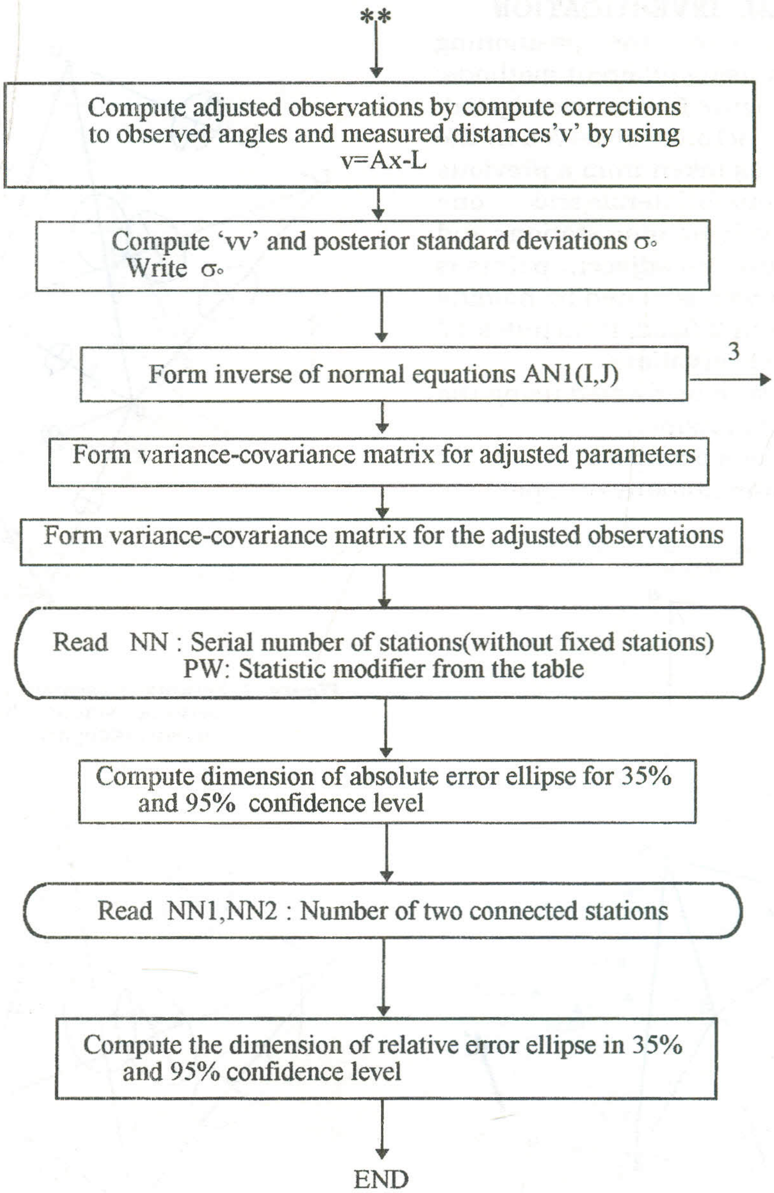
Output

- Corrected coordinates.
- A posterior standard deviations.
- Residuals of observations.
- Adjusted observations.
- Dimension of absolute and relative error ellipses at 35% & 95% confidence levels.

SUMMARIZED FLOW CHART FOR THE COMPUTER PROGRAM [7]




*
Convert observed angles into radians, store in array OLA(I)
Write I, OLA(I)
1
If (Jsqrt1.eq.2)
Yes
No
o
Compute the bearings and distances for sides of observed angles (triangulation)
Write I,CD(I,1), FI(I,1), CD(I,2), FI(I,2)
2
Form observation equations
Jsqrt1=1
Yes
No
Read (NND(I,J),J=1,2) OLD(I), SD(I) :
Distance name (2 stations), as well as, observed distances in meters and standard error.
(repeat ND times)
Write (I,(NND(I,J), J=1,2), OLD(I), SD(I)
Compute the bearings and lengths for measured distances (trilateration)
Write I,CDD(I),FID(I)
2
Form observation equations
Write A(NR,NCC): Array of observation equations
Form normal equations
Write I,(C(I,J),J=1,NCC+1): Array of normal equations
Solve simultaneous equations (array c) using the Gauss Jordan algorithm
Write I,C(I,NCC+1): the solution vector
Compute corrected coordinates
Write I, X(I), Y(I)
**



Sub - Program

1	Angles from degree to radian
2	Compute bearings and distances for side
3	Form inverse of normal equation coefficients

PRACTICAL NUMERICAL INVESTIGATION

In order to evaluate the positioning accuracy of nw stations using different methods; the network shown in Figure 6 was investigated. This network was not actually observed in the present work, but it was taken from a previous work. It consists of two quadrilaterals and one center point polygon involving nine stations and the distance between any two adjacent points is about (1200-5700)m. It was adjusted by holding the stations number 1 and 2 fixed. It includes 17 distances and 28 angle observations.

Four distinct solutions were performed using the current computer program, namely:

(i) Hybrid with all stations occupied
(ii) Hybrid with minimum stations occupied
(iii) Trilateration
(iv) Triangulation

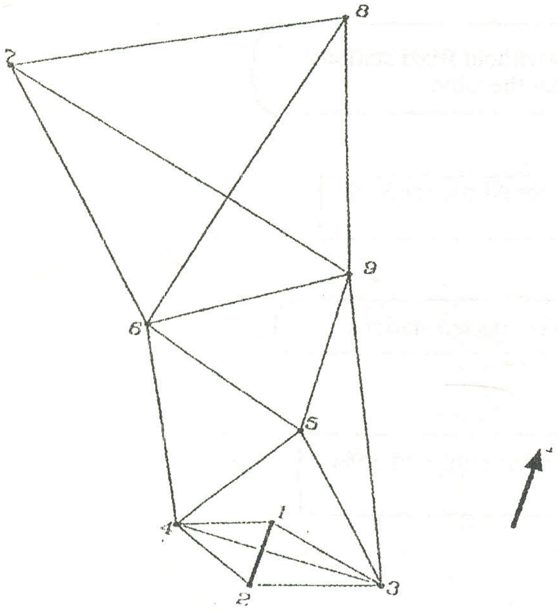


Figure 6 Network 1

Figures 7-10 show plans for the spatial distributions of relative error ellipses in cases(i),(ii),(iii),(iv),respectively. In order to study the results from these figures in a more concise form, the values of the semi-major axes of relative error ellipses (which are the worst-case criteria) were summarized in tables (not given) and ploted against distances in all cases, Figure 11. The values of the semi-major axes in this plot are represented as part per million of their respective lengths.

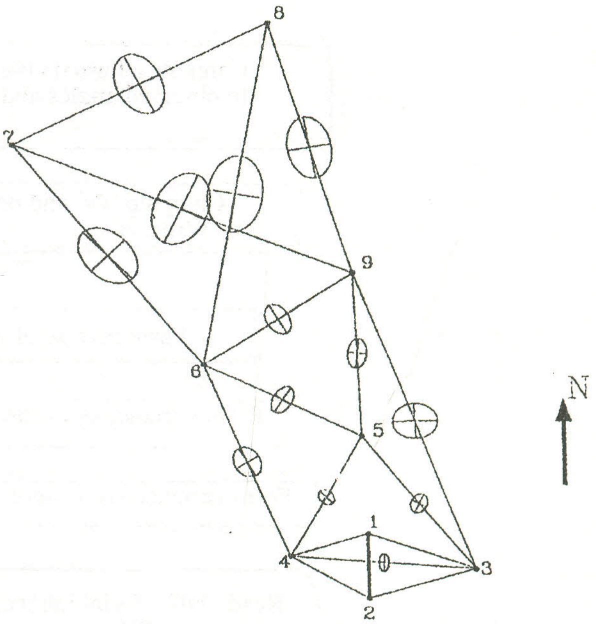


Figure 7 Network I Relative error ellipses between the network stations by hybrid system with all stations occupied.

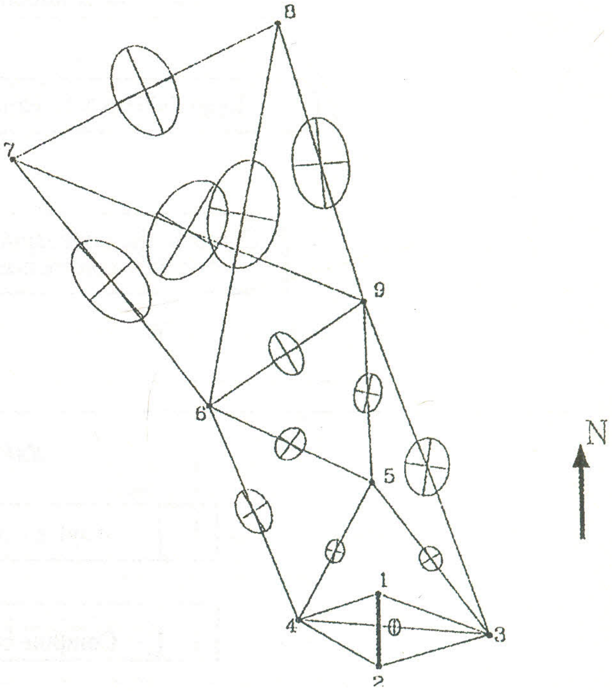


Figure 8 Network I Relative error ellipses between the network stations by hybrid system with minimum number of occupied stations.

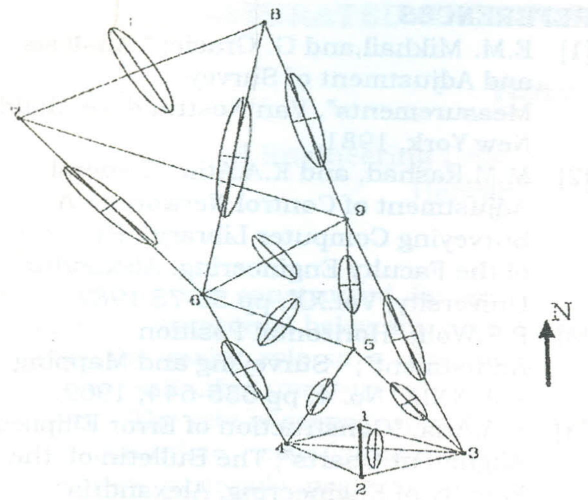


Figure 9 Network I- Relative error ellipes between the net work stations by trilatcration system.

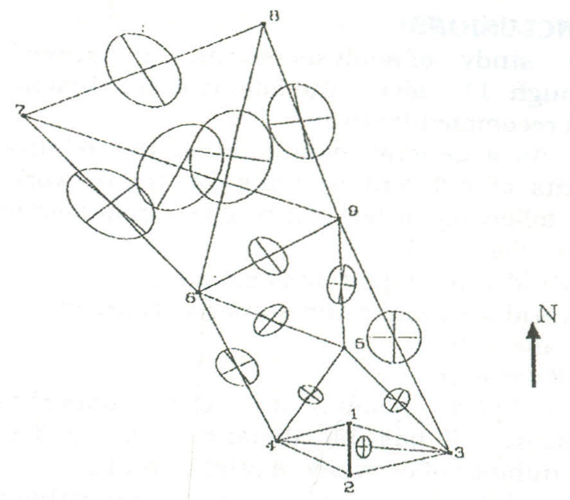


Figure 10 Network I-Relative error ellipses between the network stations by triangulation system.

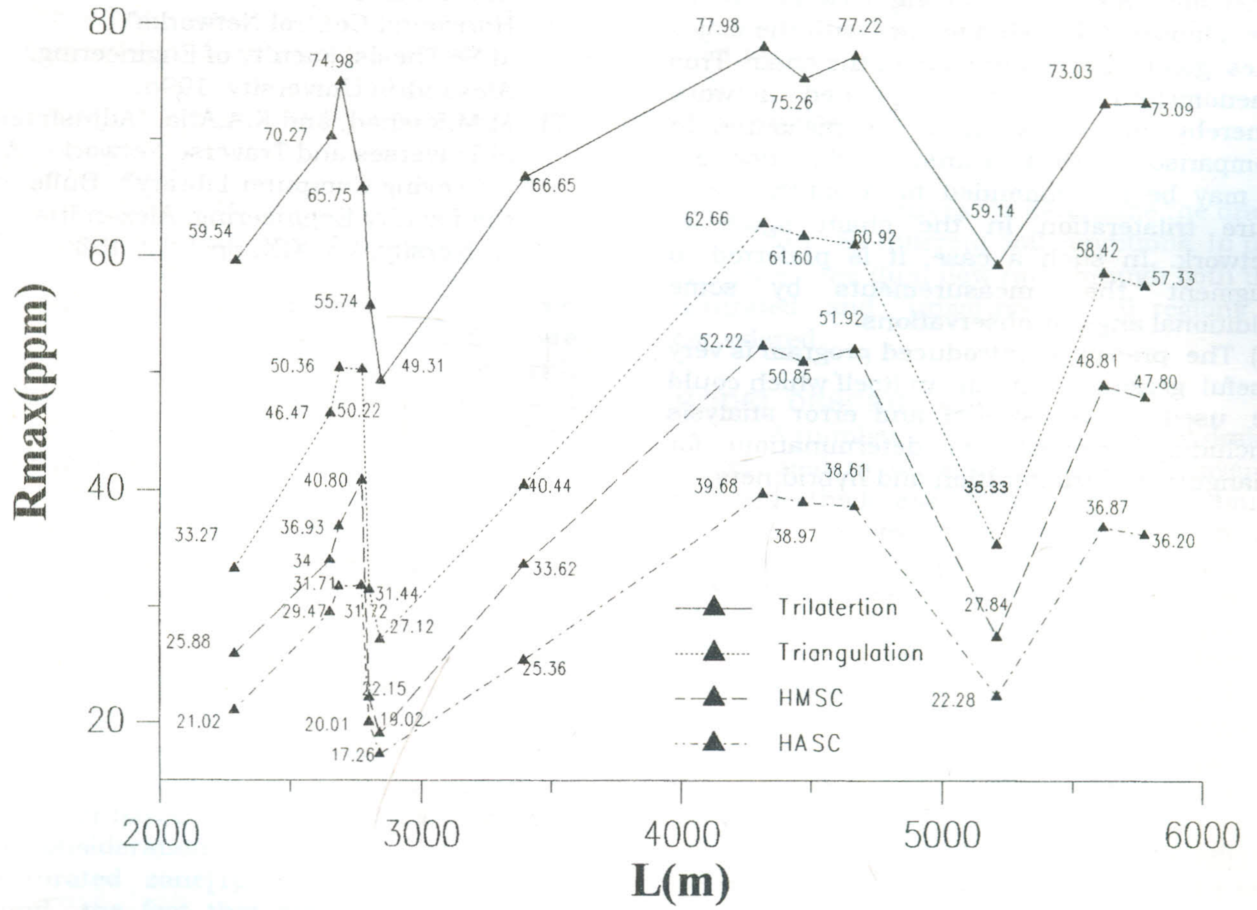


Figure 11 Network I- Semi major axis of relative error ellipse (ppm) vs. distance

CONCLUSIONS

The study of analysis results and Figures 7 through 11 give the following conclusions and recommendations :

(1) As a general evaluation of the relative merits of different systems for the network, the following order can be given from best to worse Figure 11

- Hybrid with all stations occupied.
- Hybrid with minimum stations occupied.
- Triangulation.
- Trilateration.

(2) In hybrid system, it is not economical to measure all possible distances and angles. the number of occupied stations can be reduced by one-third to one-quarter without significant loss of accuracy.

(3) It has been clearly noted in Figure 9 that the orientation of error ellipses in trilateration system takes the general direction of network extension. So, for an ablong network" chain" the ellipses take slim shapes with the major axes going in the direction of the chain. This phenomenon can be explained network whereby more distances are measured in comparison with the transverse direction. So, it may be recommended to avoid the use of pure trilateration in the chain figure of network. In such a case, it is preferred to augment the measurements by some additional angular observations.

(4) The presently introduced program is very useful general program in itself which could be used in adjustment and error analysis (including error-ellipses determination) for triangulation, trilateration and hybrid nets.

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