HORIZONTALLY CURVED REINFORCED CONCRETE WAFFLE SLAB BRIDGES : ESTIMATION OF REACTIONS

Tarek I. Ebeido
Structural Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

The employment of horizontally curved bridges has remarkably increased during the last few decades. This is due to the frequent demand for these geometries in modern highway networks. An effective bridge cross-section ought to be competent to transfer & distribute truck wheel loads over a wide area, so as to maintain stresses within the acceptable limits. For distributing concentrated truck wheel loads, solid reinforced concrete slabs are extremely efficient up to a certain span limit. On exceeding such limit, the dead load stress increases rendering the solid slab section uneconomical. Moreover, in the case the horizontally curved bridges the flexibility of the longer outer edge increases significantly. One good alternative for the curved solid slab bridge is the use of concrete deck slab stiffened by a system of orthogonal ribs in the radial and tangential directions. This system is called waffle slab bridge, the presence of the ribs in the radial direction is necessary for the curved structure to resist the large torsional moments. In this paper, the influence of curvature on the distribution of reactions is detected for Waffle slab bridges. The influences of other factors such as : girder spacing , bridge aspect ratio, number of lanes, number of longitudinal girders and number of intermediate ribs in the radial direction are presented. Moreover, a comprehensive parametric study is conducted on prototype curved waffle slab bridges subjected to truck loads specified in the Egyptian code for loads. The parametric study conducted in this paper comprised more than 500 bridge cases. The finite element method is employed to conduct the parametric study. Empirical formulas are suggested for the estimation of maximum reactions in a curved waffle slab bridge under concentrated truck wheel loads.

Keywords: Bridges, Curved, Reactions, Reinforced Concrete, Slab, Waffle

Notations

D	reaction distribution factor
N	dimensionless parameter
n	number of ribs in the radial direction
R	reaction of a simply supported girder
R_{max}	maximum reaction from the finite element analysis
r	radius of curvature of the bridge
S	spacing of longitudinal girders

INTRODUCTION

In former years, the usage of waffle slab construction, Figure (1), was considered incongruent to the predominantely one-way supporting system of a bridge. Consequently, bridge design engineers have discarded its usage. Due to curvature, the flexibility of the longer outer edge of the bridge increases significantly. As a result large torsional moments are developed and the presence of ribs in the radial direction becomes necessary. Several authors (Kennedy and Ghobrial [1] Kennedy and EL-sebakhy [2]) have investigated the behaviour of waffle slab bridges showing that the advantages of such bridge system are : (i). reduction in dead load moment and deflection; (ii.) easy accessibility to parts of the structure for inspection and repair; and (iii) shallow depth of its cross section. Kennedy and El-sebakhy [3] developed a yield line solution for the estimation of ultimate collapse load for such bridges. A feasibility study by Kennedy and Bakht [4] and Kennedy [5] was undertaken to investigate the structural efficiency of a waffle slab bridge system. Meli et al. [6] studied the seismic behaviour of waffle slab structures. This study was carried out after the collapse of many buildings with waffle slabs by the 1985 earthquake in Mexico city. Recently, Verma and Dey [7] carried out a static analysis of bridge superstructures which are curved in plan, having radial as well as circumferential beams. They used a variational based finite difference energy method. More recently Aly and Kennedy [8] and Aly [9] investigated experimentally and theoretically the behaviour of curved waffle slab bridges. They used the finite element method, the finite difference method, and the yield line theory. Comparatively no research efforts have been done to investigate the distribution of reactions in a curved waffle slab bridge. Due to curvature, distribution of reactions between longitudinal girders due to truck loads differs than that in a normal straight bridge. In this paper, the influence of curvature on the distribution of reactions is presented. In addition , the influences of other factors such as : girder spacing, bridge aspect ratio, number of lanes, number of longitudinal girders & number of intermediate cross- beams are presented. Moreover, a detailed parametric study is conducted on prototype curved waffle slab bridges subjected to truck loads specified in the Egyptian code for loads.[10] The parametric study conducted in this paper comprised more than 500 bridge cases. The finite element method is used to conduct the parametric study. The finite element model is verified and confirmed using experimental results found in the literature . Good correspondence is found between the experimental results found in the literature and the results from the current finite element analysis. Conclusively, the resulting data are used to develop empirical formulas for the estimation of maximum reactions in a curved waffle slab bridge under concentrated truck wheel loads.

THEORETICAL STUDY

The theoretical study in this paper includes a three - dimensional finite element modelling of curved waffle slab bridges using the finite element program NISA [11] . The program is verified and confirmed using experimental results found in the literature. The calibrated program is then used to conduct an extensive parametric study on prototype curved waffle slab bridges. The parametric study conducted in this paper included more than 500 bridge cases. The reinforced concrete deck slab was modelled using a four-node shell element with six degrees of freedom at each node. The reinforced concrete ribs in the radial and tangential directions were modelled using two alternative methods. In the first method, method A, the ribs were modelled using a four -node shell element with six degrees of freedom at each node, Figure (2), whereas in the second method, method B, ribs were modelled using three - dimensional two-node beam elements with six degrees of freedom at each node, Figure (3). The multipoint constraint option in the Nisa program, which allows constraint between different degrees of freedom, was used between the shell nodes of the deck slab and the beam element of the radial and tangential ribs. The multipoint constraint option ensured full interaction between the reinforced concrete deck slabs and the reinforced concrete ribs.

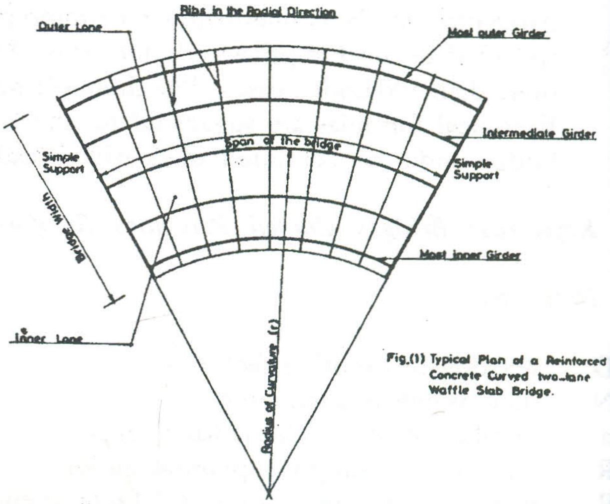


Figure 1. Typical plan of a reinforced concrete curved two-lane waffle slab bridge.

VERIFICATION OF THE FINITE ELEMENT MODEL:

Aly [9] tested thirteen curved waffle slab bridge models. The experimental results presented by Aly [9] are used herein to verify the current finite element analysis. All bridge models tested by Aly [9] are modelled herein using finite element models A& B. For brevity, results will be presented here for three bridge models only. The three bridge models considered were named by Ali as RWS, PWS, and PGS. The overall dimensions of bridge models were 0.86m. width and 1.78m. span along the longitudinal center line of the bridge model. The total connecting angle between the left and right supports was 71.9 degrees and the radius of curvature at the longitudinal center line was 1.41m. The spacing between the ribs in the tangential direction was 178mm, whereas the spacing in the radial direction was also 178mm. The slab thickness of the waffle section was 30mm and the rib depth was 101mm. Bridge model PGS was identical to bridge models RWS and PWS except for the transverse ribs. Table (1) shows a comparison between the results of the current finite element analysis and the experimental results presented by Ali [9]. The results are presented for both deflection at center and top surface strain at center. The results are presented for three loading cases. In the first loading case a single concentrated load was applied at the center of the bridge. In the second loading case the outer lane was loaded with a uniformly distributed load while the inner lane was left unloaded. In the third loading case the inner lane was loaded with a uniformly distributed load while the outer lane was left unloaded.

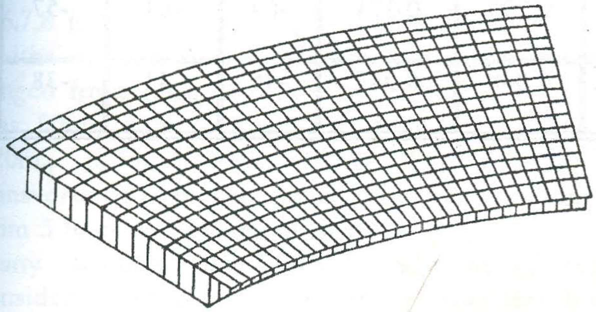


Figure 2. Finite element mesh used in method (A).

Good agreement is observed between the experimental results Ali [9] and results from current finite element analysis, for both deflections and strains. However, it is observed that the difference between the experimental results and results from method A did not exceed 6% whereas in case of using method B such a difference was up to 11% . Therefore , a shell element idealization for the longitudinal girders and the transverse ribs is more reliable than a beam element model . For this reason, method A was used for the parametric study presented in the following sections.

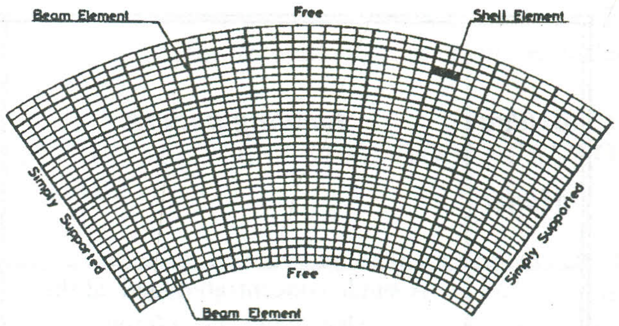
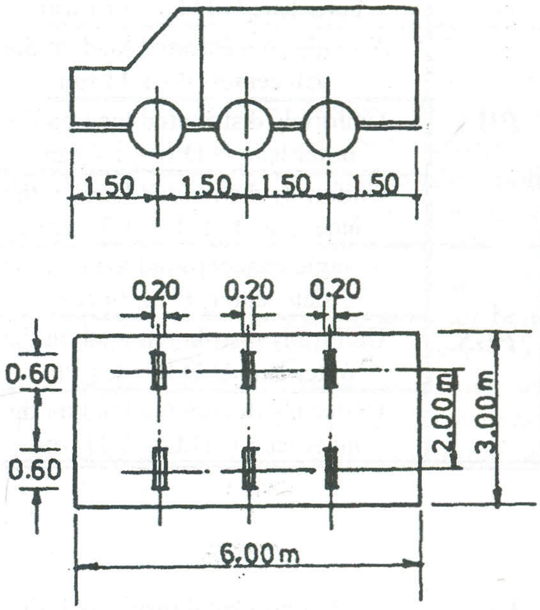


Figure 3. Finite element mesh used in method (B).



Total Truck Load = 60 Ton
Wheel Load = 10 Ton
All Dimensions are in meters

Figure 4. Truck specified in the Egyptian code for loads.

Table (1): Comparison of experimental results, Ali [9] to current finite element analysis

Bridge Model	Loading pattern	Deflection at center (mm)			Top surface strain at center × 10^6 (mm / mm)		
		Exp.*	Method** A	Method*** B	Exp.	Method A	Method B
RWS	A single concentrated load at the slab center, P= 0.6 ton	0.280	0.263	0.255	-46	-44	-41
	Uniformly distributed load on the outer lane U.D.L.= 0.7t/m^2	0.203	0.191	0.185	-22	-20	-20
	Uniformly distributed load on the inner lane U.D.L.= 0.61 t/m^2	0.089	0.083	0.079	-10	-9	-9
PWS	A single concentrated load at the slab center, P= 1.14 ton	0.330	0.317	0.287	-114	-108	-103
	Uniformly distributed load on the outer lane U.D.L.= 1.4 t/m^2	0.178	0.171	0.157	-63	-59	-57
	Uniformly distributed load on the inner lane U.D.L.= 1.31 t/m^2	0.076	0.075	0.068	-40	-38	-35
PGS	A single concentrated load at the slab center, P= 1.14 ton	0.457	0.434	0.411	-147	-139	-131
	Uniformly distributed load on the outer lane U.D.L.= 1.4 t/m^2	0.305	0.293	0.271	-64	-61	-57
	Uniformly distributed load on the inner lane U.D.L.= 1.31 t/m^2	0.051	0.048	0.047	-45	-41	-38

*Exp. : Experimental results, Ali [9]
**Method A : Current finite element analysis, method A
***Method B : Current finite element analysis, method B

PARAMETRIC STUDY

An extensive parametric study is conducted, in this paper, on prototype horizontally curved reinforced concrete waffle slab bridges. The objectives of the parametric study were :(i) to investigate the influence of all major variables affecting reaction distribution between girders; (ii) to generate a database for reaction distribution factor including more than 500 bridge cases; and(iii) to develop empirical formulas for reaction distribution factors corresponding to truck loads specified by the Egyptian code [10]. The parameters chosen for this study were degree of curvature, girder spacing, bridge aspect ratio , number of lanes, number of longitudinal girders, and number of intermediate ribs in the radial direction. The parametric study was based on the following assumptions :(i) both reinforced concrete deck slab and the longitudinal girders are simply supported at both ends; (ii) all materials are elastic and homogeneous; and (iii) the effects of the curbs are ignored. The number of lanes considered was 2,3,and 4 lanes with bridge width of 9.0 m for two lane bridges, 12.5m for three lane bridges, and 16.0m for four lane bridges. The span length at center line was 15.2, 19.6 ms, 24.0ms, 28.4ms, and 32.8ms. Based on the bridge widths and the span lengths, the bridge aspect ratio (span length /width) varied from 1.0 to 3.6. The number of longitudinal girders considered was 3,4,5,6 for two lane bridges, 4,5,6,7, for three lane bridges, and 5,6,7,8 for four lane bridges. For the above bridge widths and number of girders, the girder spacing ranged from 1.4m. to 3.5m. the radius of curvature was 50m., 60m., 70m, 80m., 90m., 100m.,110m., 120m., 130m., 140m., 150m. The number of transverse ribs in the radial direction was ranged from 5 lines to 12 lines of ribs.

Many loading cases, explained below, were considered in the parametric study, using the truck specified by the Egyptian code, Figure (4). The more than 500 bridge cases considered can be classified into three main categories according to the loading patterns. The categories were as follows: (i) one or two trucks applied on the outer lanes of the bridge representing an eccentric truck loading. The trucks were applied close to one of the supports to maximize the reactions;(ii) one or two trucks applied on the inner lanes of the bridge representing an eccentric truck loading ; and (iii) a concentric truck loading in which the bridge was fully loaded by two trucks for two-lane bridges, three trucks for three-lane bridges, and four trucks for four - lane bridges. The trucks were applied close to one of the supports to maximize the reactions. It should be noted that all loading cases the trucks were moved in the transverse direction within the loaded lanes in order to yield the maximum reactions for different girders in the bridge.

THE LOAD DISTRIBUTION FACTOR CONCEPT

The evaluation of existing bridges and design of new ones require accurate prediction of their support reactions under truck loads. The load distribution factor concept allows the design engineer to consider the longitudinal and transverse effects of wheel loads as two separate phenomena, thus simplifying the analysis and design of the bridge. However using inappropriate load distribution factors may lead to extremely conservative reactions or sometimes makes the design of the bridge unsafe. The American Association of State Highway and Transportation Officials (AASHTO, [12]) has traditionally applied a load distribution factor depending only on the center to center girder spacing. However, the factor presented by the AASHTO code [12] is limited only to the estimation of the longitudinal girder bending moment. The factor presented by AASHTO also ignores several important parameters such as :(i) curvature; (ii) bridge aspect ratio; (iii) number of girders; (iv) number of lanes; and (v) number of ribs in the radial direction.

RESULTS

In order to determine the reaction distribution factor, D, the maximum reaction of a simply supported girder, R, under the effect of a line of wheel loads specified in the Egyptian code for loads was first calculated for each girder in all bridges considered in the parametric study. Based on the experimentally verified finite element model, the maximum reaction in each girder, R_{max} was obtained for each prototype bridge from the results of the current finite element analysis. The reaction distribution factor, D, was then calculated from the following relationship, AASHTO [12],:

$$D = \frac{R_{max}}{R} \qquad (1)$$

The effect of the various design parameters on the reaction distribution factor, D, is presented below.

Effect of girder spacing

The spacing of longitudinal girders, S, is one of the most important factors affecting the reaction distribution factor. This is reflected in the code of American Association of State Highway and Transportation Officials (AASHTO, [12]) where the girder moments may be calculated using the wheel load fraction of S/5.5, AASHTO [12]. Results in this study reveal that the girder spacing has a significant influence on the reaction distribution factor. The girder spacing is a function of the bridge width and the number of longitudinal girders. The bridge width can be taken as number of lanes multiplied by the lane width , the later being constant in most cases. Therefore the girder spacing could be related to the number of girders and number of lanes without reference to the lane width. Thus, it is suggested that a measure for the girder spacing be expressed by a dimensionless factor, N, defined by:

$$N = \frac{\text{Number of traffic lanes}}{\text{number of longitudinal girders}} \qquad (2)$$

The relations between the reaction distribution factor and the ratio N for an intermediate girder of a fully loaded reinforced concrete waffle slab bridge having different radii of curvature are presented in Figure (5). It can be observed that the reaction distribution factor increases with increase in the ratio N. The effect is almost the same for all radii of curvature. Similar effects were found for the most inner girder and the most outer girder and also for the case of inner lanes loaded and outer lanes loaded.

Effect of curvature

The results of the current parametric study, revealed that the curvature of the bridge is one of the most significant parameters affecting the distribution of reactions between longitudinal girders. The effect of curvature on the reaction distribution factor ,D, for the prototype reinforced concrete waffle slab bridges was examined for three types of loading : inner lanes loaded, outer lanes loaded, and fully loaded bridge. Figure (6), shows the effect of curvature on the reaction distribution factor ,D, for the most inner girder of a fully loaded two lane waffle slab bridge having different number of girders. It is observed that an increase in the radius of curvature of the bridge increased the factor D significantly for all number of girders. Figure (7) shows the effect of curvature on the factor ,D, for an intermediate girder. In this case the factor D decreases significantly with an increase in the radius of curvature. Similar relationship was found between the radius of curvature and the factor D in the case of the most outer girder as shown in Figure (8). *Results not shown herein for brevity revealed that* in the case of inner lanes loaded the factor D increases with an increase in the radius of curvature for the most inner girder and decreases with increase in the radius of curvature for an intermediate girder. In this case an uplift force exists at the most outer girder. Such a force increases with a decrease in the radius of curvature of the bridge. The results of the parametric study also showed that in the case of outer lanes loaded the factor D decreases with an increase in the radius of the bridge for an intermediate girder and for the most outer girder. In this case the uplift force exists at the most inner girder which increases with an increase in the radius of curvature of the bridge.

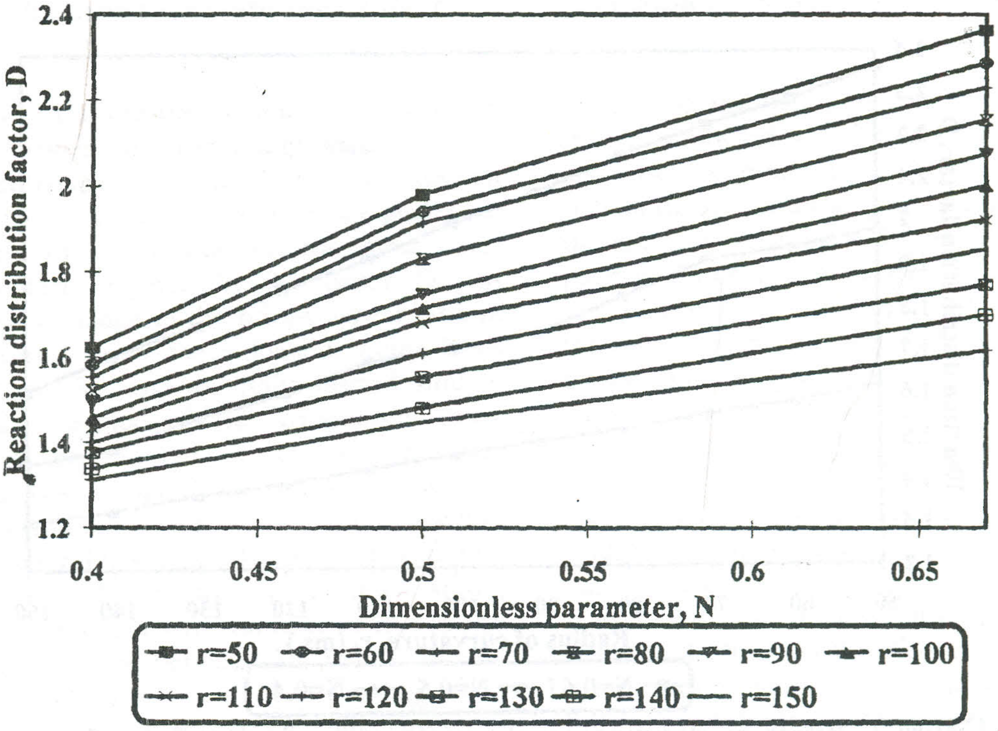


Figure 5. Effect of girder spacing on the reaction distribution factor for an intermediate girder of a fully loaded waffle slab bridge (results from the current parametric study).

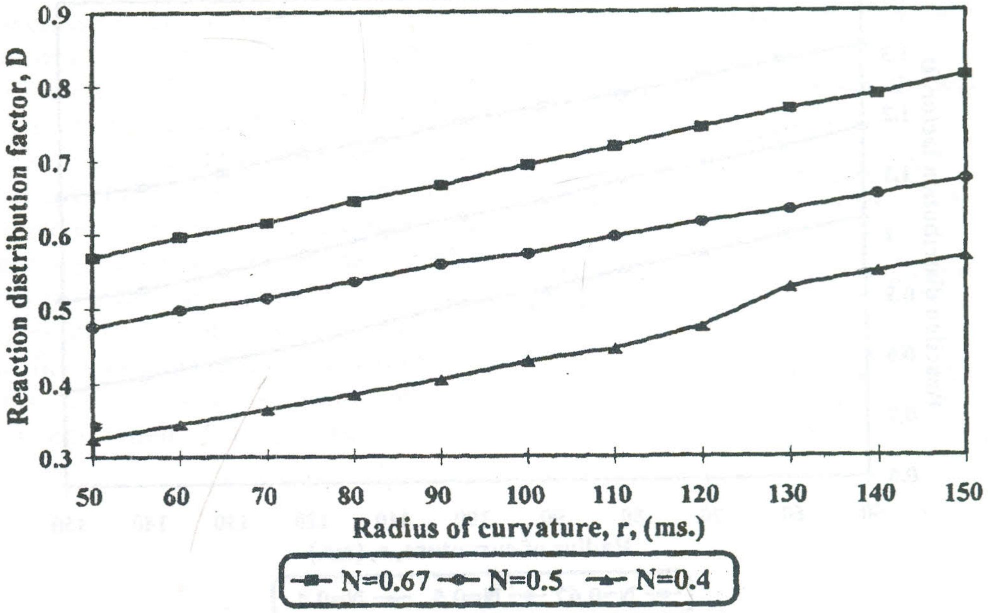


Figure 6. Effect of curvature on reaction distribution factor for the most inner girder of a fully loaded waffle slab bridge (results from the current parametric study).

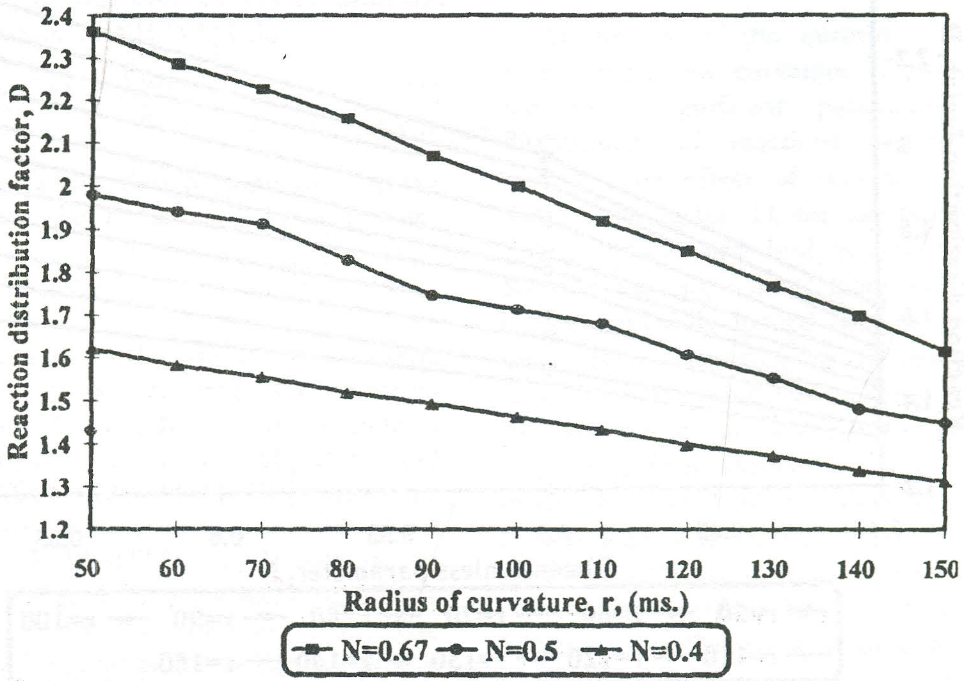


Figure 7. Effect of curvature on the reaction distribution factor for an intermediate girder of a fully loaded waffle slab bridge (results from the current parametric study).

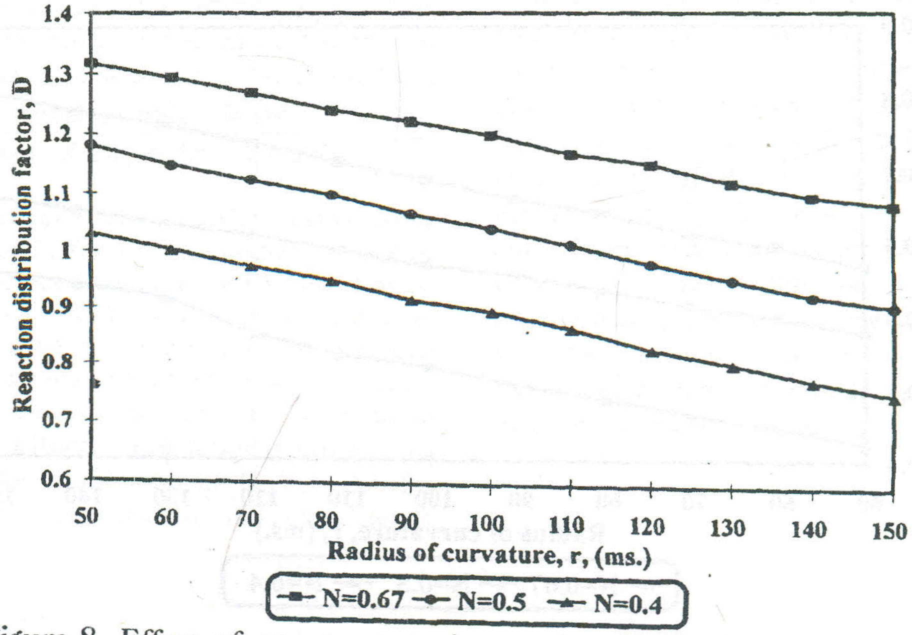


Figure 8. Effect of curvature on the reaction distribution factor for an intermediate girder of a fully loaded waffle slab bridge (results from the current parametric study).

Effect of bridge aspect ratio and ribs in the radial direction

The results of the parametric study, conducted in this paper, showed that the bridge aspect ratio is the parameter having the least effect on the reaction distribution factor, D for the most inner girder, an intermediate girder, and also for the most outer girder. Very little change in the factor D was observed with a change in the bridge aspect ratio in all loading cases: inner lanes loaded; outer lanes loaded, and in the case of fully loaded bridge. However, the results of the parametric study revealed that the number of ribs in the radial direction ,n, has a significant influence on the reaction distribution factor D. The reaction distribution factor ,D, was calculated for all prototype bridges considered in the parametric study, conducted in this paper with number of ribs in the radial direction ranging between 5 and 12 lines, the following were observed : (i) in all loading cases increasing the number of lines of ribs increases the reaction distribution factor, D for the most inner girder, Figure (9); (ii) in all loading cases increasing the number of lines of ribs decreases the reaction distribution factor, D for the most outer girder, and for an intermediate girder, (iii) in the case of outer lanes loaded increasing the number of ribs in the radial direction decreases the uplift force existing at the most inner girder; (iv) in the case of inner lanes loaded increasing the number of ribs in the radial direction decreases the uplift force at the most outer girder; (v) the effect of increasing the number of ribs in the radial direction on the reaction distribution factor was more significant in bridges having high degree of curvature than that in the case of bridges having low degree of curvature, and ; (vi) increasing the number of lines of ribs in the radial direction generally leads to a more uniform reaction distribution between longitudinal girders.

Empirical formulas for reaction distribution factor, D:

Based on the data generated from the parametric study, conducted in this paper, analyzing more than 500 cases of prototype reinforced concrete waffle slab bridges, empirical formulas are developed for the reaction distribution factor, D using a statistical package for best fit. The reaction distribution factors are determined for the most inner girder, an intermediate girder, and also for the most outer girder. The empirical formulas, deduced in this paper, are in terms of the following significant design parameters: (1) girder spacing expressed as the dimensionless parameter, N= number of lanes/ number of girders; (2) radius of curvature of the bridge, r, in meters; and (3) number of ribs in the radial direction, n. The aspect ratio of the bridge Ar, is excluded from the empirical formulas since it has very small effect on the distribution factor, D. The three loading conditions considered are as follows: fully loaded bridge case, outer lanes loaded, and inner lanes loaded. Thus the empirical formulas deduced in this paper can be presented as follows:

(I) For the most inner girder:

$$D = 0.16 \chi r^{0.33} \chi N^{0.87} \chi n^{0.28} \tag{3}$$

(II) For an intermediate girder:

$$D = \frac{9.3 \chi N^{0.6}}{r^{0.22} \chi n^{0.18}} \tag{4}$$

(III) For the most outer girder:

$$D = \frac{6.3 \chi N^{0.54}}{r^{0.23} \chi n^{0.22}} \tag{5}$$

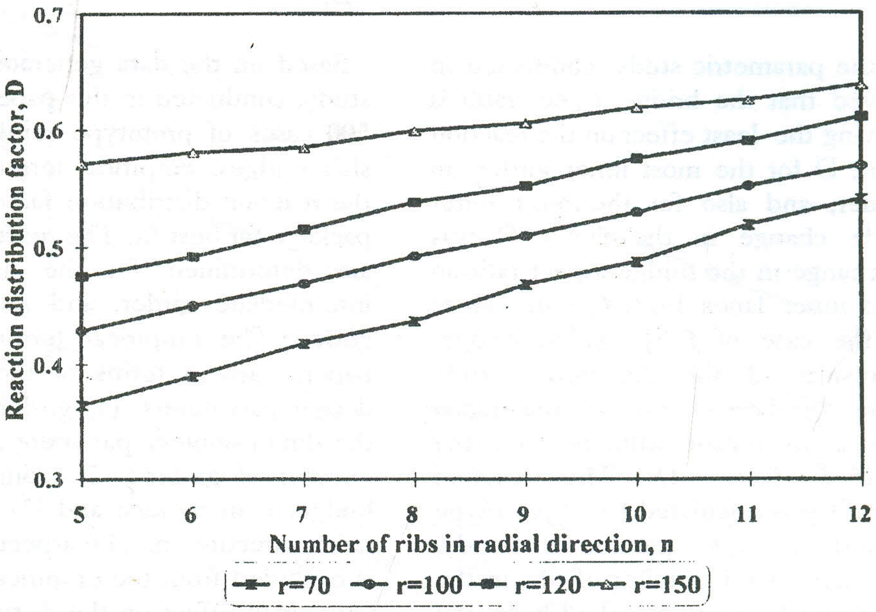


Figure 9. Effect of number of ribs in the radial direction on the reaction distribution factor, D, for the most inner girder of a two lane bridge (results from the current parametric study).

Illustrative example

Consider a three -lane horizontally curved reinforced concrete waffle slab bridge with five longitudinal girders. The bridge details are as follows: span of the bridge at center line = 24.0m; total bridge width = 11.0m; central angle of the bridge = 25°; radius of curvature of the bridge at center line = 55.0m; girder spacing = 2.25m; and number of ribs in the radial direction = 10 ribs. It is required to calculate the maximum reactions for the different girders in the bridge. The dimensionless parameter N from (2) equals (3/5) = 0.4. Applying a line of wheel loads of the truck specified by the Egyptian code for loads [10] on a simply supported girder the reaction R for the most inner girder is 27.3 ton, for an intermediate girder is 27.6 ton, and for the most outer girder is 27.7 ton. The reaction distribution factor, D, for the most inner girder using equation (3) is 0.73; therefore R_{max} using equation (1) equals 0.73 x 27.3 = 19.9 ton. The reaction distribution factor ,D, for an intermediate girder using equation (4) is 1.87; therefore R_{max} using equation (1) equals 1.87 x 27.6 = 51.6 ton. The reaction distribution factor, D, for the most outer girder using equation (5) is 1.15; therefore R_{max} using equation (1) equals 1.15 x 27.7 = 31.9 ton. The values of the reactions estimated above are used to design the longitudinal girders of the bridge for shear. These values are also used to design the abutments, and the bearing plates of the bridge. It is important to observe the large differences in reactions carried by the various girders in the bridge. Such large differences can be quite critical in the design of curved bridges in regions of high seismicity.

CONCLUSIONS

An extensive investigation is conducted to determine the effect of several variables on the distribution of reactions between girders in reinforced concrete curved waffle slab bridges. The finite element model used in this paper is verified and substantiated using experimental results found in the literature. Empirical formulas are deduced in this paper to calculate the reaction distribution factors for the different girders of the bridge. Based on this study, the following conclusions can be made:

1- The good agreement between the experimental and the theoretical results supports the reliability of using the finite element modelling to calculate the reactions of a reinforced concrete waffle slab bridge under concentrated truck wheel loads.
2- The spacing of longitudinal girders is one of the most important factors affecting the reaction distribution factor for different girders of the bridge. An increase in the suggested dimensionless parameter N increases the reaction distribution factor for all radii of curvature.
3- The degree of curvature of the bridge has a significant influence on the distribution of reactions between longitudinal girders. An increase in the radius of curvature of the bridge increases the distribution factor significantly for the most inner girder and decreases it for an intermediate girder and for the most outer girder. The large differences in reactions carried by the various girders in the bridge can be quite critical in the design of curved bridges in regions of high seismicity.
4- The number of ribs in the radial direction has a significant influence on the distribution factor. Increasing the number of ribs in the radial direction generally leads to a more uniform reaction distribution between longitudinal girders.
5- The bridge aspect ratio is the parameter having the least effect on the reaction distribution factor for all girders in the bridge. Very little change in the factor was observed with a change in the bridge aspect ratio in all loading cases.

REFERENCES

[1] J.B. Kennedy and M. Ghobrial, "Waffle slab construction for concrete bridge decks phase I", report 11-26, Industrial research institute of the University of Windsor, Windsor, Ontario, Canada, 1980.

[2] J.B. Kennedy and I.S. El-sebakhy, "Prestressed waffle slab bridges - Elastic behaviour", Journal of the structural division, ASCE, v. 106, No. St12, proc. Paper 15924, pp 2443-2462, 1980.

[3] J.B. Kennedy and I.S. El-Sebakhy, "Waffle slab concrete bridges: Ultimate behaviour", Journal of the structural division. ASCE, vol. 108, No. St6, pp. 1285-1301, 1982.

[4] J.B. Kennedy and B. Bakht, "Feasibility of waffle slabs for bridges" Canadian Journal of Civil Engineering, vol.- 10, No. 4, pp. 627-638, 1983.

[5] J.B. Kennedy, "Prestressed waffle slab bridges" Transportation research record 1118, pp. 29-38, 1987.

[6] R. Meli and M. rodrigvez, "Seismic behaviour of waffle - flat plate buildings", Concrete international and construction, No. 7, pp. 33-44, 1988.

[7] A.K. Verma and S.S. Dey, "Integrated Analysis of Curved Bridge Superstructures by variational finite difference method" Journal of computers and structures, vol., 38, No. 5/6, pp. 597-603, 1991.

[8] A. Aly, and J.B. Kennedy, Sept. "Rigidities of horizontally - curved flat structures' ASCE, Journal of structural division, 1993.

[9] A. Aly, "Behaviour of curved reinforced and prestressed concrete waffle slab bridges' Ph.D. thesis, Department of Civil Engineering, University of Windsor, Windsor, Ontario, Canada, 1994.

[10] Ministry of housing, research center for housing, construction, and planning, "Egyptian code for calculation of loads and forces in structures and buildings., pp 76, Cairo, Egypt, 1994.

[11] EMRC (NISA), Engineering Mechanics Research Corporation, center for engineering and computer technology. P.o. box 696,Troy, MI. 48099, USA, 1995.

[12] American Association of State Highway and Transportation Officials, "AASHTO standard specification of highway bridges", 15th Ed., Washington, D.C., 1994.