FRICTION HEAD LOSS IN A PIPE WITH UNIFORM LATERAL INFLOW OR OUTFLOW

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ABSTRACT

Herein, the friction head loss at a general section of a pipe with uniform lateral outflow or inflow is investigated. The accurate determination of this quantity, in case of turbulent flow in general, is an outcome of a complicated function integration. In this paper, a numerical integration, based on simplifying the complicated function, is presented. A simple polynomial relationship of the second order is presented to estimate the head loss in case of not only the outflow but also the inflow. Different from any other attempts done before, the new relationship is based on introducing three coefficients which can be estimated by using tables and charts as well. The coefficients are function of the inlet Reynold's number and the pipe roughness ratio. An evaluation analysis is done to examine the accuracy of applying the new relationship. It is shown that the relationship is not only simple but also accurate and can be applied in both cases of lateral outflow and inflow.

Keywords: Friction headloss, A pipe with lateral inflow, A pipe with lateral outflow.

Notation

- q uniform rate of outflow or inflow through the pipe
- Q_o discharge at pipe inlet
- Q_x discharge at distance x measured from the pipe inlet
- x distance measured from the pipe inlet
- X dimensionless parameter equals to qx/Q_o
- L total pipe length
- e roughness height
- D pipe diameter
- R_o Reynold's number at the inlet
- R_x Reynold's number at distance x
- f friction factor
- f_x friction factor at distance x
- a pipe cross-section area
- A a constant defined in the paper
- B a constant defined in the paper
- C a constant defined in the paper
- α a constant defined in a formula presented herein
- β a constant defined in a formula presented herein
- h_l friction headloss
- V mean velocity in the pipe
- V_x mean velocity at distance x through the pipe
- g gravity acceleration
- ν kinematic viscosity of water
- π equals 3.14
- K_1 a parameter defined in the paper
- K_2 a parameter defined in the paper
- Y a complicated function defined in the paper
- H a parameter defined in the paper

INTRODUCTION

The problem of a pipe with lateral outflow has several applications in civil engineering such as in case of drip and sprinkler irrigation. Also, the problem of a pipe with lateral inflow is very important in case of subsurface drainage system. In a pipe with lateral inflow, the upstream flow may be laminar with a friction factor dependent only on Reynold's number. At certain distance downstream, the flow may become turbulent in general at which the friction factor becomes dependent on both the Reynold's number and the pipe roughness ratio. Far downstream, the flow may become fully turbulent with a friction factor dependent only on the pipe roughness ratio. On the other hand, in case of a pipe with lateral outflow, the flow may start fully turbulent passing with the turbulent in general and end laminar. Generally, estimating the friction head loss in either the laminar or fully turbulent is easy task.

Hathoot et al. (1991) presented two different relationships, based on a simple integration, to estimate the head loss in a pipe with only lateral outflow to cover both the laminar and fully turbulent flow. In case of turbulent in general, they presented five charts to estimate the friction head loss. The curves were smooth in only two charts, however, in the rest, drops and changes in slopes were found which led to two different values of the head loss at the same location. Commenting on what was happening, the authors explained that the error might be attributed to the instructions cycles as values of the Reynold's number and/or the pipe roughness ratio became beyond the range of validity of the equation used to estimate the friction factor. Therefore, using the charts proposed by Hathoot et al (1991) may lead to inaccurate values of the head loss emphasizing the need of accurate relationships or charts which can be confidently applied to estimate the friction head loss in case of not only the lateral inflow but the outflow as well.

LATERAL OUTFLOW AND INFLOW

Figure (1a) shows a pipe with lateral outflow. However, Figure (1b) shows a pipe with lateral inflow. If it is assumed that the rate of outflow or inflow is constant, q, at distance x far downstream, the discharge in both pipes are (Q_o - xq)and (Q_o + xq), respectively.

FRICTION FACTOR

It is well known that Moody diagram is of general use in determining the friction factor coefficient, f, in pipes. It is very useful in both cases of laminar and turbulent flow. The latter, is being subdivided to three zones, the smooth, transitional and rough. For purpose of numerical calculations, the engineer is always looking for a relationship to estimate f.

Swamee and Jain (1976) presented a coefficient of friction equation that covers a significant portion of the general turbulent zone in the Moody diagram, $5000 < R < 10^8$ and $10^{-6} < e/D < 10^{-2}$, where R is the Reynold's number and e is the absolute roughness of the pipe. The explicit Eq. is

$$f=\frac{1.325}{[\ln(\frac{e}{3.7D}+\frac{5.74}{R^{0.9}})]^2} \tag{1}$$

Fully turbulent flow, $e/D > 10^{-2}$; $R > 10^8$, may occur either at the beginning of the pipe with lateral outflow or close to the end of the pipe with lateral inflow. In case of fully turbulent flow, the following Eq. can be used

$$f=\frac{1.0}{[1.14 - 0.87\ \ln(\frac{e}{D})]^2} \tag{2}$$

For the pipe with outflow, far downstream, the flow may be laminar. On the other hand, for the pipe with lateral inflow, close to beginning, the flow may start laminar. In this case f can be estimated according to the flowing Eq.

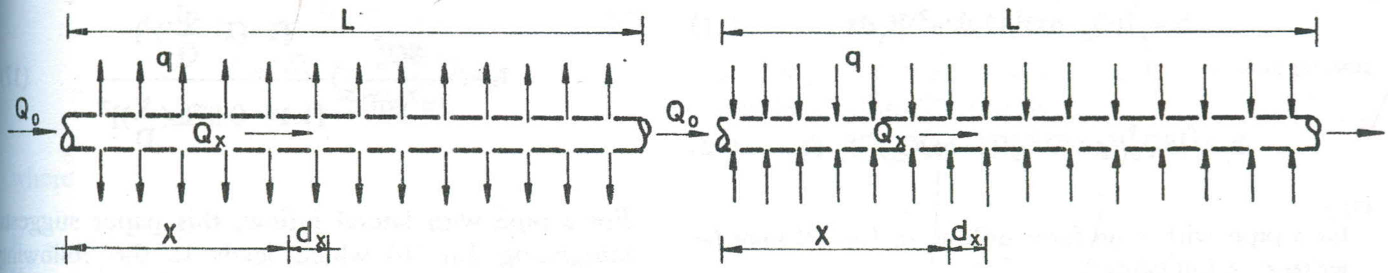
$$f = \frac{64}{R} \tag{3}$$

Churchill (1977) presented the following equation:

$$f=8[(\frac{8}{R})^{12}+\frac{1}{(\alpha+\beta)^{1.5}}]^{1/12} \tag{4}$$

to estimate f equally well for both laminar and turbulent flow conditions over all Reynold's numbers, and is good, universal equation for both smooth and rough irrigation pipes (Allen 1989)where α and β can be estimated as following:

$$\alpha = \left[2.457\ \ln\left[\frac{1}{(\frac{7}{R})^{0.9}+0.27(\frac{e}{D})}\right]\right]^{16} \tag{5}$$



(a) a pipe with lateral outflow

(b) a pipe with lateral inflow

Figure 1.

$$\beta = [\frac{37{,}530}{R}]^{16} \tag{6}$$

In computing f, α and β, double precision should be used since these factors are very sensitive to round-off error (Scaloppi and Allen (1993)).

In the following sections, the friction headloss through a pipe with either lateral inflow or outflow is estimated. Relationships, for the case of laminar and fully turbulent in a pipe with outflow, are presented as derived by Hathoot et al (1991). However, for the case of inflow, similar relationships are derived in this paper. For turbulent flow in general, a relationship for both cases of outflow and inflow is derived herein. Moreover charts are presented to prove that those presented by Hathoot et al (1991) are not accurate. Eq. 1, Eq. 2, and Eq. 3 are used to estimate f in the cases of turbulent in general, fully turbulent and laminar flow, respectively. Eq. (4), however, did not give good results in numerical integration.

FRICTION HEADLOSS IN A PIPE WITH EITHER LATERAL OUTFLOW OR INFLOW

Generally, the friction head loss can be estimated by using Darcy-Weisbach Equation as following:

$$h_l = f(\frac{l}{D})(\frac{V^2}{2g}) \tag{7}$$

where, l is the length of pipe under consideration, f is the constant friction factor for this part of the pipe and V is the average velocity. For a small longitudinal part of pipe of length dx, located at distance x from the beginning, Eq. 7 may take the following form:

$$dh_l = f_x(\frac{dx}{D})(\frac{V_x^2}{2g}) \tag{8}$$

Also, Eq. 8 may be written as following:

$$dh_l = f_x(\frac{dx}{D})(\frac{Q_x^2}{2ga^2}) \tag{9}$$

where Q_x is the discharge at distance x and a is the cross-sectional area. For the pipe with a uniform outflow, q, Eq. 9 can be

$$dh_l = f_x(\frac{dx}{D})(\frac{(Q_o - qx)^2}{2ga^2}) \tag{10}$$

The total friction headloss may be estimated by integrating Eq. 10 as following:

$$h_1=\int[(Q_o-qx)^2/(2gDa^2)]f_x dx \qquad (11)$$

$$h_1=\int[8Q_o^2/(g\pi^2D^5)][(1-qx/Q_o)^2]f_x\, dx \qquad (12)$$

for a pipe with a uniform inflow, q, Eq. 10 may be written as following:

$$dh_1=f_x(\frac{dx}{D})(\frac{(Q_o+qx)^2}{2ga^2}) \qquad (13)$$

The total friction headloss may be estimated by integrating Eq. 13 as following:

$$h_1=\int[8Q_o^2/(g\pi^2D^5)][(1+qx/Q_o)^2]f_x\, dx \qquad (14)$$

LAMINAR FLOW

Hathoot et al (1991) integrated Eq. 12. f_x was substituted from Eq. 3. They proved that the headloss in a pipe with lateral outflow, laminar, can be given by the following equation:

$$h_1=[\frac{64\nu LQ_o}{\pi gD^4}][2-\frac{qL}{Q_o}] \qquad (15)$$

For a pipe with lateral inflow, this paper suggests integrating Eq. 14 which leads to the following equation:

$$h_1=[\frac{64\nu LQ_o}{\pi gD^4}][2+\frac{qL}{Q_o}] \qquad (16)$$

FULLY TURBULENT FLOW

Hathoot et al (1991) integrated Eq. 12. f_x was substituted from Eq. 2. They proved that the headloss in a pipe with lateral outflow, fully turbulent, can be given as following:

$$h_1=(\frac{8Q_o^3}{3\pi^2gqD^5})\frac{[1-(1-\frac{qL}{Q_o})^3]}{[1.14-0.87\ln(\frac{e}{D})]^2} \qquad (17)$$

For a pipe with lateral inflow, this paper suggests integrating Eq. 14 which leads to the following equation:

$$h_1=(\frac{8Q_o^3}{3\pi^2gqD^5})\frac{[(1+\frac{qL}{Q_o})^3\ -1]}{[1.14-0.87\ln(\frac{e}{D})]^2} \qquad (18)$$

TURBULENT FLOW IN GENERAL

Lateral Outflow

At a general section in a pipe with lateral outflow, turbulent in general, the friction factor, f_x, can be given by Eq. 1 which can be rewritten in the following form:

$$f_x=\frac{1.325}{[\ln(\frac{e}{3.7D}+\frac{5.74}{R_x^{0.9}})]^2} \qquad (19)$$

where

$$R_x=\frac{v_xD}{\nu}=\frac{Q_xD}{a\,\nu}=\frac{(Q_o-qx)D}{a\,\nu}=(1-\frac{qx}{Q_o})\frac{4Q_o}{(\pi D\nu)} \qquad (20)$$

substituting in Eq. 19 and rearrange it as following:

$$f_x=\frac{1.325}{[\ln(\frac{e}{3.7D}+\frac{5.74(\pi D\nu)^{0.9}}{[4Q_o(1-\frac{qx}{Q_o})]^{0.9}})]^2} \qquad (21)$$

looking for an easy form, Eq. 21 can be written as following:

$$f_x=\frac{1.325}{[Ln(K_1+\frac{K_2}{(1-X)^{0.9}})]^2} \qquad (22)$$

where

$K_1 = e/(3.7D)$
$K_2 = 5.74[(\pi D\nu)/(4Q_0)]^{0.9} = 5.74/R_o^{0.9}$
$X = qx/Q_o$
substituting for f_x, Eq. 12 may take the following form:

$$h_1=H\ [{}_o\int^{qx/Q_o}\frac{(1-X)^2\ dX}{[Ln(K_1+\frac{K_2}{(1-X)^{0.9}})]^2}] \qquad (23)$$

where

$$H = \frac{(10.6Q_0^3)}{(g\pi^2qD^5)} \qquad (24)$$

Lateral inflow

Similar to Eq. 23 for outflow, the following equation can be derived for inflow

$$h_1=H\ [{}_o\int^{qx/Q_o}\frac{(1+X)^2\ dX}{[Ln(K_1+\frac{K_2}{(1+X)^{0.9}})]^2}] \qquad (25)$$

It is difficult to have a quick form for the integration of Eq. 23 or Eq. 25. However, it may be easier to solve the problem numerically.

NUMERICAL INTEGRATION

To solve such an integration, Eq. 23 for outflow, it is suggested to name the complicated integration function as following:

$$Y=\frac{(1-X)^2}{[Ln(K_1+\frac{K_2}{(1-X)^{0.9}})]^2} \qquad (26)$$

similarly, for inflow, the complicated integration function of Eq. 25 can be

$$Y=\frac{(1+X)^2}{[Ln(K_1+\frac{K_2}{(1+X)^{0.9}})]^2} \qquad (27)$$

X, Y curves for both the outflow and inflow were plotted at different values of K_1 and K_2 or in other words at different values of e/D and R_o. Fitting curves has done showing that the polynomial fitting, Eq. 28, has an excellent agreement with the data of X and Y as following:

$$Y = AX^2 + BX + C \qquad (28)$$

the form of Eq. 28 can represent the complicated function integration in both cases of outflow and inflow, where values of A, B and C were determined by fitting and tabulated in Table (1). All values of A, for outflow were plotted against values of e/D at different values of R_o as shown in Figure (2). The same was done for all values of B and C as shown in Figure (3) and Figure (4). For inflow, curves of A, B and C values are shown in Figure (5), Figure (6) and Figure (7), respectively. For both outflow and inflow, values of A were independent of e/D up to certain range and after that value of A increases as e/D increases, provided that R_o is constant. The mentioned range increases as R_o decreases. Values of A were almost constant at e/D=0.01 at different values of R_o. The trends of B and C curves are also shown in Figure (3) and Figure (4), respectively.

Now, the integration of either Eq. 23 or Eq. 25 can be written as following:

$$h_1=H\ [{}_{X=o}\int^{X=qx/Q_o}(AX^2+BX+C)dX] \qquad (29)$$

Table 1. Values of A,B and C at different values of e/D and R_o for both inflow and outflow.

INFLOW					OUTFLOW				
Ro	e/D	A	B	C	Ro	e/D	A	B	C
10^4	10^-2	0.0289	0.0627	0.0332	10^4	10^-2	0.0295	-0.0629	0.0332
	10^-3	0.0166	0.0436	0.0246		10^-3	0.0191	-0.0442	0.0247
	10^-4	0.0143	0.0409	0.0235		10^-4	0.0176	-0.0415	0.0236
	10^-5	0.0144	0.0407	0.0234		10^-5	0.0174	-0.0413	0.0234
	10^-6	0.014	0.0406	0.0233		10^-6	0.0174	-0.0412	0.0234
5*10^4	10^-2	0.0286	0.0585	0.0298	5*10^4	10^-2	0.0287	-0.0585	0.0298
	10^-3	0.0151	0.0341	0.0182		10^-3	0.0158	-0.0343	0.0183
	10^-4	0.011	0.0287	0.016		10^-4	0.0128	-0.0291	0.016
	10^-5	0.0104	0.0281	0.0157		10^-5	0.0125	-0.0284	0.0157
	10^-6	0.0103	0.028	0.0157		10^-6	0.0124	-0.0284	0.0157
10^5	10^-2	0.0286	0.0579	0.0292	10^5	10^-2	0.0287	-0.0579	0.0292
	10^-3	0.015	0.0321	0.0169		10^-3	0.0153	-0.0323	0.0169
	10^-4	0.0102	0.0254	0.0139		10^-4	0.0115	-0.0257	0.0139
	10^-5	0.0092	0.0244	0.0135		10^-5	0.011	-0.0247	0.0136
	10^-6	0.0091	0.0243	0.0135		10^-6	0.0109	-0.0246	0.0135
5*10^5	10^-2	0.0286	0.0574	0.0287	5*10^5	10^-2	0.0286	-0.0574	0.0287
	10^-3	0.0148	0.0302	0.0154		10^-3	0.0149	-0.0303	0.0154
	10^-4	0.0093	0.0206	0.0109		10^-4	0.0097	-0.0207	0.0109
	10^-5	0.0074	0.0184	0.01		10^-5	0.0084	-0.0186	0.01
	10^-6	0.0071	0.0181	0.0099		10^-6	0.0083	-0.0183	0.0099
10^6	10^-2	0.0286	0.0573	0.0287	10^6	10^-2	0.0286	-0.0573	0.0287
	10^-3	0.0148	0.03	0.0151		10^-3	0.0148	-0.03	0.0151
	10^-4	0.0091	0.0195	0.0102		10^-4	0.0094	-0.0196	0.0102
	10^-5	0.0069	0.0166	0.0089		10^-5	0.0077	-0.0168	0.009
	10^-6	0.0064	0.0162	0.0088		10^-6	0.0074	-0.0164	0.0088
5*10^6	10^-2	0.0286	0.0572	0.0286	5*10^6	10^-2	0.0286	-0.0572	0.0286
	10^-3	0.0148	0.0297	0.0149		10^-3	0.0148	-0.0297	0.0149
	10^-4	0.0091	0.0184	0.0094		10^-4	0.0091	-0.0185	0.0094
	10^-5	0.0062	0.0138	0.0073		10^-5	0.0066	-0.0139	0.0073
	10^-6	0.0053	0.0128	0.0068		10^-6	0.006	-0.0129	0.0069
10^7	10^-2	0.0286	0.0572	0.0286	10^7	10^-2	0.0286	-0.0572	0.0286
	10^-3	0.0148	0.0297	0.0149		10^-3	0.0148	-0.0297	0.0149
	10^-4	0.009	0.0183	0.0092		10^-4	0.0091	-0.0183	0.0092
	10^-5	0.0062	0.0132	0.0068		10^-5	0.0063	-0.0132	0.0068
	10^-6	0.005	0.0117	0.0062		10^-6	0.0055	-0.0118	0.0062
5*10^7	10^-2	0.0286	0.0572	0.0286	5*10^7	10^-2	0.0286	-0.0572	0.0286
	10^-3	0.0148	0.0296	0.0148		10^-3	0.0148	-0.0296	0.0148
	10^-4	0.009	0.0181	0.0091		10^-4	0.009	-0.0181	0.0091
	10^-5	0.0061	0.0124	0.0063		10^-5	0.0061	-0.0124	0.0063
	10^-6	0.0045	0.0099	0.0052		10^-6	0.0047	-0.01	0.0052

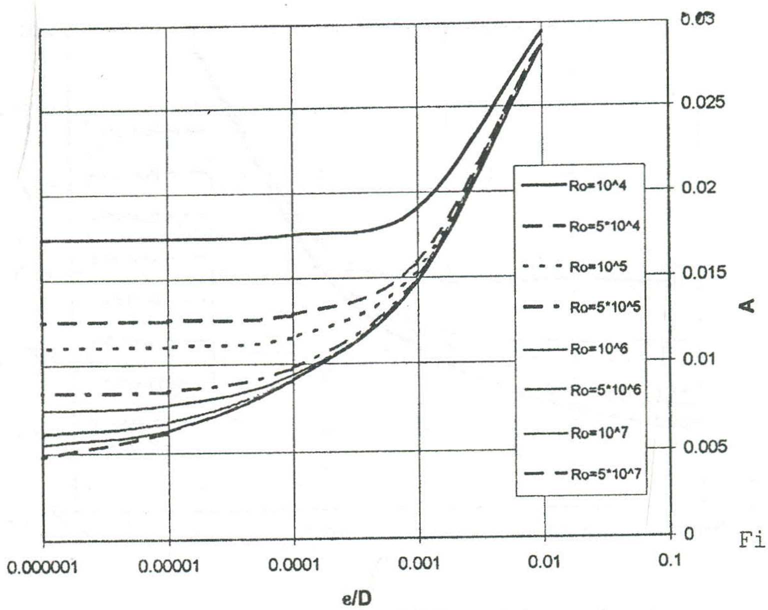


Fig. 2

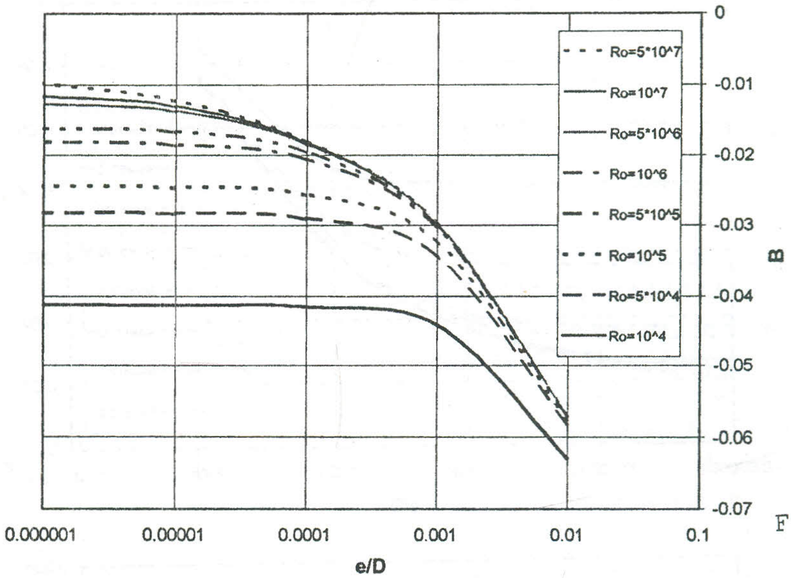


Fig. 3

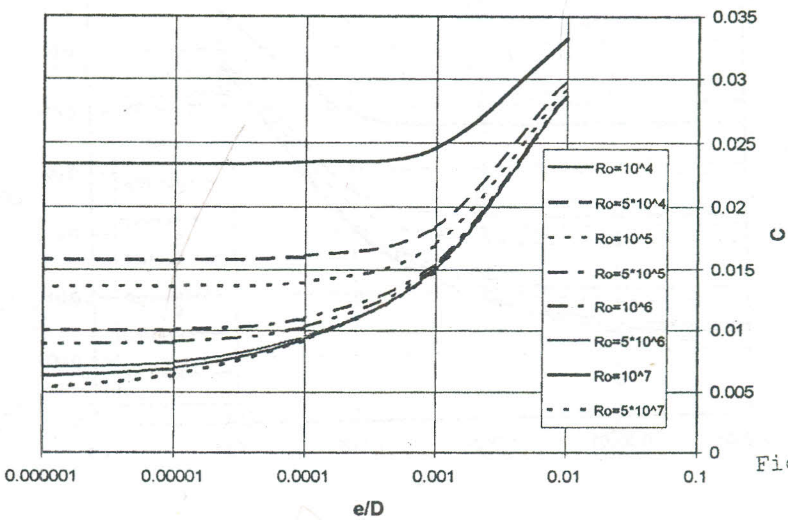


Fig. 4

the integration leads to the form

$$h_1 = H(\frac{A}{3}X^3 + \frac{B}{2}X^2 + CX) \qquad (30)$$

substituting for $X = xq/Q_o$ in Eq. 30 leads to

$$h_1 = H[\frac{A}{3}(\frac{qx}{Q_o})^3 + \frac{B}{2}(\frac{qx}{Q_o})^2 + C(\frac{qx}{Q_o})] \qquad (31)$$

Eq. 31 can be applied to estimate the headloss in both cases of inflow and outflow according to values of A, B and C which are given in Table (1). A nondimensional form of Eq. 31 can be written as following:

$$\frac{h_1}{H} = \frac{A}{3}(\frac{qx}{Q^o})^3 + \frac{B}{2}(\frac{qx}{Q_o})^2 + C(\frac{qx}{Q_o}) \qquad (32)$$

the relationship is graphically represented by Figure (8) for outflow and Figure (9) for inflow.

EVALUATION OF HEAD LOSS RELATIONSHIP

In this section, the accuracy of applying the headloss relationship, Eq. 31 is checked.

From Eq. 12, it can be concluded that

$$\frac{dh_1}{dx} = [8Q_o^2/(g\pi^2D^5)][(1-qx/Q_o)^2]f_x \qquad (33)$$

but $H = (10.6\ Q_0{}^3)/(g\pi^2qD^5)$, therefore

$$\frac{dh_1}{dx} = \frac{H}{1.325}(\frac{q}{Q_o})(1-qx/Q_o)^2f_x \qquad (34)$$

Also, from Eq. 31, it can be concluded that

$$\frac{dh_1}{dx} = H(\frac{q}{Q_o})[A(\frac{qx}{Q_o})^2 + B(\frac{qx}{Q_o}) + C] \qquad (35)$$

or in other words

$$\frac{dh_1}{dx} = H[A(\frac{q}{Q_o})^3x^2 + B(\frac{q}{Q_o})^2x + C(\frac{q}{Q_o})] \qquad (36)$$

Equating Eq. 34 and Eq. 36,leads to

$$f_x = 1.325[\frac{(AX^2+BX+C)}{(1-X)^2}] \qquad (37)$$

Similar to Eq. 37, derived for outflow, the following Equation can be derived for inflow

$$f_x = 1.325[\frac{(AX^2+BX+C)}{(1+X)^2}] \qquad (38)$$

Eq. 37 and Eq. 38 which are derived from the headloss relationship can be applied to estimate f_x in case of outflow and inflow, respectively. It is also well known that Eq. 1 can be applied to estimate f_x *in both cases of outflow and inflow according to* values of e/D and R_x. A small difference between values of f_x estimated either by applying Eq. 1 or Eq. 37 for outflow and Eq. 38 for inflow means that Eq. 31 is accurate. To check the accuracy of applying Eq. 31, for certain value of e/D and R_o, f_x can be estimated by two different methods as following:

1- according to the flow type and values of e/D & R_o, estimate values of A, B and C proposed in this paper in Table (1). Then apply either Eq. 37 for outflow or 38 for inflow to estimate f_x

2- At the same time apply Eq. 1 to estimate f_x again.

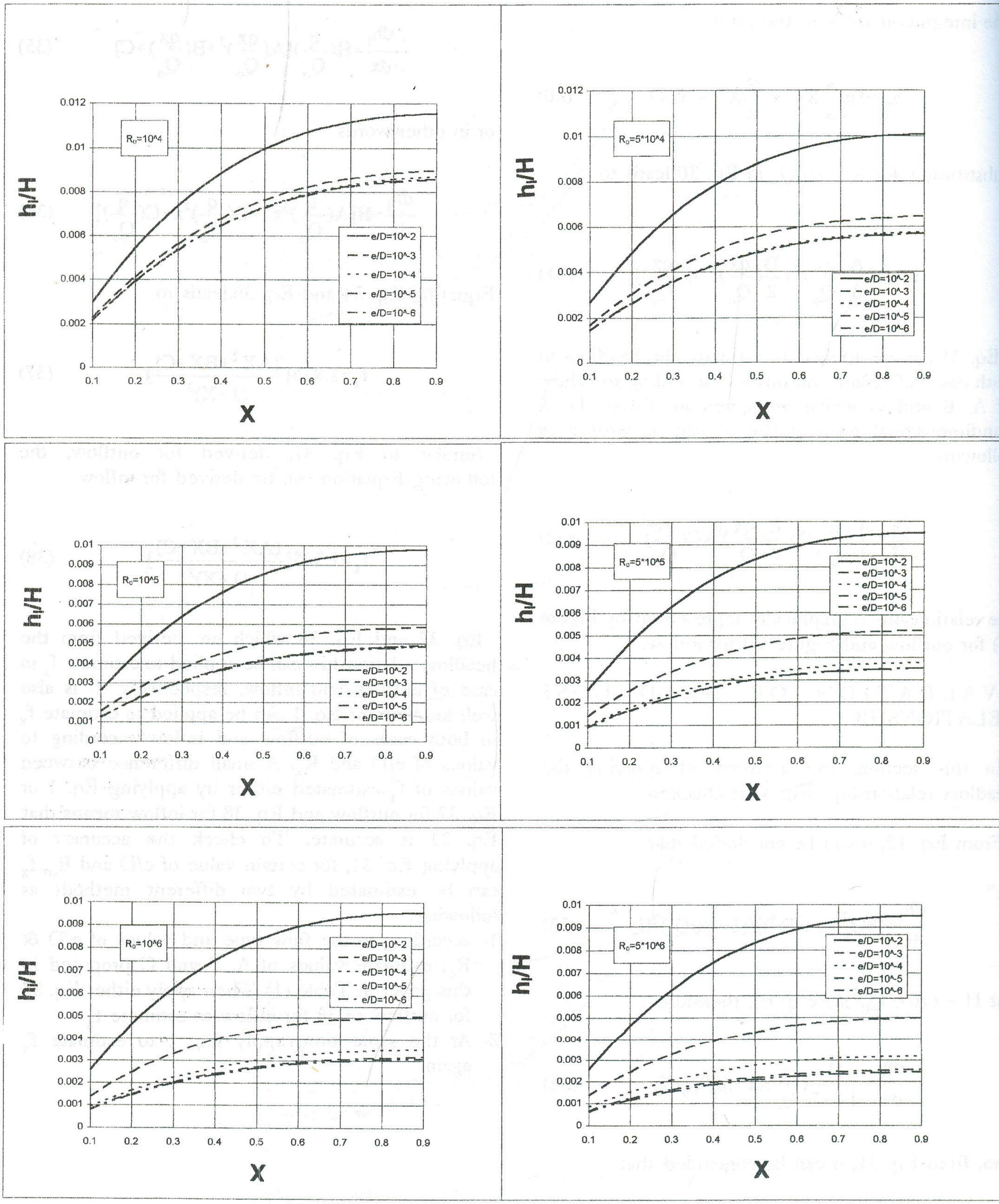


Fig. 8 Values of h_l/H versus X at different values of e/D and R_o (outflow)

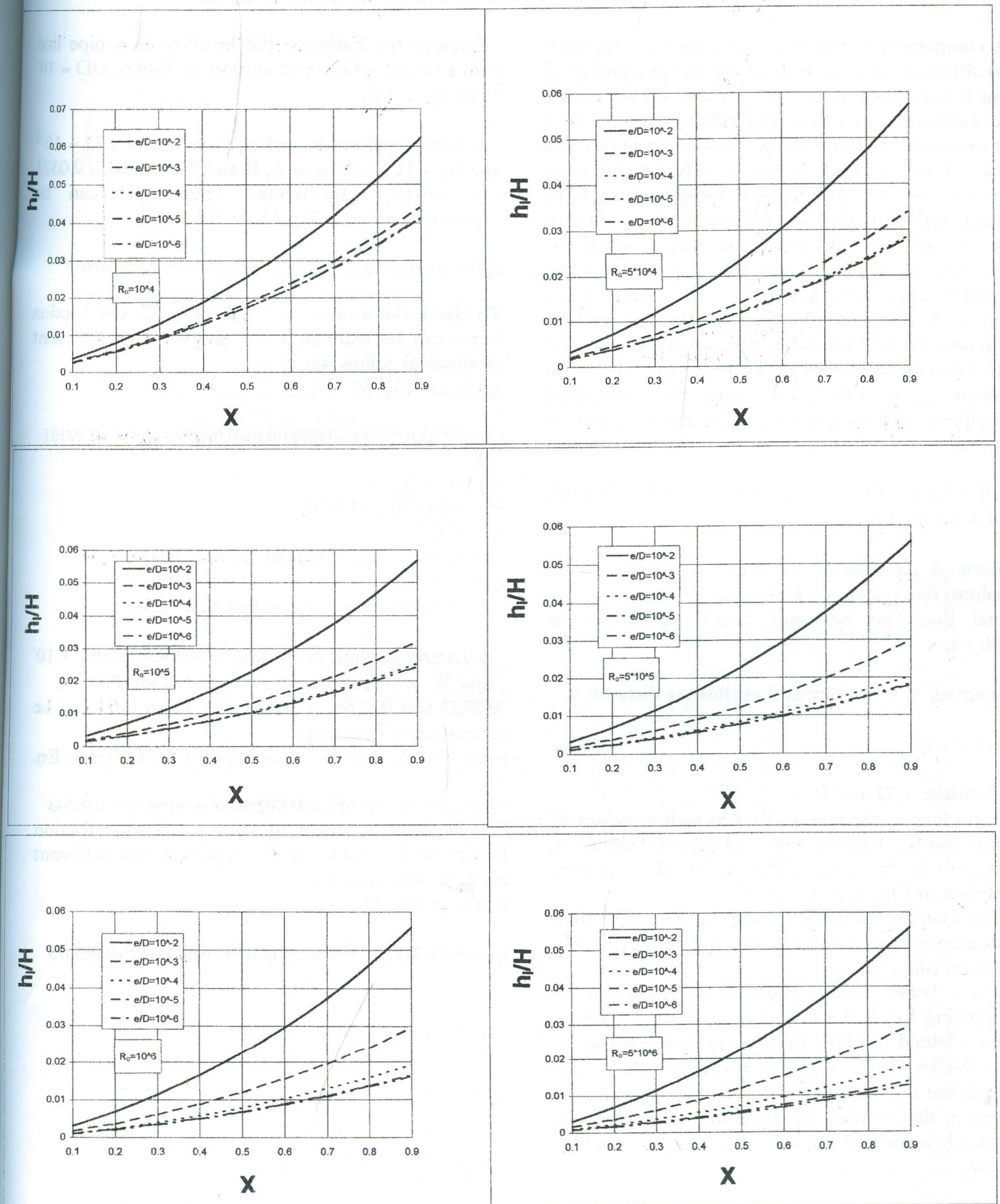


Fig. 9 Values of h_l/H versus X at different values of e/D and R_o (inflow)

A comparison between f_x estimated by applying two different methods is done and the percentage of error is estimated as shown in Table (2) for inflow and Table (3) for outflow. For inflow, the error does not exceed 0.5%. For outflow, the error is almost less than 5 % up to X=0.6, however, at X>0.6, the error increases. So, Eq. 31 proposed for estimating h_l in general turbulent flow can be accurately applied in case of inflow (X<1.0)without any restrictions. However, in case of outflow, Eq. 31 can be accurately applied up to $X \leq 0.6$. For values of X > 0.6 (i.g. X=0.9), the proposed relationship can be accurately applied following two steps, the first step is to estimate h_l at X=0.5 and the second step is to estimate h_l at x=0.4. Following the mentioned procedures will increase the accuracy as much as possible.

APPLYING THE PROPOSED HEADLOSS RELATIONSHIP

Given: A pipeline of diameter, D, discharge, Q_o, absolute roughness, e, kinematic viscosity, ν, and lateral discharge per unit length, q (inflow or outflow).

Required: h_l for inflow and outflow at distance x

Procedures

1- Estimate e/D and R_o.
2- according to the type of flow as well as values of e/D and R_o, estimate values of the coefficients, A, B, and C, by using either Table (1) or curves shown in Fig. 2, 3, 4, 5, 6, 7.
3- For a certain value of x, estimate $X=qx/Q_o$, then determine the friction headloss by applying the relationship, Eq. 31.
4- For a lateral inflow, there is no restriction on applying Eq. 31 for different X values. However, for a lateral outflow, there is no restriction as far as X<0.6. If X > 0.6, for example X = 0.9, estimate the head loss at X=0.5 then estimate the rest of the headloss at X=0.4 and determine the total headloss which will be so accurate in this case.

Example (1): Estimate the headloss in a pipe line with a lateral inflow and outflow of X=0.6, e/D = 10^{-3} and $R_o = 10^5$

1- *Lateral inflow*: According to values of e/D = 10^{-3} and $R_o = 10^5$, values of A, B and C are 0.015, 0.0321 and 0.0169, respectively. Then h_l/H can be estimated applying Eq. 32, as following:

$$h_l/H = (0.015/3)(0.6)^3+(0.0321/2)(0.6)^2+0.0169(0.6)=0.017$$

To check the accuracy of using Eq. 32, the friction factor can be estimated by applying two different methods as following:
applying Eq. 38

$$f_x = 1.325[0.015(0.6)^2 +0.0321(0.6)+0.0169]/(1+0.6)^2 =0.02151$$

applying Eq. 1
$R_x = (1+X)R_o = 1.6*10^5$
$e/D = 10^{-3}$
according to Eq. 1, similar friction factor, f_x is
$f_x = 0.021494$
It is clear that the error is less than 0.5%.

2- *Lateral outflow*: According to values of e/D = 10^{-3} and $R_o = 10^5$, values of A, B and C are 0.0153, -0.0323 and 0.0169, respectively. Then h_l/H can be estimated as following:
since X=0.6, there is no restriction on applying Eq. 32

$$h_l/H = (0.0153/3)(0.6)^3-(0.0323/2)(0.6)^2+0.0169(0.6)=0.00543$$

To check the accuracy, of using Eq. 32, the friction factor can be estimated by applying two different methods as following:
applying Eq. 37

$$f_x = 1.325[0.0153(0.6)^2-0.0323(0.6)+0.0169]/(1-0.6)^2=0.025076$$

Table (2)percentage of error between f_x estimated by two different methods for inflow

		X								
Ro	e/D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10000	0.01	0.06%	0.05%	0.04%	0.03%	0.02%	0.01%	0.00%	-0.01%	-0.02%
	0.001	0.11%	0.09%	0.10%	0.11%	0.12%	0.12%	0.12%	0.11%	0.09%
	0.0001	-0.06%	-0.06%	-0.04%	-0.01%	0.02%	0.03%	0.03%	0.02%	-0.01%
	0.00001	-0.18%	-0.23%	-0.26%	-0.28%	-0.31%	-0.35%	-0.41%	-0.48%	-0.57%
	0.000001	0.18%	0.14%	0.13%	0.14%	0.15%	0.15%	0.13%	0.11%	0.07%
50000	0.01	-0.07%	-0.05%	-0.03%	-0.02%	-0.01%	0.01%	0.01%	0.02%	0.03%
	0.001	0.17%	0.13%	0.11%	0.09%	0.08%	0.07%	0.06%	0.05%	0.05%
	0.0001	-0.21%	-0.18%	-0.13%	-0.08%	-0.04%	-0.01%	0.00%	0.00%	0.00%
	0.00001	-0.12%	-0.14%	-0.13%	-0.11%	-0.10%	-0.09%	-0.10%	-0.12%	-0.15%
	0.000001	-0.26%	-0.24%	-0.19%	-0.15%	-0.11%	-0.09%	-0.08%	-0.08%	-0.10%
100000	0.01	0.10%	0.09%	0.08%	0.07%	0.07%	0.07%	0.06%	0.06%	0.06%
	0.001	-0.20%	-0.16%	-0.13%	-0.10%	-0.09%	-0.08%	-0.07%	-0.07%	-0.07%
	0.0001	0.07%	0.04%	0.02%	0.02%	0.02%	0.01%	0.00%	-0.02%	-0.05%
	0.00001	0.09%	0.07%	0.09%	0.12%	0.14%	0.16%	0.16%	0.16%	0.14%
	0.000001	-0.18%	-0.16%	-0.11%	-0.06%	-0.02%	0.02%	0.03%	0.04%	0.02%
500000	0.01	0.13%	0.10%	0.08%	0.06%	0.05%	0.04%	0.03%	0.02%	0.01%
	0.001	-0.21%	-0.15%	-0.10%	-0.06%	-0.03%	0.00%	0.02%	0.04%	0.06%
	0.0001	0.10%	0.07%	0.05%	0.03%	0.02%	0.00%	-0.02%	-0.04%	-0.07%
	0.00001	-0.08%	-0.08%	-0.07%	-0.05%	-0.03%	-0.02%	-0.02%	-0.03%	-0.05%
	0.000001	-0.14%	-0.11%	-0.07%	-0.02%	0.02%	0.04%	0.06%	0.05%	0.04%
1000000	0.01	-0.07%	-0.06%	-0.06%	-0.05%	-0.05%	-0.04%	-0.04%	-0.04%	-0.04%
	0.001	0.04%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.0001	-0.04%	0.00%	0.04%	0.07%	0.10%	0.12%	0.13%	0.15%	0.16%
	0.00001	0.35%	0.27%	0.23%	0.20%	0.18%	0.15%	0.12%	0.09%	0.05%
	0.000001	-0.31%	-0.28%	-0.22%	-0.17%	-0.11%	-0.07%	-0.04%	-0.03%	-0.02%
5000000	0.01	0.05%	0.04%	0.04%	0.04%	0.03%	0.03%	0.03%	0.03%	0.02%
	0.001	-0.08%	-0.06%	-0.05%	-0.03%	-0.02%	-0.01%	0.00%	0.00%	0.01%
	0.0001	-0.34%	-0.26%	-0.21%	-0.17%	-0.15%	-0.13%	-0.12%	-0.12%	-0.11%
	0.00001	-0.32%	-0.24%	-0.16%	-0.10%	-0.04%	0.01%	0.05%	0.09%	0.11%
	0.000001	0.49%	0.39%	0.33%	0.29%	0.26%	0.23%	0.20%	0.17%	0.13%
10000000	0.01	0.02%	0.02%	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
	0.001	-0.28%	-0.25%	-0.22%	-0.20%	-0.18%	-0.16%	-0.14%	-0.13%	-0.12%
	0.0001	0.11%	0.09%	0.07%	0.07%	0.07%	0.07%	0.08%	0.08%	0.09%
	0.00001	0.35%	0.23%	0.14%	0.06%	0.00%	-0.06%	-0.10%	-0.15%	-0.19%
	0.000001	0.36%	0.28%	0.23%	0.19%	0.15%	0.11%	0.07%	0.03%	-0.02%
50000000	0.01	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%
	0.001	0.15%	0.15%	0.14%	0.14%	0.14%	0.13%	0.13%	0.13%	0.13%
	0.0001	-0.15%	-0.10%	-0.06%	-0.02%	0.01%	0.04%	0.06%	0.09%	0.10%
	0.00001	0.04%	0.05%	0.07%	0.07%	0.08%	0.08%	0.08%	0.08%	0.07%
	0.000001	-0.01%	0.03%	0.07%	0.10%	0.13%	0.15%	0.16%	0.17%	0.17%

Table (3)percentage of error between f_x estimated by two different methods for outflow

		X					
Ro	e/D	0.1	0.2	0.3	0.4	0.5	0.6
10000	0.01	0.17%	0.25%	0.33%	0.39%	0.44%	0.49%
	0.001	0.02%	0.21%	0.29%	0.24%	0.00%	-0.45%
	0.0001	-0.18%	-0.06%	-0.13%	-0.47%	-1.22%	-2.59%
	0.00001	0.28%	0.56%	0.72%	0.74%	0.57%	0.25%
	0.000001	0.18%	0.39%	0.45%	0.29%	-0.16%	-1.00%
50000	0.01	-0.12%	-0.16%	-0.23%	-0.34%	-0.51%	-0.83%
	0.001	-0.22%	-0.16%	-0.14%	-0.17%	-0.26%	-0.40%
	0.0001	0.07%	0.27%	0.38%	0.37%	0.21%	-0.06%
	0.00001	0.25%	0.43%	0.46%	0.26%	-0.27%	-1.31%
	0.000001	0.09%	0.32%	0.43%	0.38%	0.16%	-0.25%
100000	0.01	0.14%	0.17%	0.20%	0.22%	0.25%	0.25%
	0.001	-0.21%	-0.18%	-0.18%	-0.21%	-0.31%	-0.49%
	0.0001	0.48%	0.79%	1.08%	1.36%	1.68%	2.25%
	0.00001	-0.40%	-0.36%	-0.52%	-0.99%	-1.95%	-3.73%
	0.000001	0.13%	0.31%	0.36%	0.21%	-0.22%	-1.01%
500000	0.01	0.22%	0.31%	0.43%	0.63%	0.97%	1.61%
	0.001	-0.31%	-0.35%	-0.41%	-0.52%	-0.69%	-1.01%
	0.0001	0.27%	0.38%	0.46%	0.49%	0.45%	0.33%
	0.00001	0.25%	0.49%	0.68%	0.81%	0.92%	1.17%
	0.000001	0.11%	0.24%	0.22%	-0.02%	-0.60%	-1.71%
1000000	0.01	-0.09%	-0.10%	-0.12%	-0.14%	-0.18%	-0.24%
	0.001	0.12%	0.21%	0.34%	0.58%	1.02%	1.91%
	0.0001	-0.06%	-0.06%	-0.14%	-0.34%	-0.78%	-1.68%
	0.00001	-0.48%	-0.39%	-0.41%	-0.60%	-1.02%	-1.80%
	0.000001	0.01%	0.27%	0.48%	0.67%	0.89%	1.39%
5000000	0.01	0.06%	0.07%	0.08%	0.09%	0.11%	0.14%
	0.001	-0.13%	-0.16%	-0.21%	-0.27%	-0.36%	-0.50%
	0.0001	-0.49%	-0.55%	-0.63%	-0.76%	-0.99%	-1.38%
	0.00001	-0.36%	-0.40%	-0.56%	-0.92%	-1.65%	-3.08%
	0.000001	-0.72%	-0.79%	-1.07%	-1.69%	-2.88%	-5.13%
10000000	0.01	0.03%	0.03%	0.04%	0.04%	0.05%	0.07%
	0.001	-0.37%	-0.43%	-0.51%	-0.62%	-0.77%	-1.00%
	0.0001	0.22%	0.29%	0.38%	0.49%	0.62%	0.78%
	0.00001	0.69%	0.93%	1.24%	1.67%	2.36%	3.63%
	0.000001	0.81%	1.13%	1.45%	1.76%	2.13%	2.74%
50000000	0.01	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
	0.001	0.16%	0.17%	0.18%	0.19%	0.21%	0.23%
	0.0001	-0.29%	-0.38%	-0.51%	-0.67%	-0.91%	-1.27%
	0.00001	-0.03%	-0.09%	-0.18%	-0.32%	-0.56%	-0.99%
	0.000001	0.12%	0.30%	0.53%	0.85%	1.40%	2.55%

applying Eq. 1
$R_x = (1-X)R_o = 0.4*10^5$
$e/D = 10^{-3}$

according to Eq. 1, similar friction factor, f_x is
$f_x = 0.02495$
It is clear that the error is less than 0.5%.

Hathoot et al. (1991): According to Hathoot et al (1991), the first step is to estimate the parameter $F(E, R_x)$ by using a curve as shown in Figure (10). The parameter is function of R_x as well as e/D. The next step is to estimate the ratio ,h_l/H which is function of $F(E,R_x)$ and X. For example Figure (11) shows the curve used to estimate h_L/H for values of $0.55 < X < 0.75$. Also, Figure (12) shows the curve used to estimate h_L/H for values of $0.80 < X < 0.95$.

To solve example (1) by using the charts of Hathoot et al.(1991), for $R_x = (1-X)R_o = 0.4*10^5 = 4*10^4$, and for $e/D = 10^{-3}$, So $F(E, R_x) = 6*10^{-4}$ by using Figure (7). Using Figure (8), $h_L/H = 47*10^{-4}$ which is different than that estimated by using the proposed curves presented herein. If the value of $h_L/H = 47*10^{-4}$ is used to estimate the friction factor, f_x, it will be different than that given by Eq.1 Moreover, the curve shown in Figure (8) proposed by Hathoot et al (1991) is discontinuous between $2*10^{-4}$ and $3*10^{-4}$. Similarly, the curve shown in Figure (9) is discontinuous between the same range. This leads to errors in estimating h_l.

Example (2): Estimate the headloss in a pipe line with a lateral inflow and outflow of $X=0.80$, $e/D=10^{-3}$ and $R_o = 10^5$

1- *Lateral inflow*: According to values of $e/D = 10^{-3}$ and $R_o = 10^5$, values of A, B and C are 0.015, 0.0321 and 0.0169, respectively. Then h_l/H can be estimated by applying Eq. 32, as following:

$$h_l/H = (0.015/3)(0.80)^3 + (0.0321/2)(0.80)^2 + 0.0169(0.80) = 0.0348$$

To check the accuracy of using Eq. 32, the friction factor can be estimated as following:
applying Eq. 38

$$f_x = 1.325[0.015(0.80)^2 + 0.0321(0.80) + 0.0169]/(1+0.80)^2 = 0.02123$$

applying Eq. 1
$R_x = (1+X)R_o = 1.80*10^5$
$e/D = 10^{-3}$
according to Eq. 1, similar friction factor, f_x is
$f_x = 0.02122$
It is clear that the difference is so small.

2- *Lateral outflow*: To estimate an accurate values of f_x and consequently h_L/H, divide X to two values; the first X=0.5 and the second X=0.3
For X=0.5, hL/H = 0.005 and then estimate A, B, and C for values of $R_x = R_o(1-X) = 0.5*10^5$
A=0.0158, B=-0.0343, C=0.0183
$$h_L/H = (0.0158/3)(0.30)^3 - (0.0343/2)(0.30)^2 + 0.0183(0.30) = 0.004$$
so the total headloss, $h_l/H = 0.005 + 0.004 = 0.009$
forX=0.30,
$$f_x = 1.325[0.0158(0.3)^2 - 0.0343(0.3) + 0.0183]/(1-0.3)^2 = 0.0255$$

Applying Eq. 1
$R_x = (1-X)R_o = (1-0.8)*10^5 = 0.2*10^5$
$e/D = 10^{-3}$
according to Eq. 1, similar friction factor, f_x is
$f_x = 0.028$
the difference is not so bad.

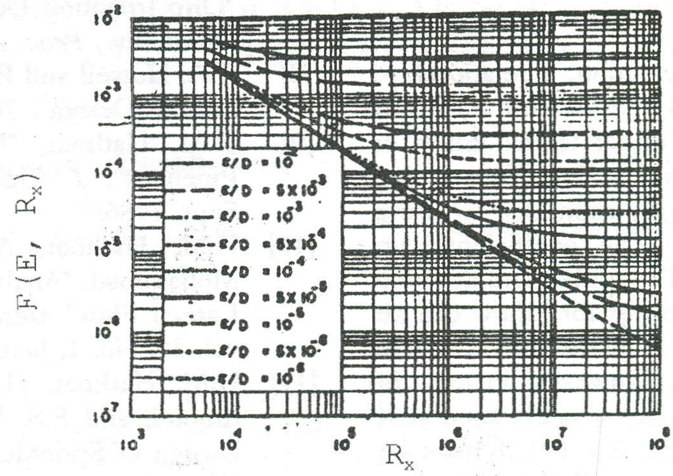


Figure 10. Chart for calculating F (E,R_x) by Hathoot et al (1991)

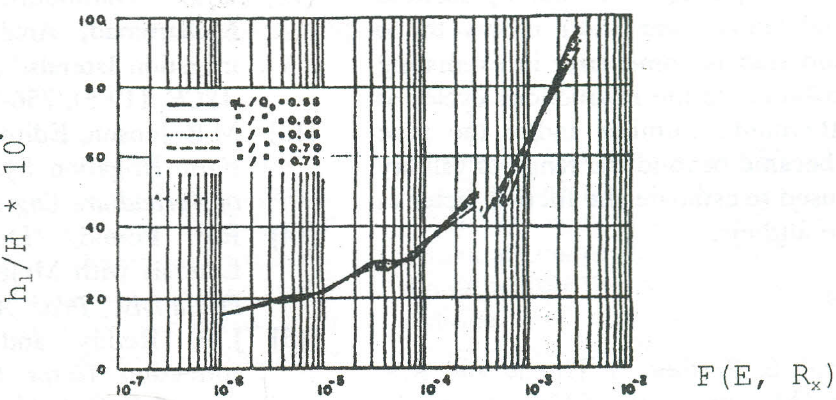


Figure 11. h_1/H versus F (E,R_x) at X = 0.55 to 0.75 by Hathoot et al (1991)

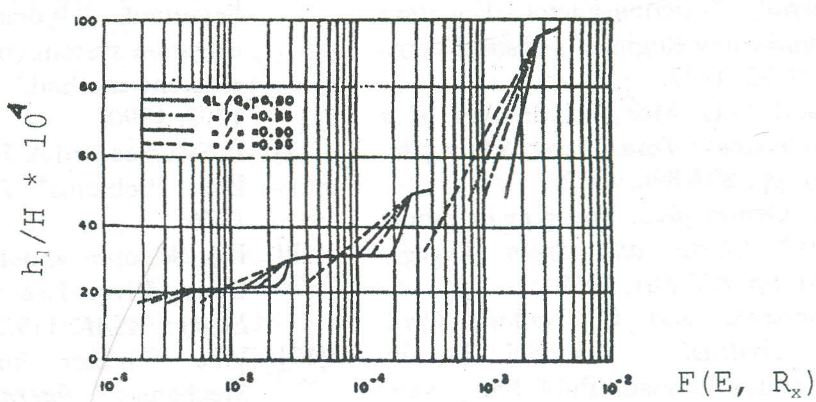


Figure 12. h_1/H versus F (E,R_x) at X = 0.8 to 0.95 by Hathoot et al (1991)

CONCLUSION

Based on a numerical integration, a relationship, Eq. 30 or Eq. 31 is presented to estimate the friction head loss through pipes of either lateral inflow or outflow. The relationship is based on introducing three coefficients which can be estimated by using tables and charts as well. The coefficients are function of the inlet Reynold's number and the pipe roughness ratio. The relationship for inflow can be accurately used to estimate h_l at distance x where the dimensionless $X < 1.0$. However, in case of outflow, the relationship can be accrately used at $X \leq 0.6$. At $X > 0.6$, for example $X = 0.9$, h_l must be estimated following two steps. The first is estimating h_l at $X = 0.6$ or 0.5 and the second is estimating h_l at $X = 0.3$ or 0.4. Comparing with charts presented by Hathoot et al (1991), the latter shown to be discontinuous and lead to some error in estimating h_l. That was attributed to the instructions cycles as values of the Reynold's number and/or the pipe roughness ratio became beyond the range of validity of the equation used to estimate the friction factor as explained by the authers.

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