NEW APPLICATIONS OF SHELL THEORY TO SHIP STRUCTURES

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ABSTRACT

Many applications of the theory of open cylindrical shells have been demonstrated in various literature. However, these applications have already been established in some simple problems of ship structures by generating a circular plate to a flat one. The purpose of this paper is to show the potential of the theory of open cylindrical shells in demonstrating further complex applications of this generated theory to ship structures. In this paper, this theory is generalized to solve complicated problems of ship structures such as a box barge , a circular bilge plate and a midship section of a small boat. A computer program is developed to calculate the stresses, moments and resulting deflections in the structural members. The results are compared with those of other theoretical methods. The obtained results have shown good agreement with those methods.

Keywords: Shell theory, Cylindrical shell, Flat plate, Circular bilge, Ship structure.

Nomenclature

A	area of unit length of shell section.
E	Young's modulus.
I	second moment of area of unit length of shell thickness.
L	longitudinal shell length.
M_x	longitudinal moment acting along the transverse edge of shell section and is defined as moment per unit length of shell edge.
M_ϕ	transverse moment acting along the longitudinal edge of shell section and is defined as moment per unit length of shell edge.
$M_{x\phi}$, $M_{\phi x}$	torsional moments acting in the transverse and longitudinal section respectively and are expressed as moments per unit length of the shell.
N_x	normal force acting on the transverse section of the shell and is defined as force per unit length of the shell edge.
N_ϕ	normal force acting on the longitudinal section of the shell and is defined as force per unit length of the shell edge.
$N_{x\phi}$, $N_{\phi x}$	torsional moments acting in the transverse and longitudinal sections respectively , and are expressed as moment per unit length of the shell.
n	order of Fourier term.
R	shell radius.
t	shell thickness.
u	displacement parallel to x-axis.
v	displacement parallel to y-axis.
w	displacement parallel to z-axis.
X,Y,Z	external applied forces and are expressed as forces per unit area of the shell surface.
x,y,z	coordinate axes.
υ	Poisson's ratio.
ϕ	half angle in degree.
φ	pitch angle in degree.

1. INTRODUCTION

The purpose of this paper is to demonstrate the high capability of the open cylindrical shell theory to analyze different components of ship structures.

The classical analysis of plate structures subjected to in-plane and lateral loads may be performed with two different methods. In the first method of analysis, an equilibrium equation is generated as a function of in-plane forces and transverse moments.

While in the second method, the membrane stresses are superimposed to the flexural stresses that have developed due to pure bending of plates.

The first method of analysis has been fully explained by Flugge, the calculations are straightforward and are discussed in detail in [1]. The second method of analysis, which takes into account membrane and out-of plane stresses, leads to two separate compatibility equations. The solutions of these equations have to satisfy the prescribed boundary conditions. The solution procedure of the second method is similar to that applied in the theory of open cylindrical shells. The difference being that in the case of shells, the conditions of equilibrium of the shell element yield an intermediary equation to be established between the flexural and membrane conditions. As a result, the two separate equations of the plate theory defining such conditions are reduced to a single one called the shell theory compatibility equation. The shell compatibility equation is however much more amenable to the prescribed boundary conditions. Solving the compatibility equation, stresses and forces acting on the shell element may be determined.

In this paper, the shell theory is degenerated and applied to flat plate structures. Then the theory is generalized to solve complicated problems of ship structures such as a box barge , a circular bilge plate and a midship section of a small boat. A computer program is developed to calculate the stresses, moments and resulting deflections in the structural members. The results of analysis are compared with other theoretical methods. The results have shown good agreement with those methods.

2. THE THEORY OF OPEN CYLINDRICAL SHELLS

2.1 *Assumptions and Description of Shell Element*

In this section, the assumptions and the description of the shell element are presented. As mentioned before, the theory of circular cylindrical shells is applied to thin shells with small displacement. The following assumptions are taken into account when solving the compatibility equations.

1. Points initially lying normal to the middle surface of the shell remain normal when the shell is bent.
2. The normal stresses in the transverse direction to the shell are neglected.

Figure (1) illustrates a typical section of a shell of thickness 2t, the middle surface of radius R is circular in the transverse direction and straight in the longitudinal direction.

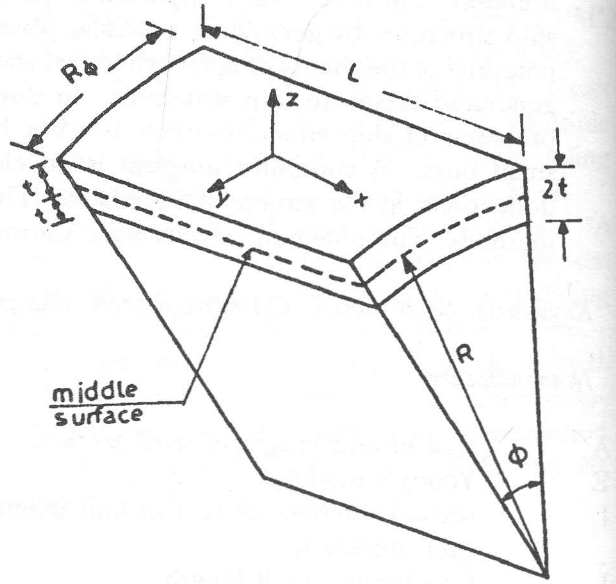

Figure 1. Description of shell element.

2.2 *Forces and Moments Acting on Shell Element*

The forces and moments acting on the shell element are assumed as shown in Figures (2a), (2b) and (2c). The forces and moments per unit length may be explicitly defined as follows,

N_x, N_ϕ	the forces on transverse and longitudinal sections of the shell respectively ,
$N_{x\phi}$, $N_{\phi x}$	the shear forces acting on the surface of the shell.
Q_x, Q_ϕ	the transverse shear forces acting on transverse and longitudinal sections
M_ϕ	the transverse moment acting along the longitudinal section of the shell element.

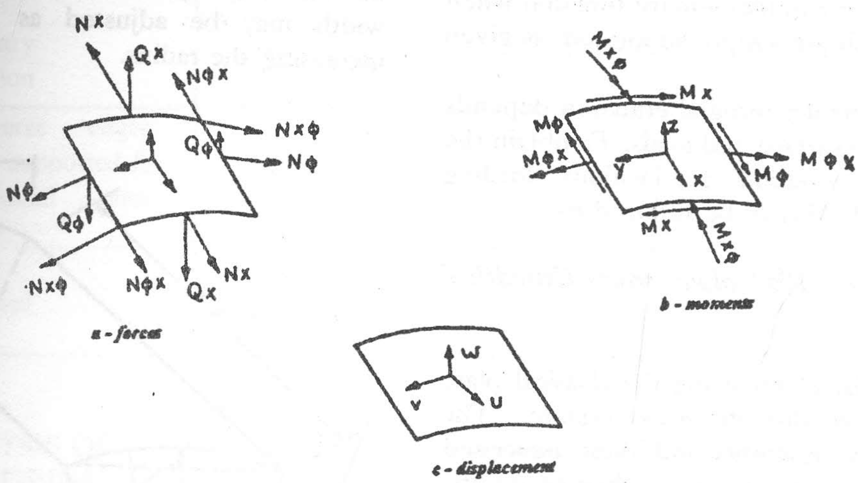

Figure 2. Positive forces, moments and displacements.

M_x the longitudinal moment acting along the transverse section of the shell element.

$M_{x\phi}$, $M_{\phi x}$ the torsional moments acting on the transverse and longitudinal sections, respectively.

The displacements *u,v,w* as shown in Figure (2c) are parallel to the coordinate axes x , y (=Rϕ), and z respectively. X, Y and Z are the external applied forces acting on the shell element and are defined as forces per unit area of the shell surface. In the analysis, the shell element will be assumed to be simply supported at its ends.

2.3 *The Shell Compatibility Equation*

The shell compatibility equation can be obtained by introducing the equilibrium equations, stress-strain relationships and strain-displacement relationships as a function of the displacement , *w*, in the z direction [2]. This equation may be expressed as follows,

$$R^8\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\phi^2}\right)^4 w+R^6(1-\nu^2)\frac{A}{I}\frac{\partial^4 w}{\partial x^4}=$$

$$\frac{R^6(1-\nu^2)}{EI}\left\{\begin{aligned}&R^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\phi^2}\right)^2 Z+\frac{\partial}{\partial\phi}\left((2+\nu)\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\phi^2}\right)Y\\&+R\frac{\partial}{\partial x}\left(\left(\nu\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\phi^2}\right)X\right)\end{aligned}\right\} \quad (1)$$

By solving this equation for *w*, the stress resultants and displacements can be determined.

A complete solution of Eq.(1) is obtained by combining the particular integral for given surface loads X,Y and Z with the complementary function. The complementary function is the solution of the equation with the right-hand side equated to zero. The second part is the solution of the particular integral i.e. particular solution of the equation as a whole.

Firstly, the complementary function may be expressed by equating the right-hand side of Eq.(1) to zero as follows,

$$R^8\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{R^2\partial\phi^2}\right)^4 w+R^6(1-\nu^2)\frac{A}{I}\frac{\partial^4 w}{\partial x^4}=0 \quad (2)$$

The deflection function w may be expressed as follows,

$$w = A_m e^{m\phi} \cos kx \quad (3)$$

where, $k = \frac{n\pi}{L}$

The solution of the complementary function when the ends of the shell are simply supported is given in [3].

Secondly, the particular integral equation depends on the nature of the surface and loads. To obtain the solution to this equation for various loading conditions, [2] and [3] may be referred to.

2.4 *Degeneration of Flat plate from Cylindrical Surface*

The analysis of flat plates using the classical plate theory considers two discrete stress systems. The first, in which only moments and their associated cross shear are assumed to act is known as the bending theory. The second, in which only in-plane or membrane stresses are assumed to act is known as the membrane theory. These two systems lead to two sets of stress equilibrium equations which are not interlinked and which have to be solved independently.

The equations of equilibrium for cylindrical shells are however almost identical to those for plates except that the curvature allows the bending and membrane equations to be linked. Now if the cylindrical surface is degenerated into a flat surface, the analysis previously developed for cylindrical shells may then be used to analyze flat plates.

The cylindrical surface may be caused to degenerate into a flat surface by reducing the half angle ϕ and by suitably increasing the value of its radius so that the required breadth of the flat surface is maintained. In order to establish the validity of using the circular cylindrical shell theory for the analysis of flat plates the following conditions must be fulfilled ,

1. The degenerated open cylindrical surface is considered flat for all practical cases.
2. The numerical solutions for plate problems using classical flat plate theory agree with the cylindrical shell degenerate theory.

By examining Figure (3), it is clear that a flat plate may be generated from a cylindrical one by reducing the half angle ϕ to a very small value. The span width may be adjusted as desired by suitably increasing the radius.

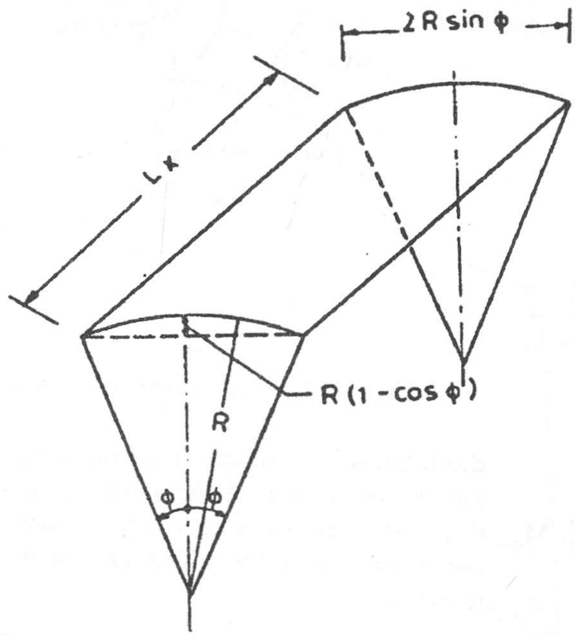

Figure 3. Generation of flat surface.

$$\text{Flatness ratio} = \frac{\text{rise of arc}}{\text{span width}} = \frac{R(1-\cos\phi)}{2R\sin\phi} = \frac{1-\cos\phi}{2\sin\phi}$$

When the angle ϕ is taken 1° for instance, the above ratio yields 1/175 and it happens that the resulting element may be considered flat for most practical cases [4].

With a half angle of 1° ,the required radius of a degenerate cylindrical surface which is to be equivalent to a flat surface of width L_y will be given by,

$$R = L_y / 2\sin(1°)$$

The degenerate theory which was introduced by Gibson [4] implies that the half angle must be equal to 1°, but Salem [3] obtained limiting parameters for generating flat surfaces from cylindrical ones as shown in Table (1).

Table. 1 Limiting Values of Half Angles and Ratio of Span Width to Rise at Center of Arc.

Boundary Condition	Aspect Ratio $\beta = L_x / L_y$	Half Angle ϕ (degree)	Span Width / Rise of Arc
Transverse edges simply supported & longitudinal ones built-in.	$1 \leq \beta \leq 4$	1/4°	920
Simply supported edges.	$1 \leq \beta \leq 4$ $2 < \beta \leq 4$	1/2° 2°	460 115

3. STRESS ANALYSIS OF MULTI-CYLINDRICAL SHELL

Ship structural units mainly consist of two types. These are,

1. Panels connected directly with each other such as the stringer plate with the sheer strake, double bottom, and the corrugated bulkheads.
2. Panels between edge beams having longitudinal stiffeners and girders.

The theory of circular cylindrical shells can be used efficiently in the analysis of such structures. It is the purpose of this paper to illustrate the use of this theory in solving structural problems of the first type. By considering the first type of structures, multi generated flat plates may be constituted. The versatility of using the shell theory in the analysis of such structures may be shown to supersede other methods. However, the shell edge conditions have to be generated. This will be explained in the next section.

3.1 General Shell Edge Condition

Considering Figure (4), the edges of the rth shell may be defined with reference to the center line, one edge at $+\phi_r$ and one at $-\phi_r$. . The displacement of the shell edges will be u,v,w and θ. In order to have a common reference direction for these sets of displacements it is convenient to refer them to fixed axes x_j,y_j,z_j as shown in Figure (4) in which z_j is vertical, y_j is horizontal and x_j is parallel to the axis of rotation of the cylindrical surface.

The position of the transverse section may then be defined by the inclination angle φ of its center line to the vertical as shown in Figure (4). The boundary conditions for the two outer edges of the structures at $+\phi_1$ and $-\phi_N$ are accounted for by providing two corresponding sets of four shell equations. Continuity of the structure is dependent on the equality of displacements , moments and forces at each junction between two shell components, [2] and [3].

The continuity of displacements at the junction between shell r and shell r+1 are as follows,

$$\begin{aligned} uj)_r &= uj)_{r+1} \\ vj)_r &= vj)_{r+1} \\ wj)_r &= wj)_{r+1} \\ \theta j)_r &= \theta j)_{r+1} \end{aligned} \tag{4}$$

The continuity of moments and forces at the same junction are as follows,

$$\begin{aligned} M_\phi j)_r &= M_\phi j)_{r+1} \\ Vj)_r &= Vj)_{r+1} \\ Hj)_r &= Hj)_{r+1} \\ N_{x\phi} j)_r &= N_{x\phi} j)_{r+1} \end{aligned} \tag{5}$$

where

Vj = resultant forces in the vertical direction.
Hj = resultant forces in the horizontal direction.

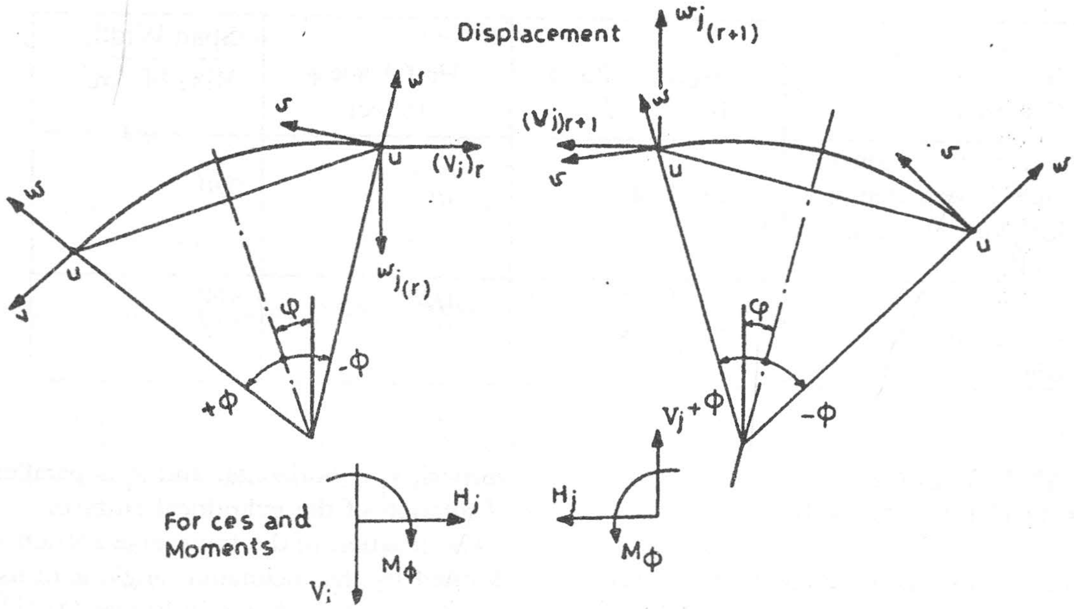

Figure 4. Equilibrium between two shells.

Equations (4) and (5) are sufficient to ensure the continuity for the junction between shells r and r+1. In general for structures having N shell components there will be 8N arbitrary constants and in order to solve the problem uniquely a number of 8N simultaneous equations involving these constants must be sought. As there are 8N-1 junctions then continuity of displacements, moments and forces provides 8N-8 equations. A further two sets of four equations (i.e. eight equations) will also be provided at the outer edges of the structure. In all , the total number of equations available will be 8N equations.

The last preceding method has been expounded in a computer program , this program is so generalized that multi-plate, shell panels may take any configuration desired with individual panels having any thickness and being subjected to any arbitary normal pressure.

4. CASES OF STUDY

After the presentation of the shell theory, the theory will be applied to some cases of ship structures. A computer program was developed to estimate stress resultants and deflections in complicated structural members. Also the results obtained by using such a theory will be compared with those obtained by using other methods. In the following cases, the modulus of elasticity and Poisson's ratio are 2.1×10^7 N/cm^2 and 0.3 respectively. These cases are as follows,

1- box barge
2-circular bilge plate.
3- study of the effect on strength of deck camber.
4- a deck with discontinous distributed load .
5- midship section for small boat

It must be noted here, that in order to illustrate the versatility of the shell theory, the following examples are considered without stiffeners and to compensate for the effect of the stiffeners on the strength, the plating is taken rather thick and the deck loads are kept small so that the conceived results will be realistic.

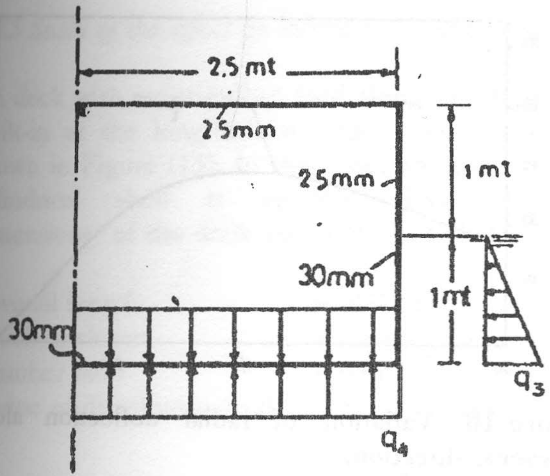

Figure 5. Configuration and loading condition of box barge.

4.1 *Box Barge*

The hull of a ship is considered as a box girder, with the longitudinal and transverse bulkheads dividing it into compartments or multi-cells. The shell theory will be applied to a simply supported box barge of a rectangular cross section as shown in Figure (5) in which the loading conditions are given. The box barge particulars are as follows,

- length between two transverse bulkheads = 5 m
- rectangular section = 5 x 2 m^2
- deck plate = 25 mm
- side shell plating= 25 mm above load water line.
- bottom plate = 30 mm below load water line.
- hydrostatic pressure = 1.025 N/cm^2

This box girder will be considered as a multi shell problem. The analysis of the box girder as a multi-shell problem is shown in Figure (6) in which the pitch angles are given. Figure (7) shows distribution of transverse bending moment along the transverse direction using both the shell theory and the slope deflection method. It is clear that results obtained by using both methods are in good agreement.

4.2 *Circular Bilge Plate*

A typical bilge structure with the particulars as given below has been analysed using the shell theory. The boundary conditions in this case are built-in for the sides and simply supported for the ends as shown in Figure (8). The particulars of the bilge plate are as follows,

- bilge radius = 320 cm
- longitudinal span = 500 cm
- half angle = 45°

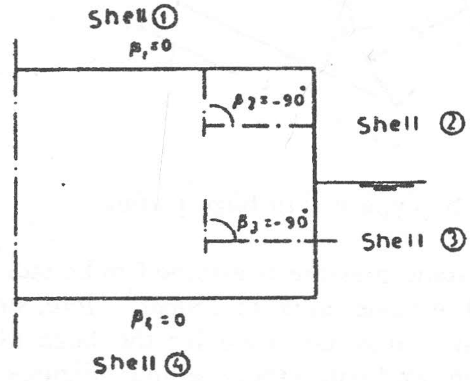

Figure 6. Analysis of box barge as a multi shell problem.

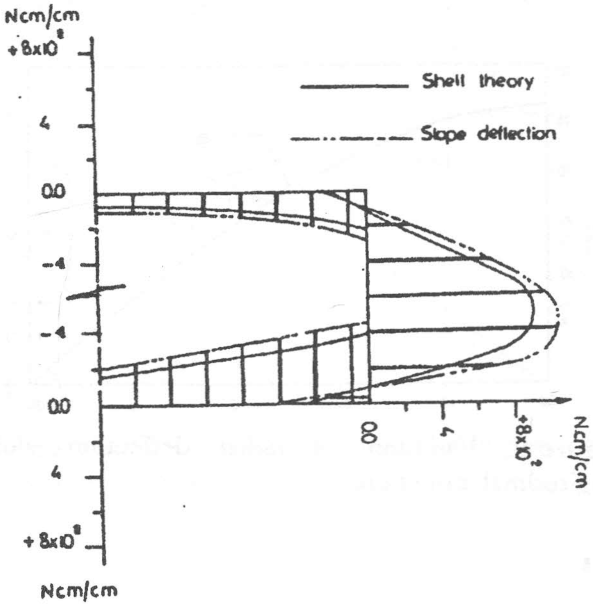

Figure 7. Distribution of transverse bending moment along transverse direction.

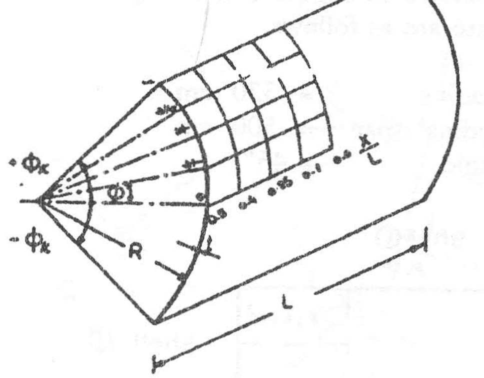

Figure 8. Typical ship bilge plating.

The hydrostatic pressure is assumed to be radially uniform and of value equal to $2N/cm^2$. Here, small shell elements may be used for the bilge plate model as compared with other methods. Figures (9) and (10) show the variation of radial deflection along longitudinal and transverse directions, while Figures (11) and (12) show the variation of transverse bending moment along longitudinal and transverse directions.

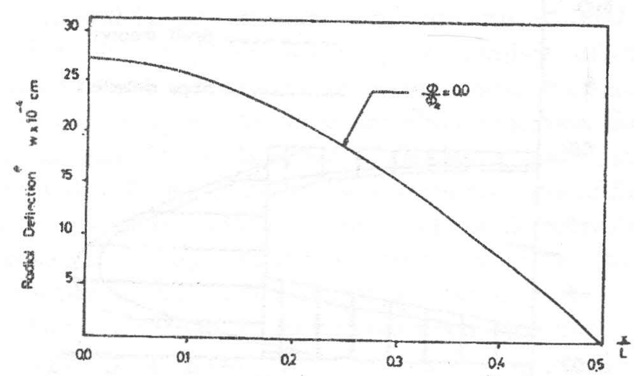

Figure 9. Variation of radial deflection along longitudinal direction.

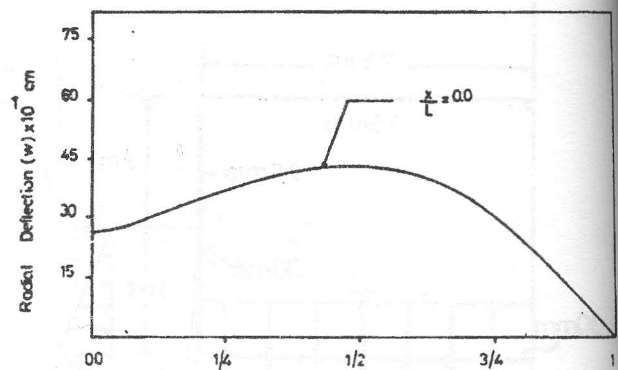

Figure 10. Variation of radial deflection along transverse direction.

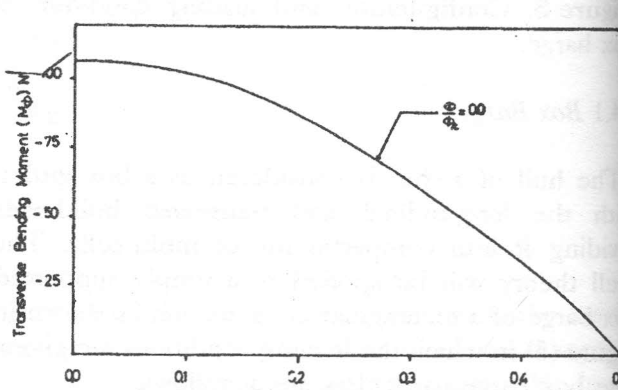

Figure 11. Variation of transverse bending moment along longitudinal direction.

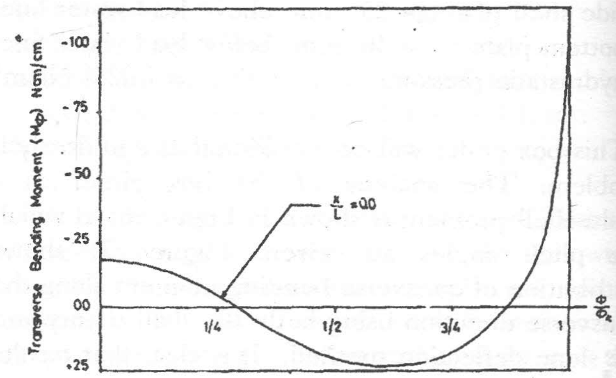

Figure 12. Variation of transverse bending moment along transverse direction.

4.3 *Study of the effect on strength of deck camber*

A deck with a distributed load along its length and built-in at the longitudinal edge is considered as shown in Figure (13). In this case, an open circular cylindrical shell is applied. The particular dimensions of the deck are taken as follows,

- overall length = 600cm
- deck thickness = 25mm
- camber ratio = 0.02
- value of distributed load = $0.5 N / cm^2$

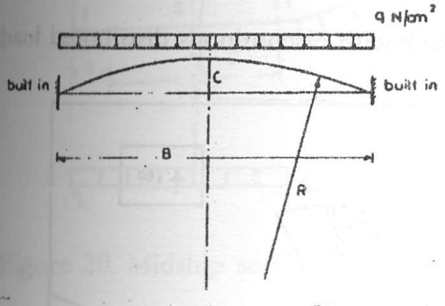

Figure 13. Deck with continuous distributed load.

The calculated results are obtained for two cases , namely,

1. when the deck is flat with width 500 cm .
2. when the deck is a cylindrical surface with radius of 3130 cm ; the value of this radius is calculated for a camber ratio as given above. Figures (14) and (15) show the distribution of radial deflection along longitudinal and transverse directions, while Figures (16) and (17) show the distribution of transverse bending moment.

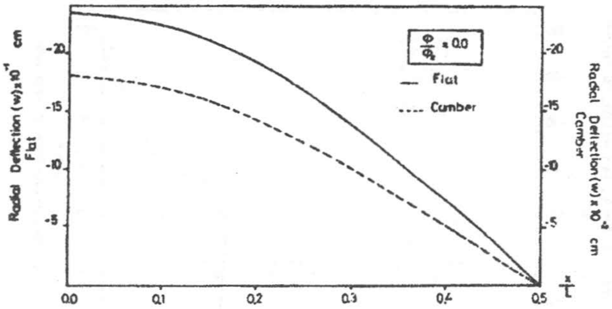

Figure 14. Variation of radial deflection along longitudinal direction.

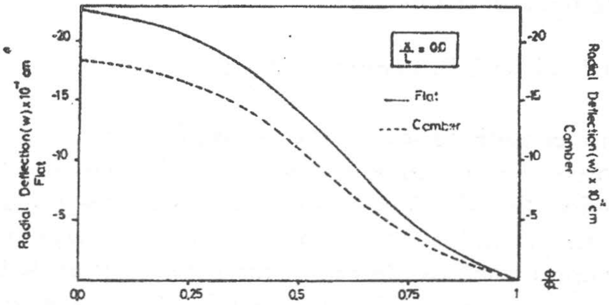

Figure 15. Variation of radial deflection along transverse direction.

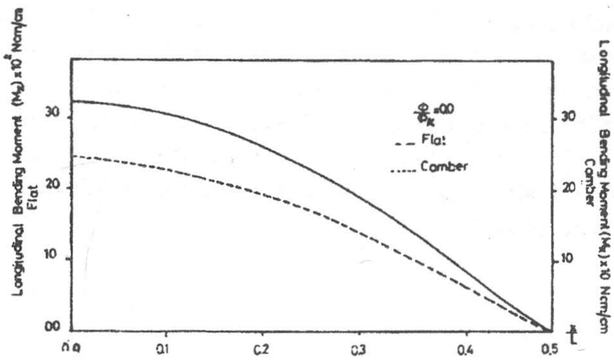

Figure 16. Variation of transverse bending moment along longitudinal direction.

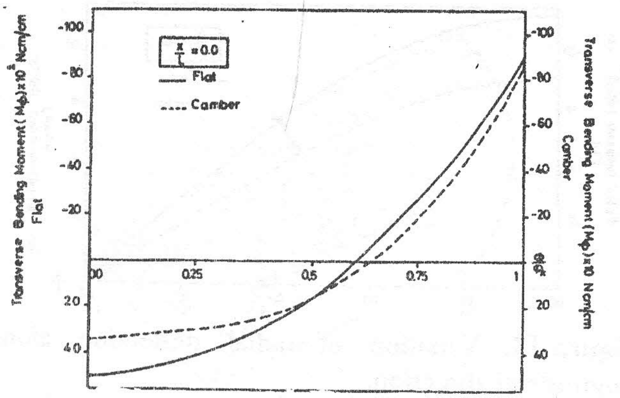

Figure 17. Variation of transverse bending moment along transverse direction.

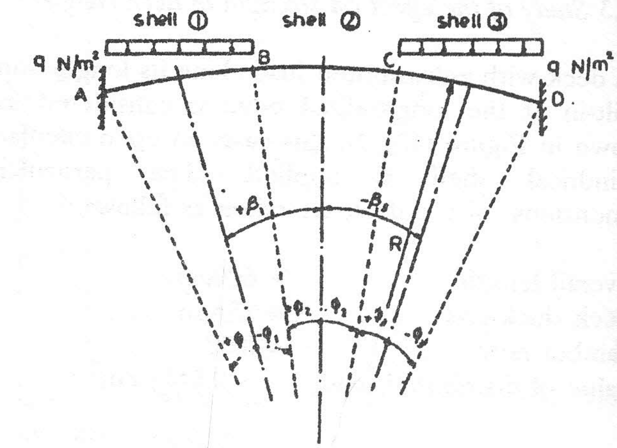

Figure 18. Deck with discontinous distributed load.

4.4 *Deck with Discontinuous Load*

The considered deck shown in Figure (18) is exposed to discontinuous loading in the transverse direction.The problem may be solved by considering that the complete deck consists of three bays of cylindrical shell as shown in the figure. Provided that suitable values of pitch angle for three bays are selected , it is easy to see from the geometry of the cross section that shell (1) must have an angle ϕ_1 of 1.52° and a pitch angle of 3.052°, the distributed load being 0.5 N/cm^2. In the case of shell (2) , ϕ_2 must be 1.526° and the pitch angle equals zero, with no load. Finally, shell (3) must have ϕ_3 = 1.526° , a pitch angle of -3.052° and a load equal to 0.5 N / cm^2. The transverse bending moment is plotted along the transverse direction as shown in Figure (19).

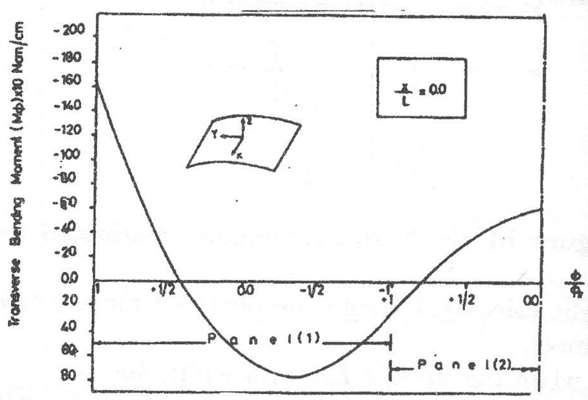

Figure 19. Distribution of transverse bending moment along transverse direction.

4.5 *Midship Section of Small boat*

A further application of the shell theory has been used to analyse a midship section for a small boat by considering it as a multi shell problem. The midship section configuration is shown in Figure (20). The loading and boundary conditions are shown in the same figure. Figure (21) shows the distribution of transverse bending moment along the transverse direction.

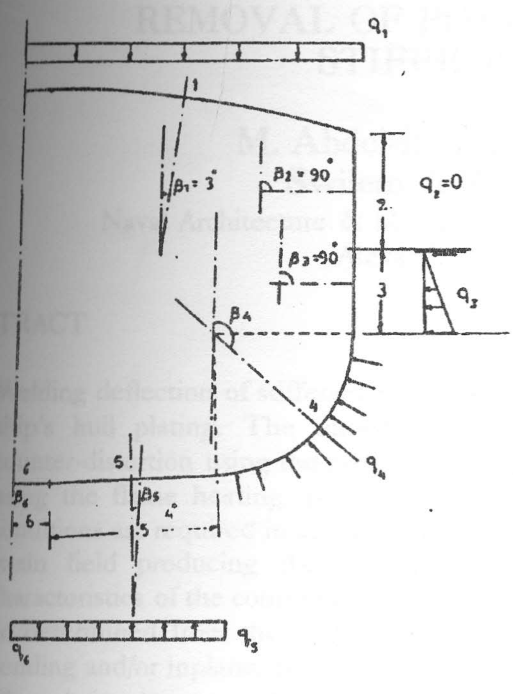

Figure 20. Midship section for small boat.

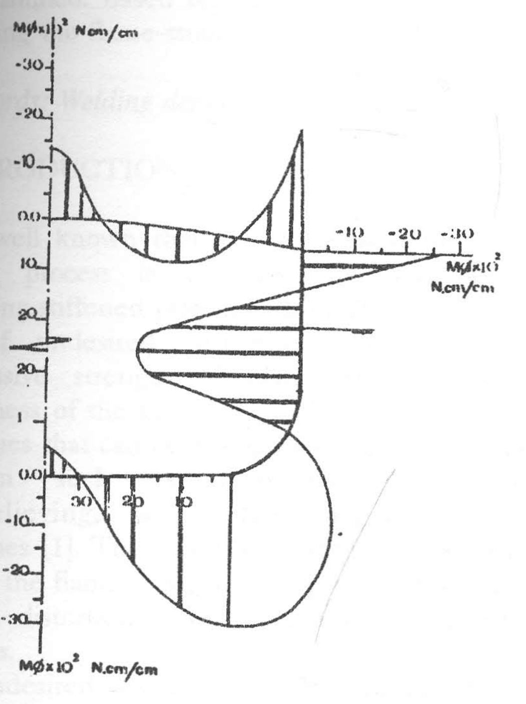

Figure 21. Distribution of transverse bending moment along transverse direction at (x/L=0.0).

5. CONCLUSIONS

After the presentation of the shell theory and how it may be applied to some ship structures, it is seen that it is easier in application from other methods due to the following advantages:

- The governing equation of compatibility is much more responsive to solutions (in respect to prescribed boundary conditions), than the two compatibility equations of the plate theory.
- It is applicable to any geometrical boundary conditions and material variation.
- It is easier to be programmed where only a small number of input data is required.

Considering the above advantages, the following may be concluded,

1. By generating a flat surface from a cylindrical one, it is possible to use the shell theory in the analysis of flat plates.
2. The accuracy of results attained for plates under lateral loads is equal to that attained by the classical bending theory of plates.
3. The shell theory may be used for the analysis of plate panels with the advantage that such panels may be either flat or circular cylindrical.
4. A further application of shell theory would be the analysis of multi-shells when more than two shells are connected at one joint. Also by developing the orthotropic shell theory, orthogonally stiffened plate problems can be solved with remarkable ease as compared with other methods.

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