MICRO-BUCKLING OF REINFORCED ELEMENTS EMBEDDED IN SOILS

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ABSTRACT

The problem of buckling of fiber reinforced elements embedded in bonded contact with a supporting soil has been analyzed. The final buckling load is expressed, implicitly in terms of buckling wave number, and explicitly in terms of all other concerned parameters, e.g. properties of fiber reinforced elements, and the supporting soil. We have proved that the 1st buckling mode prevails for wide spacings between the reinforced elements. The 3rd mode is the sole dominating mode, when reinforced elements are grouped in small spacings. The buckling mode abides to the 2nd one, if the range of h_m value lies between the above two extremes. Another parameter has shown to have a strong impact on the buckling mode, which is the soil shear stress μ_m. As such, the buckling load is linearly increasing with the increase in the soil shear stress value.

Keywords: Micro- Buckling, Reinforced Soils, Soil - Structure Interaction.

INTRODUCTION

The present analysis considers the problem of micro-buckling of fiber reinforced elements embedded in bonded contact with an isotropic elastic soil continuum. Fiber elements are assumed to be arranged unidirectional and equally spaced. The proposed model is suitable to the analysis of a unidirectional fiber reinforced soils, under the action of compressive forces.

The category of problems deal with the interaction between an embedded structural element and elastic continuum has several useful engineering application [1]. The interaction between the reinforced elements and the soil domain in reinforced soils falls in this category.

When fiber reinforced soils are subjected to compressive loads, the mode of failure is the fibers buckling. The problem of buckling of fibers embedded in elastic continuum has been studied analytically by many investigators. Among those, Selvadurai [1], Sadowsky [2], and Chung [3]. Selvadurai, and Sadowsky assumed an infinite extended elastic isotropic continuum. Therefore, their analyses are limited to a class of low - volume percentage of reinforcement soils. Chung developed a two dimensional model for a unidirectional reinforced continuum. The critical buckling load was evaluated numerically while some physical assumptions were imposed. Greszczuk [4] conducted an experimental study where, the influence of volume fraction, end fixity, thickness, geometry, and continuum properties on the buckling strength of a unidirectional reinforced continuum were investigated.

In the present two dimensional model, the displacement and the stress fields in the elastic medium are presented by Love's strain function. The reinforced element buckling equation and the medium field equation are solved together using the separation of variables. Finally, the buckling load is given explicitly in terms of reinforced elements and supporting soil properties, as well as the domain geometry.

Mathematical Formulation

The present model considers the reinforced elements as elastic beams of width h_f and flexural rigidity E_fI_f embedded in adhesive contact with $2h_m$ by L isotropic elastic medium. The Possion's ratio and shear modulus are denoted by ν_m and μ_m

respectively. The reinforced elements are assumed to be uniformly spaced. Figure (1) depicts the geometry of the problem.

Each reinforced element carries a compressive force P which may be given in terms of the total applied load Po, soil properties and geometry in the form [3],

$$P=-\frac{P_o}{(1+2h_m/nh_f)} \tag{1}$$

where $n = E_f/E_m$. E_f and E_m are the elastic modulii for the reinforced element and the medium, respectively.

The behavior of the elastic isotropic domain can be readily formulated in terms of Love's strain function $\Psi(y,z)$ [5], which satisfies the biharmonic equation,

$$\nabla^4\Psi(y,z)=0$$

$$\text{where} \quad \nabla^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \tag{2}$$

The displacement and the stress of interest in terms of Ψ are [5],

$$u_{y_m}=-\frac{1}{2\mu_m}\frac{\partial^2\Psi(y,z)}{\partial y\,\partial z} \quad \sigma_{y_m}=\frac{\partial}{\partial z}\left(\nu_m\nabla^2-\frac{\partial^2}{\partial y^2}\right)\Psi(y,z) \tag{3}$$

where m denotes the soil domain.

The interaction between the reinforced element (RE) and the supporting medium is assumed to be due to an interaction normal stress mechanism which acts at the (RE) - medium interface and tends to restrain the transverse motion of the RE. Therefore, the governing differential equation of the (RE) lateral displacement has the form [2],

$$E_fI_f\frac{\partial^4u_y(h_m,z)}{\partial^4 z}+P\frac{\partial^2u_y(h_m,z)}{\partial^2 z}-2h_f\sigma_y(h_m,z)=0 \tag{4}$$

The continuity condition at the interface requires that,

$$u_{y_m}=u_{y_f}, \qquad \sigma_{y_m}=\sigma_{y_f} \tag{5}$$

where f denotes the (RE).

Equation (5) contains boundary conditions for Eq. (2). Substituting Eq. (3) into (5), and use the result into Eq. (4) yields,

$$E_fI_f\frac{\partial^4}{\partial z^4}\left[-\frac{1}{2\mu_m}\frac{\partial^2\Psi(h_m,z)}{\partial y\partial z}\right]+P\frac{\partial^2}{\partial z^2}\left[-\frac{1}{2\mu_m}\frac{\partial^2\Psi(h_m,z)}{\partial y\partial z}\right]$$
$$2h_f\frac{\partial}{\partial z}\left(\nu_m\nabla^2-\frac{\partial^2}{\partial y^2}\right)\Psi(h_m,z)=0 \tag{6}$$

Operating on Eq. (6) by Laplace's operator and then use Eq. (2), we get

$$\left[K\frac{\partial^6}{\partial z^5\partial y}+p\frac{\partial^4}{\partial z^3\partial y}+\frac{\partial^3}{\partial z\partial y^2}\right]\Phi(h_m,z)=0$$
$$\text{where } K=\frac{E_fI_f}{4\mu_m h_f},\ p=\frac{P}{4h_f\mu_m},\ \Phi=\nabla^2\Psi. \tag{7}$$

The solution to the above equation can be assumed as,

$$\Phi(h_m,Z) = Y(h_m)Z(z) \tag{8}$$

Substitution of Eq. (8) into (7) gives,

$$KZ^{V}Y'+pY'Z'''+Z'Y''=0 \tag{9}$$

where the number of (') defines the order of differentiation.

Equation (9) could be rewritten as,

$$K\frac{Z^{V}}{Z'}+p\frac{Z'''}{Z'}=c \tag{10-a}$$

$$\frac{Y''(h_m)}{Y'(h_m)}=-c \tag{10-b}$$

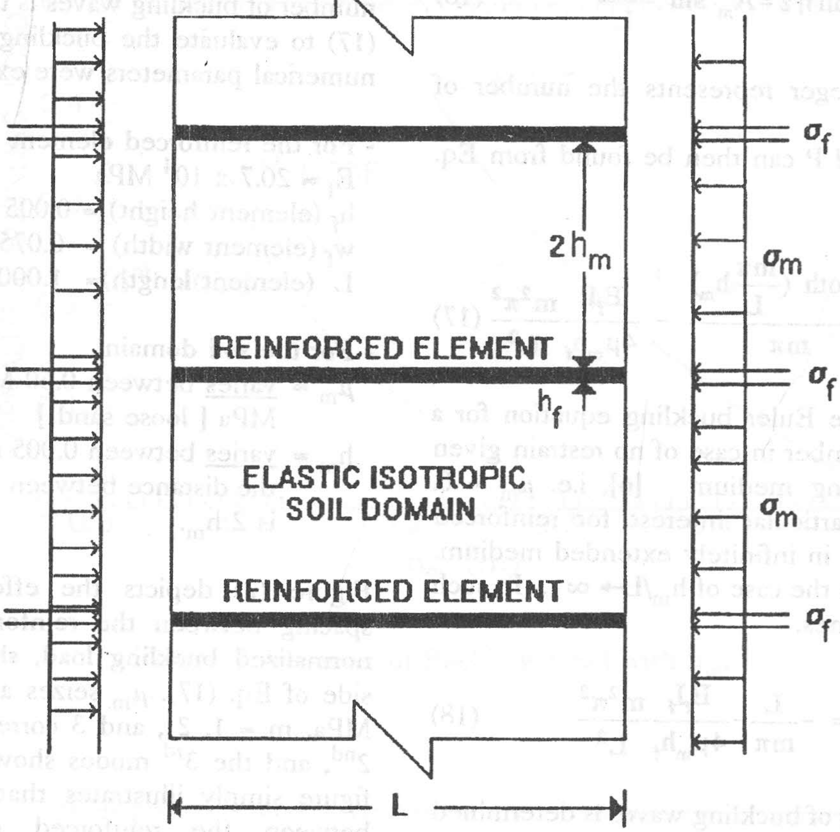

Figure 1. Geometry and Loading.

where c is a separation constant.

To obtain the constant c, we utilize Eq. (2) which in terms of Φ is

$$\nabla^2\Phi \quad = 0 \tag{11}$$

Equation (8) shall be used as a the solution of Eq. 11 with y replacing h_m. This leads to the following equations

$$\frac{Z''}{Z}=-\eta^2, \tag{12-a}$$

$$\frac{Y''}{Y}=\eta^2 \tag{12-b}$$

η^2 in Eq. (12) is a separation eigenvalue. We now define the boundary value problem with Eq. (12-b) as a governing differential equation and Eq. (10-b) as its boundary condition. The solution of Eq. (12-b) could be written as,

$$Y=B_\eta \cosh \eta y \tag{13}$$

Using the boundary condition (10-b) leads to the following relationship,

$$c=-\eta \coth \eta h_m \tag{14}$$

Substituting the value of c into Eq. (10-a) gives,

$$KZ^V + pZ'''=-(\eta \coth \eta h_m)\, Z' \tag{15}$$

The solution to Eqs. (15) and (12-a) may be assumed in the form

$$Z = A_\eta \sin \eta z = A_m \sin \frac{m\pi z}{L} \qquad (16)$$

where m is an integer represents the number of buckling waves.

The buckling load P can then be found from Eq. (15), which is

$$\frac{P}{4h_f\mu_m} = -\frac{L \coth\left(\frac{m\pi}{L}h_m\right)}{m\pi} - \frac{E_f I_f}{4\mu_m h_f}\frac{m^2\pi^2}{L^2} \qquad (17)$$

Equation (17) is the Euler buckling equation for a beam - column member in case of no restrain given from the surrounding medium [6], i.e. $\mu_m = 0$. Another case of particular interest for reinforced element embedded in infinitely extended medium. This corresponds to the case of $h_m/L \rightarrow \infty$. In such case Eq. (17) becomes,

$$\frac{P}{4h_f\mu_m} = -\frac{L}{m\pi} - \frac{E_f I_f}{4\mu_m h_f}\frac{m^2\pi^2}{L^2} \qquad (18)$$

The critical number of buckling waves is determined by solving the equation

$$\frac{\partial P}{\partial m} = 0 \qquad (19)$$

After some algebraic manipulations, Eq. (17) reduces to the following form

$$\frac{\coth\theta}{\theta^3} + \frac{1}{\theta^2 \sinh^2\theta} = \Gamma \qquad (20)$$

Θ and Γ in Eq. (19) are non-dimensional quantities equal to $m\pi h_m/L$ and $E_f I_f/2h_f\mu_m h^3{}_m$ respectively.

Analysis

To show the essence of Eq. 17, the variation of the buckling load with spacing between reinforced elements, and the soil shear stress is investigated. The root of Eq. 20 is found numerically using the Bisection Method, James et al (1977). The integer number of buckling waves is then substituted in Eq. (17) to evaluate the buckling load. The following numerical parameters were exercised in the analysis

- For the reinforced element
  - E_f = 20.7 x 10^4 MPa
  - h_f (element height) = 0.005 m
  - w_f (element width) = 0.075 m
  - L (element length)= 1.000 m

- For the soil domain
  - μ_m = varies between 0.60 MPa [soft clay] → 10.0 MPa [loose sand]
  - h_m = varies between 0.005 m → 0.50 m. Note that, the distance between the reinforced element is 2 h_m.

Figure (2) depicts the effect of increasing the spacing between the reinforced elements on the normalized buckling load, shown in the left hand side of Eq. (17). μ_m seizes an average value of 6.0 MPa. m = 1, 2 , and 3 corresponds to the 1st, the 2nd, and the 3rd modes shown in the figure. The figure simply illustrates that the range of spacing between the reinforced elements grants the existence of one mode of buckling and denounces the other modes.

With wide spacing between the elements, the 1st mode prevails upon. For lower range of spacings, higher modes predominate the buckling shape.

Figure (3) characterizes the effect of soil shear stress on the buckling load of the reinforced elements. As long as h_m holds a value of 0.30 m, the buckling mode is always the 1st, whilst the buckling load changes linearly with μ_m. For h_m equals 0.15 m, and μ_m is less than about 3.o MPa, the 1st mode is still controlling the buckling shape. At higher value of μ_m, the 2nd mode prevails. The 3rd mode governs the buckling shape, for h_m equals to 0.05 m. Apparently, the solution for this case does not exist for soil shear stress of values less than about 4.0 MPa.

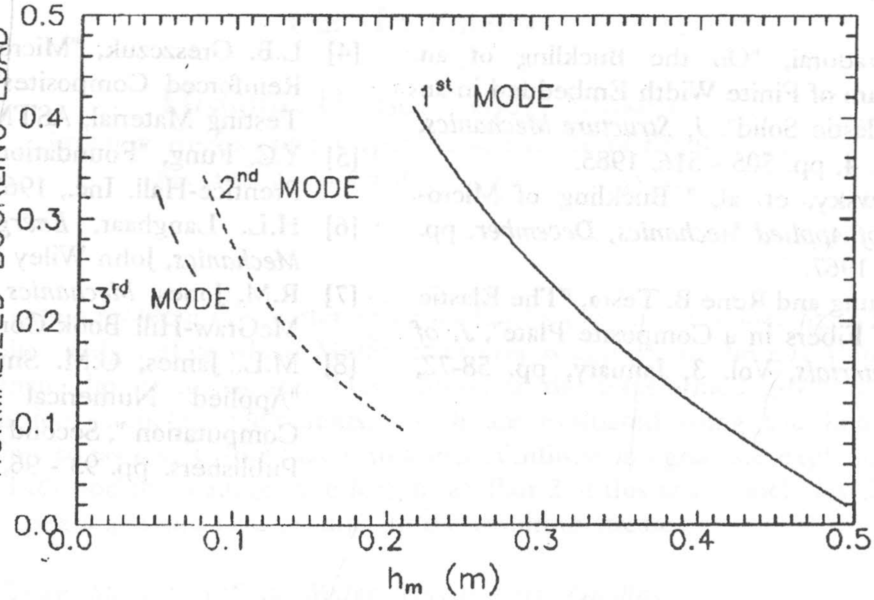

Figure 2. Variation of Buckling Load with h_m.

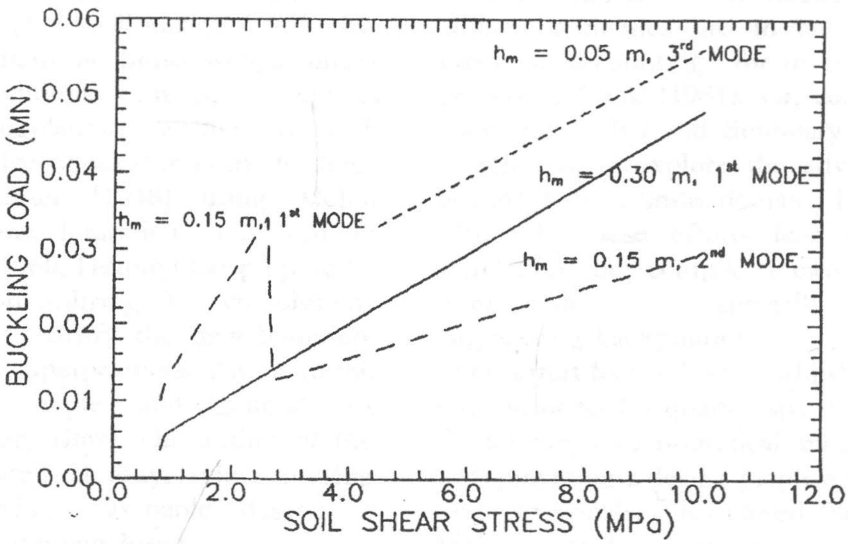

Figure 3. Variation of Buckling Load with Soil Shear Stress.

CONCLUSION

The article has demonstrated the formulation of the buckling equation of a reinforced fiber elements embedded in bonded contact with the supporting soils. The elements are assumed to be unidirectional arranged with equal and uniform spacings. We have shown the explicit influence of element spacings and soil shear stress on the buckling load and mode.

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