A HYDRODYNAMIC LUBRICATION MODEL FOR RECIPROCATING RUBBER SEALS

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ABSTRACT

The development of engineering models for the hydrodynamic lubrication of reciprocating seals is a vital task in achieving a design for a long lasting seal with low friction and minimal leakage. In this paper, the actual shapes of reciprocating lip seals with worn edges have been proposed to justify the generation of hydrodynamic pressures under such conformal tribological contacts. A direct numerical scheme known as the Thomes Algorithm has been used to solve the boundary value problem formulated for such a model. Comparison with the Gauss Seidel iterative method reveals the fact that the direct technique is more reliable, accurate and much more faster in terms of computational time. Performance charts for the lip seal have been developed to provide estimates of the contact radial force, the frictional force resisting the sliding motion of the sealed rod and the amount of leakage from the pressurized side to the atmosphere.

Keywords: Hydrodynamic theory, Rubber lip seals, Finite difference method, Thomas Algorithm.

Nomenclature

ai- di Central coefficients in finite difference equations
B Lip width
f Friction factor = $6F/^{h}{}_{m}$ Pa
F Friction force per unit circumferential length (N/ m)
G Term describing film shape in the modified Reynolds equation
h Film thickness (m)
H Nondimensional film thickness $=h/h_m$
i Nodal point location in the domain, (1<i<N)
n Exponent describing the worn lip shape
N Number of nodal points
p Pressure (N/m^2)
P Nondimensional pressure = (P - Pa) / Pa
q Leakage factor = $12\, Q\mu\, B / h_m^{\ 3}\, P_a$
Q Axial discharge ($m^3/s/m$)
R Wedge action term in the modified Reynolds equation.
S Transformed pressures = $PH^{1.5}$
w Radial load factor = W/BP_a
W Radial load per unit circumferential length (N/m)
x Axial coordinate
X Nondimensional location = x / B
Z Half thickness of lip (inclosing the super ellipse)
Λ Bearing number
μ Fluid dynamic viscosity
ζ Lip thickness to film thickness ratio = z/m
ε Convergence criterion in iterative technique

Subscripts

a Atmospheric
m Minimum
new Refers to the current iteration
old Refers to the previous iteration

INTRODUCTION

Rubber lip seals are still considered the most efficient systems used for sealing rotary shafts and reciprocating rods. The basic function of these elastomeric elements is to ensure the reliability of the tribological system by preventing the transfer of either lubricant from, or contaminant to the protected volume of the machine, Figure (1). During the last two decades lip seals design has gradually evolved. Most of the development took place in the technology of rubber compounding where the thermal and chemical stability of elastomers was drastically improved. Recently, the availability of computer codes capable of dealing with the nonlinear deformation of elastomeric bodies has also provided a powerful tool for seal design, [1-4]. However, relatively little has been accomplished in the filed of seal lubrication which mode and mechanism are still not well defined by researchers, [5-6]. For reciprocating seals, widely used in shock absorbers, their dynamics and efficient performance are linked with the ability of the designer to predict and control the fluid film development in the sealing contact region. Due to the radial load and the compliance of the lip we may assume that, at least at rest, the two surfaces are fully conformal and parallel to each other. Under such a condition no hydrodynamic pressures can build up to support the sealing load when the rod begins to slide. However, during the running-in period and due to the rod surface roughness, the edges of the seal installed around the sliding rod will be subjected to a wear process leaving round edges rather than sharp ones. At this stage and along the consequent strokes, the lip will act as a slider due to the new developed contact shape. In fact it is difficult to describe mathematically the contour of the used seal at the contact zone. In the present work a function having the form $(x/(B/2))^n + (y/z)^n=1$ known in the literature as a super ellipse, [7] is proposed to describe the contact contour of a worn lip under operation, Figure (2). This function may be used to describe the shape of similar conformal tribological contacts loosing their sharp edges during continuous operation in relatively starved lubrication conditions such as compression piston rings in internal combustion engines and electrical motor brushes.

With such a generated profile axisymmetrical hydrodynamic pressures will arise depending on the sliding speed in the axial direction. Their values are usually calculated by solving Reynolds equation using iterative techniques, [8]. Due to the sharp variation in the lubricant film thickness at the leading edge these numerical methods call for a large number of iterations with tight convergence criteria to yield accurate solutions. The numerical strategy used here, is a direct method of solution based on the Thomas Algorithm, a special case of the Gauss elimination method for tri-diagonal matrix equations, [9- 10]. As shown in Figure (2) the problem to be solved consists of a one dimensional boundary value problem with the pressure value known at the oil side and zero at the location of the trailing edge of the fluid film where the possibility of cavitation exists. The proposed method has been used successfully to get the generated pressures and to locate the start of the cavitated region. Comparing the results with that obtained by the Gauss-Seidel iterative method reveals the fact that the proposed method is more reliable in calculating the exact pressures and much more faster in terms of computational time

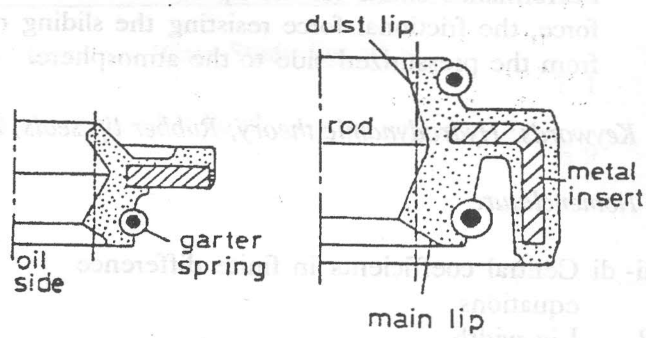

Figure 1. Design features of lip seals.

FINITE DIFFERENCE EQUATIONS FOR THE LIP MODEL

The Reynolds equation for an incompressible and isothermal lubricant flowing in the axial direction can be written in a nondimensional from such as, [8]:

$$\frac{d^2p}{dx^2} + \frac{3}{H}\frac{dh}{dh}\frac{dp}{dx} = \frac{\Lambda}{H^3}\frac{dh}{dx} \quad (1)$$

where

$$P = (p - p_a) / p_a$$

$$X = h / h_m$$

$$X = x / B$$

With the bearing number

$$\Lambda = 6\mu\, U\, B/\,(p_a h_m{}^2) \qquad (2)$$

Using the transformation $S{=}PH_{1.5}$ primarily because gradients of S are less severe than those of P, hence the numerical analysis is more accurate, [11]. Equation (1) after being modified becomes:

$$\frac{d^2S}{dX^2} + G\,S = R \qquad (3)$$

with $R = \Lambda \dfrac{dH}{dX}/H^{1.5}$

and

$$G = -\frac{3}{4}\left[2[(\frac{d^2H}{dX^2})/H] + [(\frac{dH}{dX})/H^2]\right] \qquad (4)$$

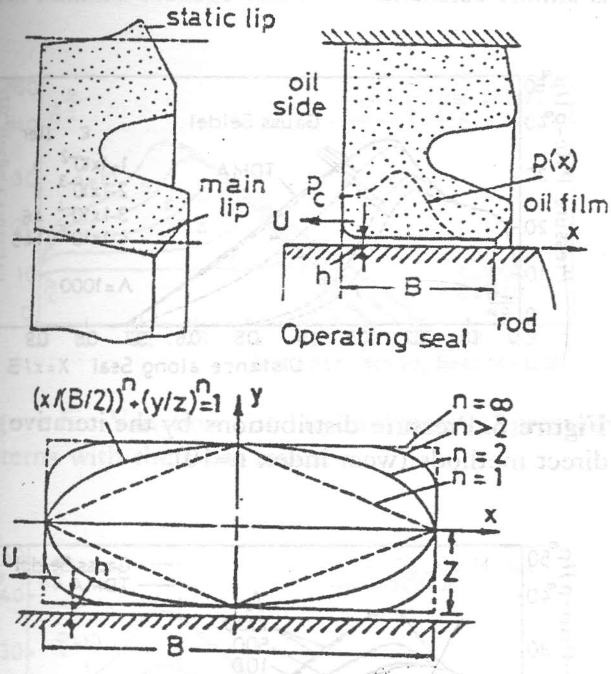

Figure 2. Geometry of worn lip seal.

The film shape and the different derivatives with respect to X are derived in appendix (A).
The approximate finite difference expression of the first term of equation (3) is:

$$\frac{d^2S}{dX^2} = [S_{i-1} - 2S_i + S_{i+1}]/\Delta X^2 \qquad (5)$$

After substitution, a set of simultaneous finite difference equations can be written in the form:

$$c_i\, S_{i-1} + a_i\, S_i + b_i\, S_{i+1} = d_i,\ (1 < i < N) \qquad (6)$$

This set of equations can be solved using the Gauss-Seidel iterative technique by calculating the modified pressure values at each nodal point from :

$$S_i\ [\ d_i - c_i\, S_{i-1} - b_i\, S_{i+1}]\ /a_i \qquad (7)$$

The solution can be obtained by setting a convergence criterion such as

$$\left|\sum \frac{S_{iold} - \sum S_{inew}}{\sum S_{iold}}\right| < \epsilon = 1 \times 10^{-3} \text{ to } 1 \times 10^{-5} \qquad (8)$$

Referring to appendix (B), the set of the finite difference equations (6) can rather be solved using the Thomas algorithm which is a direct and efficient method for this class of boundary value problems [11- 12] . For high sliding speeds in the axial direction cavitation may exists near the trailing edge. In the iterative methods, the Christopherson cavitation approximation is usually used, [13]. This simply demands that the calculated pressures never be allowed to become negative. The scheme of the Thomas algorithm calls for the calculation of α and η coefficients at each station i, appendix (B). These coefficients depends only on the leading edge location hence they can be precalculated independently of the trailing edge location. The start of the cavitated region was found to start at the nodal point where α changes sign from positive to negative. In this fashion the cavitation location is determined without iteration. A backward sweep using the β and α coefficients along the full film region generates all the required S_i values, hence the P_i values used for consequent performance analysis.

PERFORMANCE FACTORS

The radial force generated by the hydrodynamic film between the lip contact surface and the sliding rod per unit circumferential length can be calculated from:

$$w = \int_0^B p\,dx = \int_0^1 BP_a (S/H^{1.5})\,dX \qquad (9)$$

Thus, a load factor can be expressed as:

$$W = \frac{w}{Bp_a} = \int_0^1 (S/H^{1.5})\,dX = \int_0^1 P\,dX \qquad (10)$$

The viscous shear force created during the sliding motion can be evaluated from:

$$F = \int_0^B \left(\frac{\mu U}{h} + \frac{h\,dp}{2dx}\right) dx$$

$$= \int_0^1 \left(\frac{\mu UB}{Hh_m} + \frac{Hh_m}{2} p_a \frac{dP}{dX}\right) dX \qquad (11)$$

After rearranging terms a friction factor can be deduced as follows:

$$f = \frac{6F}{p_a h_m} = \int_0^1 \left(\frac{\Lambda}{H} + 3H\frac{dP}{dX}\right) dX \qquad (12)$$

The leakage rate can also be evaluated by evaluating the pressure gradient at the parallel section of the lip as follows:

$$Q = \frac{h^3}{12\mu}\frac{dp}{dx} + \frac{Uh}{2} = \frac{H^3 h_m^3}{12\mu B} p_a \frac{dP}{dX} + \frac{UHh_m}{2} \qquad (13)$$

Similarly a discharge factor can be expressed as :

$$q = \frac{12Q\mu B}{h^3 p} = H^3\frac{dP}{dX} + \Lambda H \qquad (14)$$

DISCUSSION OF RESULTS

To verify the solution for the generated hydrodynamic pressures with the proposed lip seal shape, the two computational schemes have been used to obtain the numerical results. The first is the Gauss-Seidel iterative method with a successive over relaxation factor = 1.8 and the second is the direct Thomas algorithm. Figure (3) shows different pressure distributions obtained by the first method with different conversion criteria. It is clearly shown that the solution obtained by the direct technique coincide closely with that obtained by the iterative one with a tight conversion criterion (ϵ=1x10^{-5}, N=41 points). The use of the lower values yields nonreliable pressure distributions taking in consideration that favorable e values are, in general not specified for such classes of solutions for boundary value problems. In addition to the pressure magnitudes the film trailing edge location is also obtained successfully by using the direct technique at X = x / B= 0.73 for a bearing sliding number Λ=1000. Referring to Figure (4), the solutions obtained for low values of (Λ) show a decrease in the pressure magnitudes with a linear distribution over the mid portion of the lip where the film thickness is almost constant.

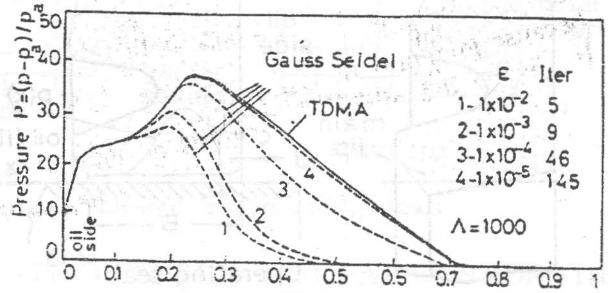

Figure 3. Pressure distributions by the iterative and direct methods (wear index n=10).

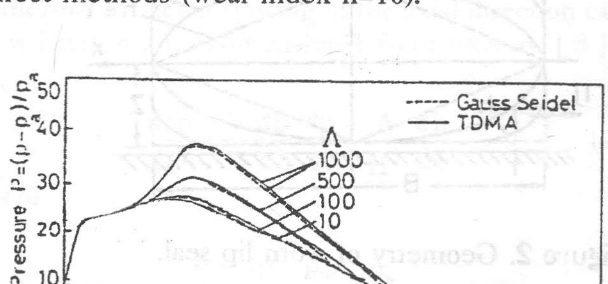

Figure 4. Variation of hydrodynamic pressures for different bearing numbers (n=10,ζ=1000).

In this region the lip seal acts as a long constant-gap annular flow restrictor. The efficiency of this restrictor in reducing leakage is optimal for new seals where wear of the sharp edges is minimum (high values of the index n). This is clearly illustrated in Figure (5) where the pressure gradients at the trailing edges decrease as the index n increases. For extreme convex shapes (n= 2) the efficiency of the seal in controlling the leakage is unacceptable and the oil side pressure drops sharply within a narrow portion at mid span of the contact where conformity between the contacting surfaces is uncertain. Figure (6) shows the variations of the hydrodynamic pressure patterns along the length of the seal for different oil side pressures. In the region 0.3 ≤ X ≤ 0.7 a linear pressure drop is observed irrespective to the oil side pressures. For Pc=0 the seal acts as a plane slider and considerable hydrodynamic pressures are shown to be generated in the parallel contact region maintaining conformity with minimal leakage rates.

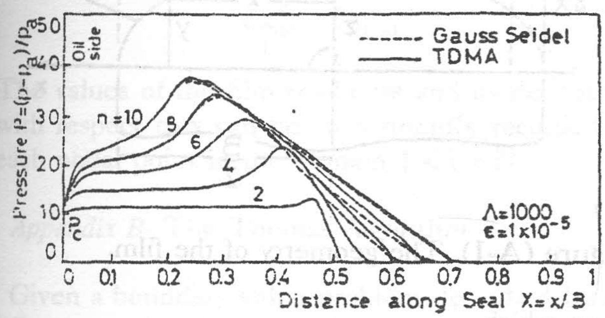

Figure 5. Variation of hydrodynamic pressure patterns with the wear index.

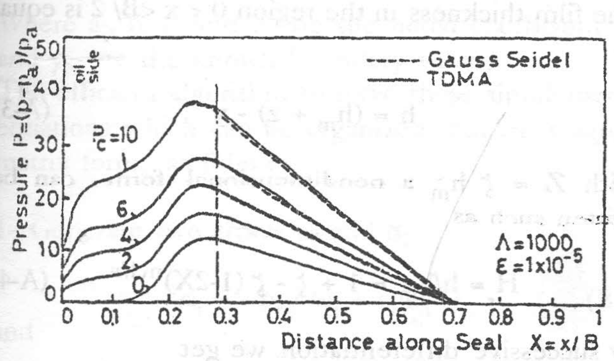

Figure 6. Hydrodynamic pressure patterns for different oil side pressures.

Using the pressure values obtained from the direct solution scheme, the performance of the lip seal along its service life have been evaluated. Nondimensional factors have been calculated per unit circumferential width after integrating the pressures as expressed in equations 10, 12 and 13. Figure (7) shows clearly that tight contact between the seal and the rod is desired for newly mounted seals (index n ≥ 14), where radial hydrodynamic loads are comparatively high. The spring elements usually integrated with the seal designs can provide such necessary reactive radial loads in their early stages of operation. The loss of their elasticity after continuous running should be considered according to the oil side pressure values where leakage can be relatively high. This is demonstrated in Figure (8) where the leakage factor is acceptable for new seals but increases with high rates as the sharp edges wear out after prolonged operation, (n = 4 to 6). Figure (9) shows different plots for the friction factor at different oil side pressure values. It is interesting to notice that the sliding friction shows optimal values for n ≈ 9 where frictional forces are maximum. Beyond this value, a new lip will operate as a plane parallel slider with minimal friction. The frictional force estimates do not count for the cavitated regions where laminar flows are questionable.

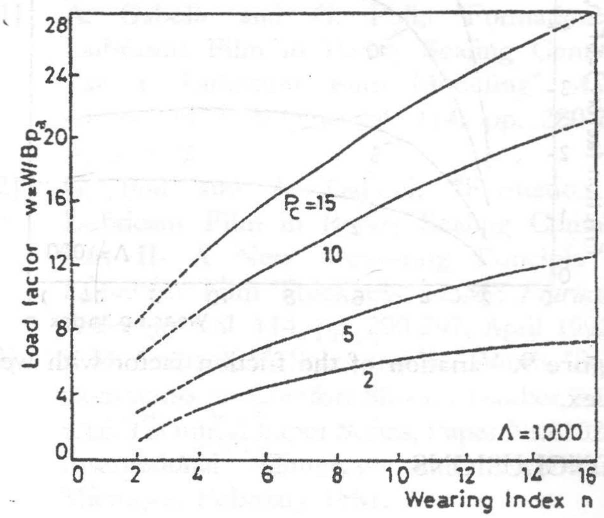

Figure 7. Variation of the load factor with wear index.

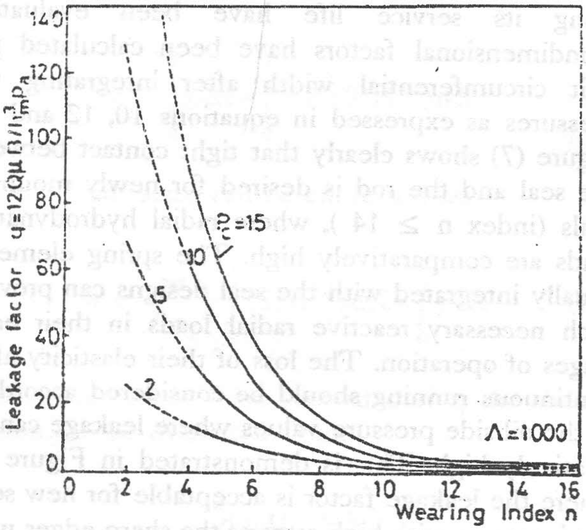

Figure 8. Variation of the leakage factor with wear index.

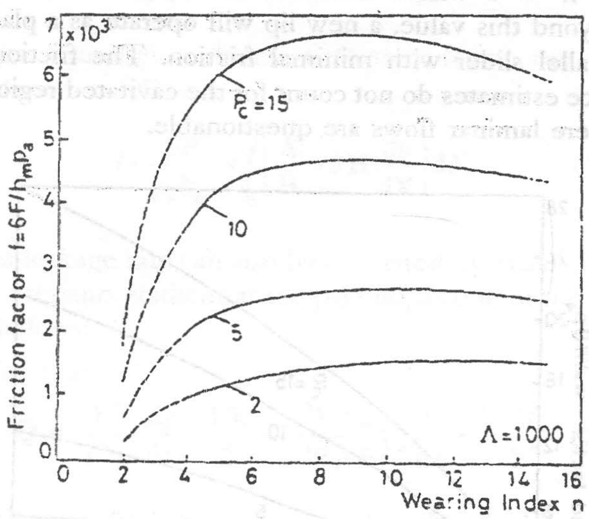

Figure 9. Variation of the friction factor with wear index.

CONCLUSIONS

The present analysis has asserted the possibility of hydrodynamic pressures generation in a conformal reciprocating lip seal through an engineering model which reflects the actual shapes of the contact during the operating life. The conformity depends mainly on the rubber quality used and its elasticity in the radial direction. The use of the fast and direct numerical scheme provided reliable solutions when compared with the usual Gauss-Seidel iterative technique. In addition, the edges of the separating fluid film have been specified accurately. Nondimensional performance charts have been presented to describe the efficiency of the lip seal over its operating life in terms of leakage control, frictional resistance and contact loads versus lip edge shape.

APPENDIX A- Film Geometry

Referring o Figure (A-1) and considering the first quadrant of the symmetrical shape we have:

$$(x/(2B))^n + (y/z)^n = 1, \quad 1, < n < \infty \qquad \text{(A-1)}$$

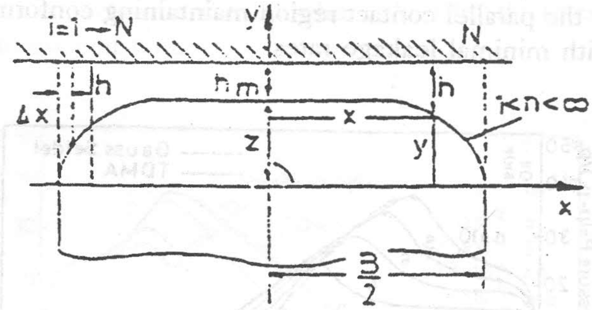

Figure (A-1). The geometry of the film.

From which

$$(y = Z\,(1-(2X)^n)^{1/N}, \qquad X = x/B \qquad \text{(A-2)}$$

The film thickness in the region 0 < x <B/ 2 is equal to:

$$h = (h_m + z) - y \qquad \text{(A-3)}$$

With $Z = \zeta\, h_m$ a nondimensional form can be written such as

$$H = h/h_m = 1 + \zeta - \zeta\,(1-2X)^n)^{1/n} \qquad \text{(A-4)}$$

By successive differentiation we get

$$\frac{dh}{dX} = 2\zeta\left(1-(2X)^n\right)^{\frac{1-n}{n}}(2X)^{n-1} \qquad \text{(A-5)}$$

and

$$\frac{d^2H}{dX^2} = 4\zeta(n-1)\left[\left(1-(2X)^n\right)\right]^{\frac{1-n}{n}}(2X)^{n-2} + \left(1-(2X)^n\right)^{\frac{1-2n}{n}}(2X)^{2n-2} \qquad \text{(A-6)}$$

Due to symmetry around the y axis we have

$$h_{(x)} = h_{(-x)}$$

$$\frac{\partial h}{\partial x_{(x)}} = \frac{\partial h}{\partial x_{(-x)}}$$

and

$$\frac{\partial^2 h}{\partial x_{(x)}^2} = \frac{\partial^2 h}{\partial x_{(-x)}^2} \qquad \text{(A-7)}$$

The values of the film thickness and its derivatives with respect to x can be consequently recorded for each nodal point in the domain $1 < i < N$.

Appendix B- The Thomas Algorithm

Given a boundary value problem described by the following finite difference equations:

$$(c_i\, y_{i-1} + a_i\, y_i + b_i\, y_{i+1} = d_i,\ (1 < i < N) \qquad \text{(B-1)}$$

Where a_i, b_i c_i and d_i are the nodal coefficients y_1 and y_N are the known boundary values.
The efficient algorithm to solve those simultaneous equations which can be organized into tri- diagonal matrix form, as follows:

1- Generate two arrays α_i and β_i

$$\alpha_i = -\, b_i \,/\, (a_i + c_i\, \alpha_{i-1}) \qquad \text{(B-2)}$$

and

$$\beta_i = (b_i - c_i \beta_{i-1}) \,/\, (a_i + c_i\, \alpha_{i-1}) \qquad \text{(B-3)}$$

with $\alpha_1 = 0$ and $\beta_1 = y_1$ as starting values.

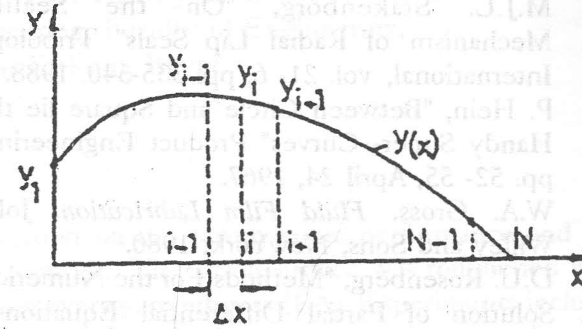

Figure B-1. The discretized domain.

2- Get the y_i values by back substitution

$$y_i = \alpha_i\, y_{i+1} + \beta_i \qquad \text{(B-4)}$$

Form the nodal point N- 1 back to point 2 in the discretized domain.

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