TWO-DIMENSIONAL CUTTING STOCK ALGORITHM BASED ON DIRECTED HEURISTIC

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ABSTRACT

A new approach for the two-dimensional cutting stock problem is presented in this work. The proposed algorithm incorporates human intuitive thoughts in allocating pieces on a rectangular stock sheet. Automating such thoughts in a computer algorithm, not only overcomes the human inability to produce adequate results in cases of large scaled problems, but also reduces allocation time, and the need of a skilled operator becomes uncritical. A number of such intuitive thoughts has been programmed and tested in this paper. Simulation results presented demonstrate the potential of such an approach.

Keywords: Cutting stock problem, Heuristic techniques.

INTRODUCTION

Proper utilization of raw material by minimizing trim losses is one of the main objectives in various industries. It is a major problem in sheet metal textile, furniture and flat glass industries, just to mention a few applications. In these applications, to achieve the previously mentioned objective, effective allocation of a set of two dimensional shapes (rectangular and/or irregular) onto a relatively larger stock sheet of finite dimensions should be carried out.

However, a comprehensive allocation algorithm which is suitable for various applications is not easily achievable, since each industry has its own special allocation restrictions and patterns [1]. Even for a single application, the problem is often considered to be difficult, since very large number of cutting patterns may be generated in the allocation process [2,3].

Surveying the literature, various approaches were proposed to define model and solve the problem [3]. Such techniques include restricted mathematical optimization [4], tree search [5], heuristics, [6,7], Neural Networks [1], and simulated annealing [8,9].

It is well known that the two-dimensional layout is a large scale combinatorial problem, as well as it is NP-complete [10]. Consequently, simplifying assumptions and restrictions were imposed in order to make it possible to formulate the problem, and thereby make it amenable to mathematical solution within practically acceptable implementation algorithms and time.

Some of the commonly used assumptions are: the guillotine patterns cut [4,5,11,12,13], limited number of patterns variety [13], constraint on the maximum number of each type in a pattern [3,5]. Some of the above assumptions could be accepted in practice, while others impose real limitations to the applicability of the used optimization technique. In addition, such approaches require excessive, and some times prohibitive amount of computing [11], especially for medium to large scale problems.

Tree-search and related algorithms were alternative approaches that have been used to tackle the allocation problem. However, to make the two-dimensional problem amenable to efficient solution by these algorithms,the size of tree search is limited by deriving and imposing necessary conditions for the cutting pattern to be optimal, as well as including designed node evaluation functions to give bounds for driving the search [5,14]. Such imposed conditions and bounds reflect negatively on the optimality of the solution.

Achieving close to optimal solutions with reasonable computing time is the objective of numerous heuristic approaches reported [3, 6, 8, 11, 15, 16]. Various heuristics were adopted to tackle the two-dimensional allocation problem, such as: Bottom-left [17]. Decreasing length perpendicular strip packing [6], the bin packing heuristic [2, 6] just to mention few methods. As with all allocations algorithms heuristics schemes are very dependent on the particular problem being solved, hence the allocation mechanism used must be derived from the problem environment. However, multiple-objectives can be incorporated within such heuristic techniques [3].

Human intervention, even if limited, has demonstrated that it had improved the quality of some of the above mentioned solution approaches substantially [7]. It is also the belief of the authors that human intervention would improve results, even if used with approaches where human intervention is not part of their development, such as simulated annealing. This can be seen by analyzing the allocation results obtained by such methods, such as that to Dagli [8]. However, although human intervention has demonstrated improvements, the efficiency and time required for such intervention depends on the skill of the operator, the size of the problem, and thereby the number of sheets to be cut. As well as the characteristics (features & variety) of the bill of the material (BOM) [19]. Kopardekar et.al. [18] focused on identifying human intuitive thought process in laying out of irregular parts. In that work,instead of adapting the human intervention to improve the outcome of a basic heuristic scheme, the human strategies of allocating the whole sheet were suggested to constitute a heuristic approach. However, human strategies in allocating the whole sheet may not be effective or easy to comprehend if BOM characteristics are wide. Hence to overcome such hurdle, an underlying allocation algorithm could be used, and then human strategies could be applied to continue the allocation process.

The identification and computer implementation of various human strategies in filling spaces generated from an underlying allocation algorithm is the objective of this work, whereby this approach direct human intervention is eliminated but not the intuitive thoughts. Heuristic and non-heuristic algorithms could be adopted as an underlying allocation technique, however the first fit decreasing length perpendicular (FFDLP) is used here, because of its simplicity, and for comparison with other approaches.

THE PROBLEM TACKLED

The pattern allocation process considered here is related to sheet metal industry, in which a standard-size rectangular stock sheet is available with unlimited number, is to be cut into demanded rectangles of smaller sizes as specified by a BOM.

The following constraints and specifications are to be taken into account:

1- The cuts to be performed are not restricted by the guillotine constraint and they should by orthogonal.

2- The orientation of a piece is not fixed, it could be rotated 90 degrees.

3- The pieces to be allocated should not overlap, and should be entirely packed within a sheet's area.

4- The demand of each type of pieces should be met exactly.

The pieces to be cut from the stock sheets, should be allocated in such way that would produce minimum waste, or equivalently reduce the number of needed stock sheets, the work is directed towards relatively large scaled problems with relatively large pieces variety.

Human reasoning in space recognition and filling.

In this section various techniques, which are actually representative among others to the way a human would think about filling confined spaces, which are generated after applying the FFDLP are presented.

1. **First-fit decreasing:** It is the very simple bin-packing heuristic that deals with the available gaps as separate bins or subspaces and works on filling them with the first piece in the BOM that fits in the space as illustrated in Figure (1). Obviously for any skilled or "intelligent" operator this approach is hardy used since the first piece which can be fitted may not be the best (optimal) piece in filling the considered space. However, it represents a simple and fast, but short sighted rule to fill the gaps. It is included here to show the gradual increase in depth of thought about the problem.

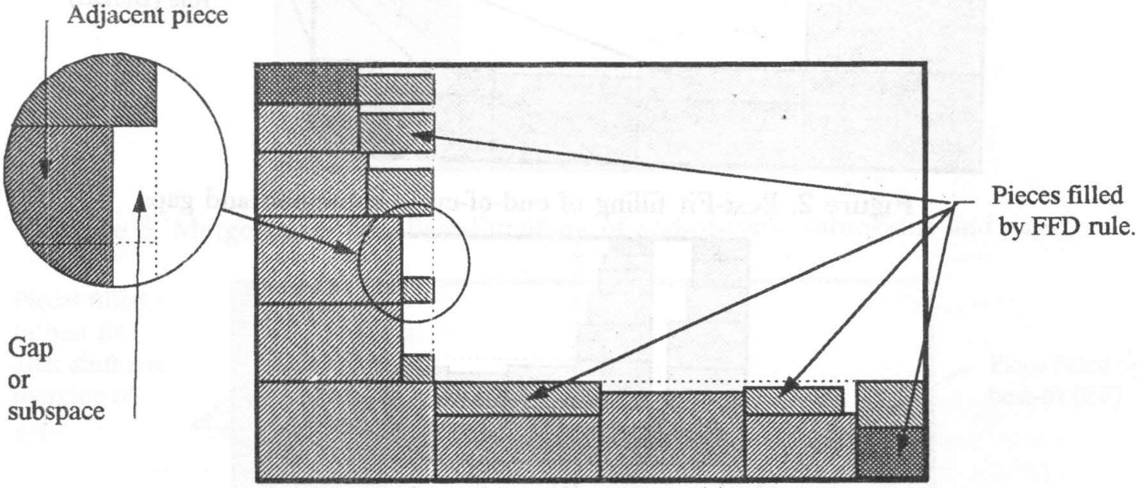


Figure 1. First-Fit filling of end-of-cycle subregion and gaps.

2. **Best-fit decreasing:** A human thought would naturally suggest such strategy in filling of gaps, where for a given space a human operator would look for a piece with the largest area that would cover as much as possible of a gap, such that minimum waste is produced. In implementing such heuristic rule, 90° rotation of a piece is also permitted to test for best filling, and thus provide the chance for better results as illustrated in Figure (2).

3. **Shift:** Different types of shifts could be considered by a human operator (Figure (3)), however, the one shown in Figure (4) is the implemented type, whereby this approach the gap spaces generated from previous allocations of pieces are reduced, and the recursive nature of the whole algorithm is preserved. This restricted type of shift is applied when the set of pieces on one edge of the sheet-excluding the corner piece- are less in their dimension than the corner piece.

4. **Merging of gaps into large rectangles:** By this approach the human ability to recognize contiguous space is realized and implemented in a simplified manner, where spaces sharing at least one common dimension are joined together, then the merged space are reconsidered for filling by rule (1) or (2) as Figure (5) shows.

5. **Largest rectangle:** The largest rectangular space that could be realized in an irregularly bounded unfilled space is searches by this heuristic, and considered for allocation with largest pieces, then the remaining spaces around it are considered for filling by other heuristic.

6. **Strips:** When a human operator recognizes at least one common dimension between pieces he tries to join them together and then allocate them as a single unit (a strip) of two or more pieces. This approach has the benefit of keeping edges of pieces and spaces aligned together which introduces some homogeneity to edges and thereby has potential for reduced waste.

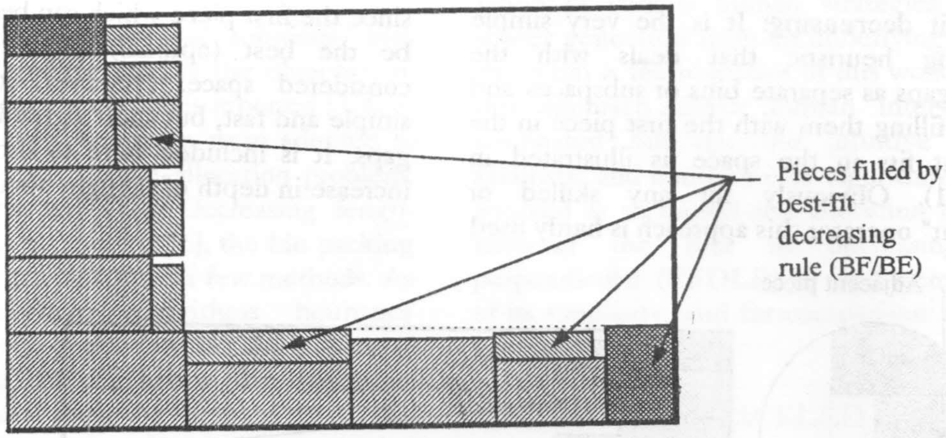


Figure 2. Best-Fit filling of end-of-cycle subregion and gaps.

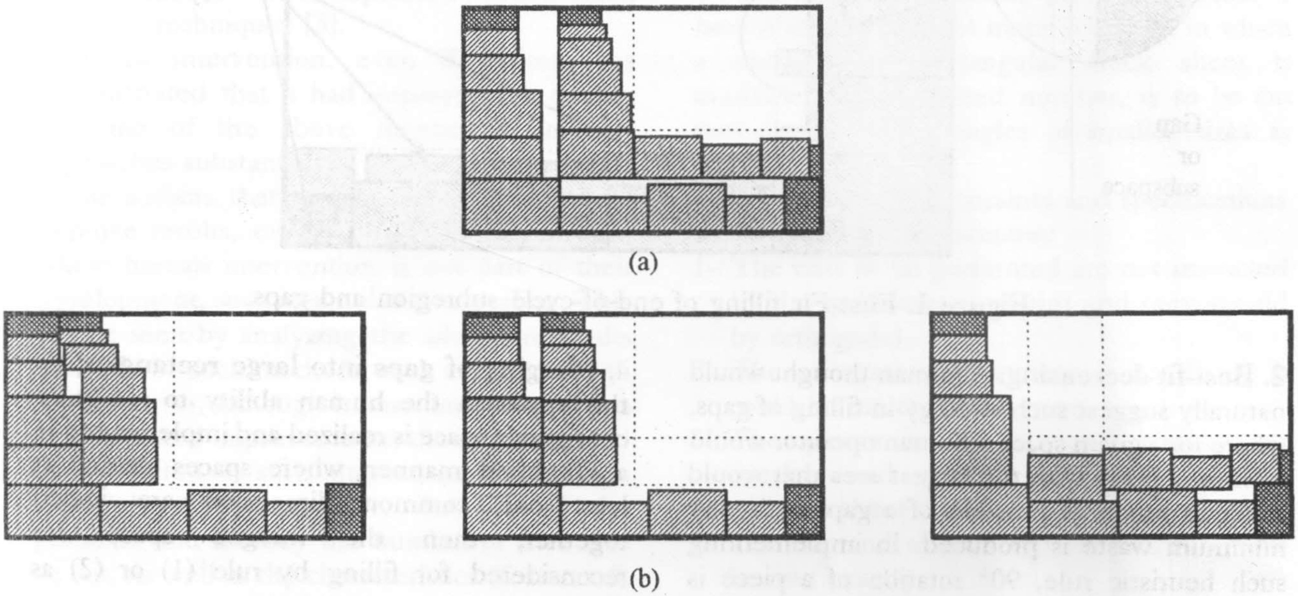


Figure 3. a) Before applying shift. b) After applying some of the different possible types of shift.

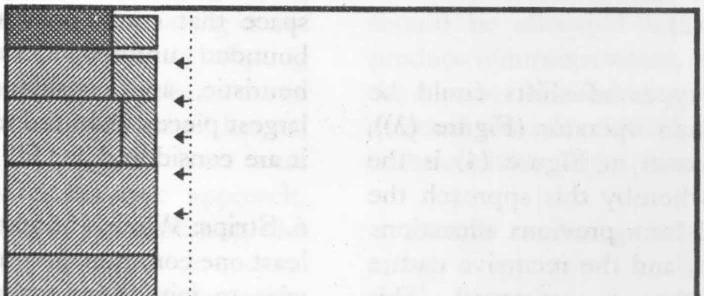


Figure 4. Shift, then best-fit filling of end-of-cycle subregions and gaps.

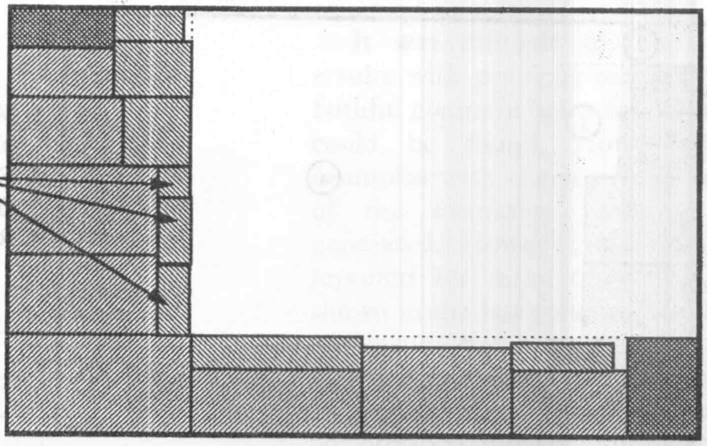


Figure 5. Merge-gaps, then Best-Fit filling of end-of-cycle subregions and gaps.

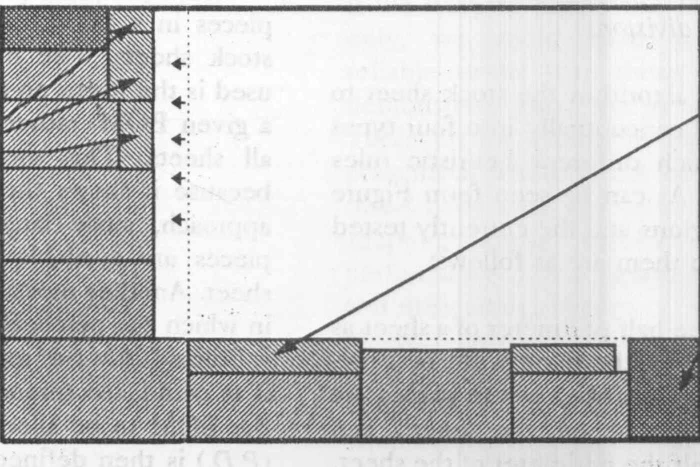


Figure 6. Shift, Merge-gaps, then Best-Fit filling of end-of-cycle subregions & gaps.

7. Other underlying rules and aspects:

- Combinations of the previous heuristic could also be used to take advantage of different characteristics of each, such as applying the shift strategy at some position then typing to fill the produced gap by (2), or the shift could be applied, then merging of spaces is considered, and after that the gaps are filled with best-fit decreasing as shown by Figure (6).
- Sorting the BOM according to nonincreasing length or width and starting with larger pieces in terms of area, length or width is also considered as a preallocation process on the BOM, with the aim of consuming larger pieces (relative to the stock sheet) ar earlier stages of allocation and preserving smaller ones for filling of possibly generated gaps and reduced sheet area at later stages.
- Deciding upon the rule or combination of rules to be applied at specific points of allocation, based on different characteristics of the original BOM or the remaining pieces to be allocated in conjunction with the space to be filled, could also be implemented. This decision making process which represents and inference engine of the approach is not implemented yet, and research is being pursued in its direction. However, in this work the heuristic or their combinations to be used in a specific allocation instance are left to the operator to be preselected before the allocation starts. The process is performed off line, a user could apply all possible heuristic combinations and selects the one which gives best results for practical implementation.

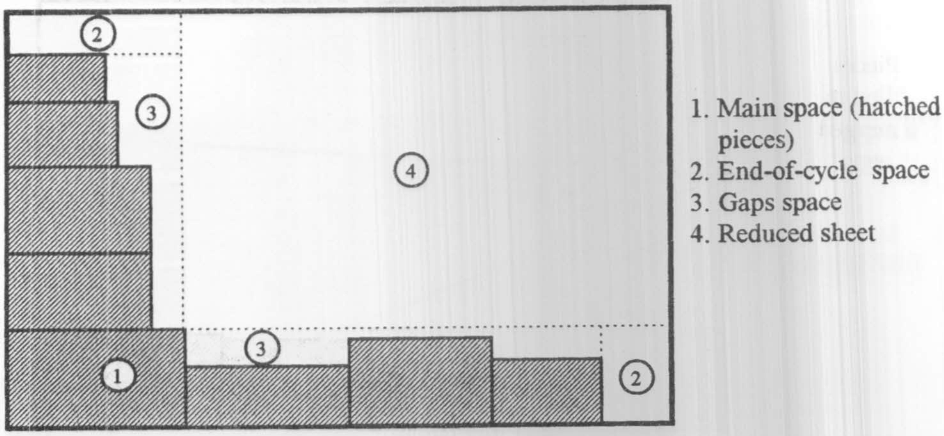


Figure 7. Sheet space division into four subregions.

Sheet Conceptual divisions:

In the presented algorithm the stock sheet to be filled is divided conceptually into four types of regions, on which different heuristic rules will be applicable, As can be seen form Figure (7), the four subregions and the currently tested heuristic related to them are as follows:

(1) Main-space: the half perimeter of a sheet as suggested by [6] to be filled with the First-Fit Decreasing heuristic, where pieces are alternately allocated on the length and width to fill half the perimeter of the sheet.
(2) End-of-cycles space: To be filled by the First-Fit or Best-Fit heuristic.
(3) Gaps between cycles: To be filled by a selected heuristic or combination of heuristic from the following set: 1) First-fit-decreasing, 2) Best-fit-decreasing. 3) Shift, 4) Merge spaces, 5) Largest rectangle, 6) Strips.
4) Reduced sheet: Where the divisions above are reconsidered in a recursive manner.

The flowchart of Figure (8) represents the major steps of the proposed algorithm.

TEST OF THE APPROACH

To assess the performance of the approach, several indices for measuring solution quality could be used. The most obvious one is the total waste produced after allocating all the pieces in a BOM, calculated over full packed stock sheets. The other index that could be used is the waste on the first sheet allocated for a given BOM, rather than the total water over all sheets. This measure is usually adopted because it shows the ultimate potential of the approach, since large number of unallocated pieces are available at that stage to fill that sheet. Another measure is the packing density, in which the practical concept of reusable scrap is utilized. A waste region is considered reusable if it could accommodate the average piece in the BOM to be allocated [7]. Packing Density (*P.D.*) is then defined as the ratio of total area of pieces allocated to stock sheet area used and expressed as follows:

$$P.D. = \frac{\sum_{i=1}^{n} a_i}{N.A - S} \quad (1)$$

where a_i is the area o a rectangular shape in the BOM to be allocated. *N* is the number of stock sheets used, *A* is the area of a single stock sheet, and *S* is the total reusable scrap generated.

Furthermore, computational time of allocation could be used to assess the performance of the algorithm, especially when comparing it with approaches based on human manual intervention in automatic layout solution approaches, or with fully manual layouts production.

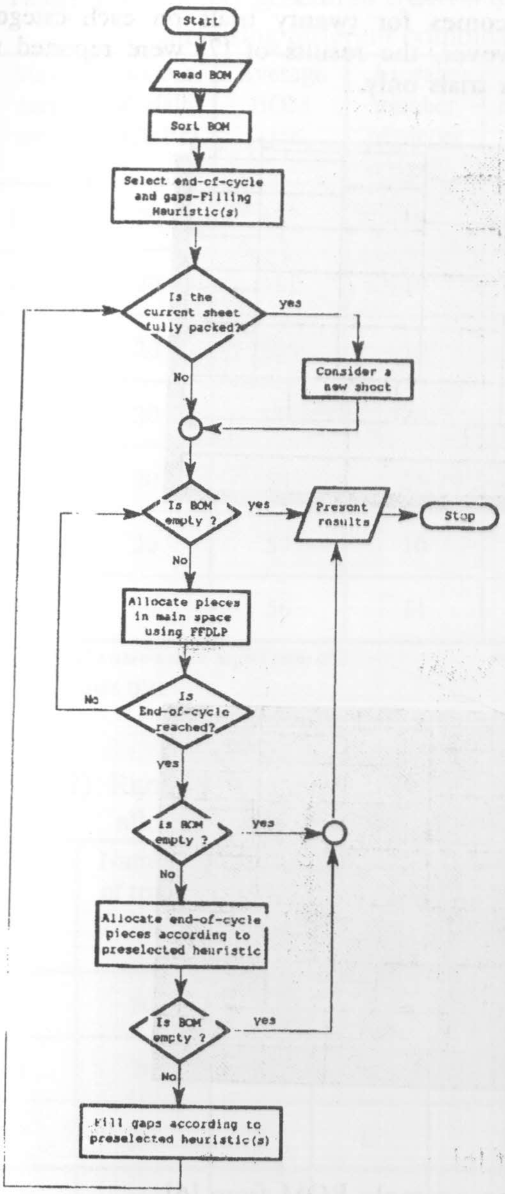


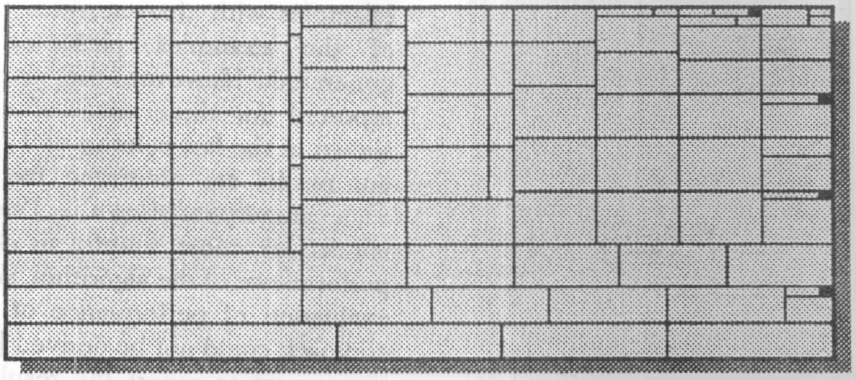
Figure 8. Allocating algorithm flow chart.

A random BOM generator based on a uniform distribution has been used to generate test samples with specific size, maximum piece are to sheet ratio, maximum aspect ratio of pieces, and maximum permissible demand of a piece type. Different individual heuristics and combination of heuristics were the applied to these BOM's. It was clear from the obtained results that combinations of heuristics showed to out perform individual heuristics over the range of tested samples of BOM's.

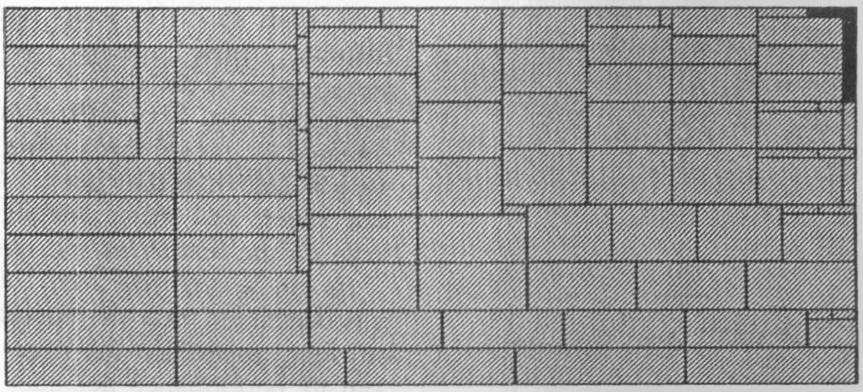
It was difficult to directly compare our results with previous research results since no faithful common bases for comparison purposes could be found. However, a number of examples with characteristics that match those of the examples presented in [6] were generated. However, the results of [6] were reported for three trials of each category as shown in the last column of Table (1). Whereas our results are presented for twenty trials of each category (as shown in Table (1)) since this is-relatively-more faithful in characterizing the performance of the algorithm, since an analytical evaluation of performance of such a heuristic approach could not be derived [7]. Furthermore, if we were to report the results of three trials only, we could have seen better-but not reliable-results than those reported in [6]. To demonstrate this, the detailed runs given in Table (3) for maximum area ratio of 1/100 are presented, where it can be seen from the first three trials that the scrap rate is zero. Then adapting these results to represent the performance of the algorithm is totally incorrect and misleading. Figure (9) shows the allocation obtained by our approach using a BOM that was used by [6], were they get a 0.46% scrap on the first sheet, and using our algorithm a 0.14% scrap on the first sheet was obtained.

As can be deduced from Table (1), generally it is concluded that the approach is practically efficient, especially for large scale problems. And its performance degrades as the piece-to-sheet are-ratio increases, which is natural, since the ability to maneuver larger area pieces is more restricted than smaller pieces, furthermore size mismatch among larger pieces produces larger scrap regions that could not be filled by other large pieces. But still the obtained figures for scrap rate are well below those practically accepted in industry [7].

Another set of BOM's were generated by sampling from a Beta distribution to control the area distribution of pieces in a sample and to also control the aspect ratio distribution in the same sample. The samples were categorized into nine classes that represent the different possibilities of cases for area aspect ratio. The categories and the corresponding results are given in Table (2). The same not for the number of trials given for the first set of results holds here also when comparing our results with the results of [7]. Where we report our outcomes for twenty trials on each category however, the results of [7] were reported for four trials only.



(a)



(b)

Figure 9. A fully packed sheet using a sample BOM from [6].
(a) Allocation using the algorithm proposed in this paper. Scrap rate is 0.14%
(b) Allocation using the algorithm of [6]. Scrap rate is 0.46%.

It is apparent that the presented approach gives remarkably better results in cases of large number of sheets, which is an expected result, since a human operator could not comprehend such large scale problems effectively. It is also worth mentioning that the performance of the algorithm becomes superior when the distribution of pieces areas is smooth, i.e., with no abrupt changes in areas.

In both sets of tests the algorithm proves to be efficient in terms of computational time, especially for large scale problems as the reported results for the time of allocation in both Table (1) and (2) show. This is contrasted to the fact that the time required for laying out pieces when human intervention is permitted, is highly dependent on the skill of the operator, and ranges from ten seconds to few minutes per stock sheet, and may extend to hours if large number of sheets with different sizes are to be tried [20]. Furthermore the time losses in such cases could be significantly large.

Table(1): Randomly generated BOM's from a uniform distribution and their allocation results using different combinations of filling heuristics*.

Max. area ratio	Number of trials (BOM's)	Average BOM size	Average number of pieces types	Average number of sheets	Average allocation time (sec)**	Average packing density%	Average scrap% ***	Average Scrap% by:[6]♣
1/200	20	487	10	2	1.61	99.90	0.14	-
1/100	20	311	19	2	1.20	99.03	0.84	1.36
1/50	20	265	20	3	1.20	97.08	2.84	0.62
1/20	20	132	20	4	0.49	94.51	4.97	-
1/10	20	84	14	5	0.39	92.14	7.06	5.74
1/6	20	57	10	6	0.28	87.44	13.13	9.04
1/4	20	56	11	9	0.28	85.08	16.20	11.96

* All BOM's have a max. aspect ratio of 3.
*** Scrap % is calculated on fully packed sheets over 20 trials.
** On a PC 486 DX2
♣ Scrap % is calculated on fully packed sheets over 3 trials.

Table(2): Randomly generated BOM's from Beta-distribution and their allocation results using different combinations of filling heuristics*.

#	Number of trials (BOM's)	Area ratio status	Aspect ratio status	Average number of pieces types	Average number of sheets	Average allocation time (sec) **	Average packing density %	Average packing density by (LI) *** [7]
1	20	S	S	61	3	0.40	93.65	94.30
2	20	S	M	73	3	0.40	95.71	97.59
3	20	S	L	59	4	0.46	96.00	97.28
4	20	M	S	83	15	0.63	89.05	89.82
5	20	M	M	90	14	0.55	91.81	91.99
6	20	M	L	53	15	0.71	92.76	92.43
7	20	L	S	60	37	0.68	72.39	70.13
8	20	L	M	73	29	0.71	80.47	76.54
9	20	L	L	29	26	0.63	88.92	85.35

S: Small M: Medium L: Large
* All BOM's have a size of 100 pieces. ** On a PC 486 DX2
*** (LI) Decreasing length perpendicular strip packing with human intervention. Packing Density calculated over 4 trial / category.

Table(3): Detailed results for Allocating 1/00 area-ratio pieces:

#	BOM Size	Variety	Total Scrap% **	Packing Density%	No. of Sheets	Time (sec) •
1	357	19	00.00	99.37	2	1.258
2	407	19	00.00	99.33	2	1.984
3	394	19	00.00	99.86	2	1.641
4	279	20	00.00	99.25	2	0.930
5	279	19	00.00	99.30	2	0.980
6	236	19	00.18	99.45	2	0.719
7	222	19	00.18	99.74	2	0.602
8	324	19	00.46	99.36	2	1.539
9	309	19	00.50	98.62	2	1.211
10	329	19	00.64	99.16	2	1.102
11	224	19	00.68	98.96	2	0.652
12	454	19	01.00	99.49	3	2.301
13	381	19	01.04	98.88	2	1.375
14	232	18	01.32	98.57	2	0.820
15	194	19	01.46	98.14	2	0.488
16	292	19	01.75	98.84	2	1.219
17	331	19	01.82	98.45	2	1.313
18	327	19	01.89	97.97	2	1.383
19	253	19	01.96	98.36	2	0.934
20	400	19	01.98	99.50	3	1.594
Averages	311	19	00.84	99.03	2	1.20

• On a PC 486 DX2 ** Scrap % is calculated on fully packed sheets .

CONCLUSION AND FUTURE WORK:

An algorithm for solving a version of the rectangular cutting stock problem has been presented. The results obtained show its success in eliminating the manual human intervention usually utilized in solving such a problem, but not his intuitive thoughts. Simultaneously the scrap, and utilization figures are nearly kept at the same levels of previous works if not enhanced. The approach is much more promising for large scale problems and could be developed to include irregular patterns.

Only a few of used human strategies were tested in this work to show its potentials, and to compare it with already published results. As for future work research is currently being pursued to identify more human strategies, and to automate the selection of filling heuristics at different strategic points of allocations (Fuzzy Logic could be used for such reasoning process).

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