# SHIP YAW AND SWAY CONTROL USING LQR AUTOPILOT

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ABSTRACT

A combined simplified linear model of the horizontal ship motion in sway and yaw, together with the steering system, in state space form is presented. A solution to the problem of autopilot design is carried out with LQR (Linear Quadratic Regulation) approach and comparisons of performance made with conventional PID (Proportional-Integral-Derivative) and PD controllers. Through a numerical example, the performances of the resultant control system, applied to a large container ship, are illustrated, and the advantages of using the LQR approach are demonstrated.

*Keyword: Ship Modeling, Digital Control, Linear Quadratic Regulation Technique.*

*Nomenclature*

| | | | |
|---|---|---|---|
| $A$ | Combined system matrix | $N_{\dot r}$ | $\partial N/\partial \dot r$ |
| $A_c$ | System matrix of PID controller | $N_v$ | $\partial N/\partial v$ |
| $A_m$ | System matrix for ship motion | $N_{\dot v}$ | $\partial N/\partial \dot v$ |
| $A_s$ | System matrix of steering mechanism | $N_\delta$ | $\partial N/\partial \delta$ |
| $b$ | Combined control vector | $Q$ | Symmetric non-negative definite matrix |
| $b_c$ | Control vector of PID controller | $q(k)$ | State vector of control system |
| $b_m$ | Control vector for ship motion | $q_c$ | State vector of PID controller |
| $b_s$ | Control vector of steering mechanism | $r$ | Rate of yaw |
| $c^T$ | Combined output vector | $\dot r$ | Yaw acceleration |
| $c_c^T$ | Output vector of PID controller | $T$ | Sampling time |
| $c_m^T$ | Output vector for ship motion | $T_1, T_2, T_3$ | Time constants |
| $c_s^T$ | Output vector of steering mechanism | $u_0$ | Nominal ship velocity |
| $d_c$ | Transmission value of PID controller | $v$ | Ship velocity in transverse direction |
| $e(k)$ | Error signal $= \psi(k)-\psi_d(k)$ | $\dot v$ | Sway acceleration |
| $H$ | Positive definite real, symmetric constant matrix | $x$ | Combined state vector |
| | | $x_G$ | x-position of center of gravity |
| $h$ | Control vector of the digital control system | $x_m$ | State vector for ship motion |
| | | $x_s$ | State vector of steering mechanism |
| $I_z$ | Mass moment of inertia of ship at about a vertical axis through G | $Y$ | Hydrodynamic force acting on ship in transverse direction |
| $J$ | Quadratic performance index | $Y_r$ | $\partial Y/\partial r$ |
| $K^T$ | State feedback vector | $Y_{\dot r}$ | $\partial Y/\partial \dot r$ |
| $K_D, K_I, K_P$ | PID controller's parameters | $Y_v$ | $\partial Y/\partial v$ |
| $K_s$ | Gain of transfer function of steering system | $Y_{\dot v}$ | $\partial Y/\partial \dot v$ |
| | | $Y_\delta$ | $\partial Y/\partial \delta$ |
| $m$ | Ship's mass | $\delta$ | Rudder angle |
| $N$ | Hydrodynamic moment acting on ship about a vertical axis through G | $\theta_c$ | Rudder command angle |
| | | $\lambda$ | Positive scalar weighing factor |
| $N_r$ | $\partial N/\partial r$ | $\phi$ | Transition matrix of digital control system |

| | |
|---|---|
| $\psi$ | Yaw angle |
| $\psi_d$ | Desired change in ship's course |

## INTRODUCTION

Simulation is an important tool which enables the design and development of digital steering control systems. Simulation models enable the analysis of control systems in both the frequency and time domains, in order to achieve the desired dynamic control performance. The model should be sufficiently representative of the vessel to be controlled, so that the dynamic performance can be established during the design process before actual installation onboard ship. The simulation model should represent the motions to be controlled. Automatic ship steering aims at controlling the ship's motion with respect to the desired course. In what concerns the problem of ship dynamics and manoeuvering, mathematical modeling and numerical estimation of the forces, moments and coefficients are presented in [1-4]. Horizontal motions and simultaneous heading control have been analyzed by numerous authors, [5-7]. Since the steering system plays an important role in the directional stability and the dynamic behavior of the ship during manoeuvering, it is important to consider the steering mechanism together with the ship dynamics model for a thorough study of the design of the autopilot.

Many researchers in recent years have investigated the problem of autopilot design [5-10]. Most of these authors have generally proposed systems based on modern control theory, with the relatively simple PID controller, pole assignment design or adaptive control using continuous time techniques [5-8]. An alternative autopilot control system based on neural networks has also been treated [9,10].

PID controllers are widely used on ships. But the real problem, is that the true relationship between the optimal controller parameters values and the whole system dynamics are not easy to identify without considerable experience of ship performance [9].

One of the modern optimal control design techniques that has found general practical application is the linear digital regulator approach. Although the regulator problem is defined with reference to a system with zero reference inputs, the optimal linear digital regulator design assures that the resultant system is stable, and possesses certain damping characteristics, so that the performance of the system will be satisfactory in practice even if the inputs are nonzero. This paper presents a possible alternative digital autopilot controller design by LQR technique. In order to demonstrate the optimal properties of that controller, conventional PID and PD controllers are also designed to perform the same course keeping under the same conditions. Comparison of performance is included.

## LINEAR MODEL OF SHIP MANOEUVERING

The main components of the course keeping control loop, in its most common configuration, are shown in Figure (1). The autopilot shown uses the heading error as the control signal to reach the required heading. The ship model used in the simulation of ship manoeuvering is based on ship kinematics and basic hydrodynamics [1]. The linearization of the equations of ship motion for the plane motions of the ship leads to uncoupling the yaw and sway equations from that for surge [3]. The linearization of the equations of motion is considered about a nominal condition at constant forward speed with negligible external disturbances from waves and wind. The remaining linear model representing sway and yaw motions will be

$$\begin{aligned}(m-Y_{\dot{v}})\dot{v}+(mx_G-Y_{\dot{r}})\dot{r}&=Y_v v+(Y_r-mu_0)r+Y_\delta.\delta\\ (mx_G-N_{\dot{v}})\dot{v}+(I_z-N_{\dot{r}})\dot{r}&=N_v v+(N_r-mx_G u_0)r+N_\delta.\delta\end{aligned} \tag{1}$$

where

$$r=d\psi/dt. \tag{2}$$

The motions described by equations (1) and (2) can be conveniently described as a linear SISO in state space form as:

$$\begin{aligned}\dot{\boldsymbol{x}}_m(s)&=\boldsymbol{A}_m.\boldsymbol{x}_m(s)+\boldsymbol{b}_m.\delta(s)\\ \psi(s)&=\mathbf{c}_m.\boldsymbol{x}_m(s)\end{aligned} \tag{3}$$

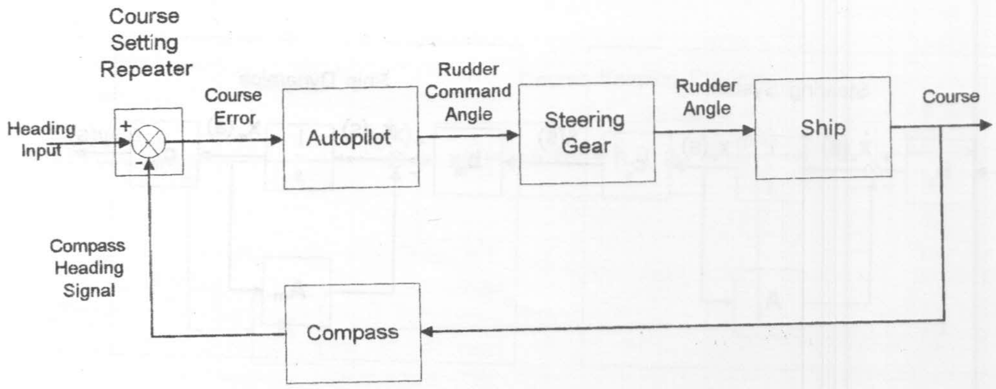


**Figure 1.** The course control loop.

This model describes the ship response to rudder deflection. The state vector $x_m(s)=[v(s)\ r(s)\ \psi(s)]^T$ is constituted by the sway velocity v, yaw rate r and the yaw angle $\psi$; the control input is constituted by the rudder angle $\delta$. The coefficients of the matrices $A_m$ and $b_m$ are given in detail in [4], Appendix 1. The dynamic analysis of the electrohydraulic steering system was carried out in [8]. For such a model a third order realization was found to be satisfactory in the form

$$\dot{x}_s(s)=A_s.x_s(s)+b_s.\theta_c(s)$$
$$\delta(s)=c_s.x_s(s) \tag{4}$$

where $x_s(s)=[x_1(s)\ \ x_2(s)\ \ x_3(s)]^T$ is an intermediate state vector, the input is the rudder command angle $\theta_c$ and the output is the rudder angle $\delta$. The coefficients of the matrices $A_s$ and $b_s$ depend on the characteristics of the mechanism construction, usually parameterized in terms of the rudder servomotor, solenoid and a three state hydraulic valve.

It is possible to obtain the linear model in a more compact form, expressed in terms of the joint state vector $x(s)=[x_m(s)\ |\ x_s(s)]^T$;

$$\dot{x}(s)=A.x(s)+b.\theta_c(s)$$
$$\psi(s)=c.x(s) \tag{5}$$

where

$$A=\begin{bmatrix} A_m & b_m c_s \\ 0 & A_s \end{bmatrix}, \tag{6}$$
$$b=[0\ \ b_s]^T\ ,c=[c_m\ \ 0]$$

The resulting open loop block diagram, shown in Figure (2), illustrates the ship being regarded as a linear SISO system having the rudder command angle $\theta_c$ as the control input and the heading angle $\psi$ as the output variable.

## OPTIMAL DIGITAL CONTROLLER DESIGN

In discrete form, the model of the process to be controlled, equation (5), can be represented by the following state space equations [11]:

$$x(k+1)=\phi.x(k)+h.\theta_c(k)$$
$$\psi(k)=c.x(k) \tag{7}$$

where

$$\phi(T)=e^{AT},$$
$$h(T)=-\int_{\tau=T}^{\tau=0}\phi(\tau).b.d\tau \tag{8}$$

and T is the sampling time.

According to these equations, it is possible to formulate the autopilot design problem within the framework of LQR theory [12,13]. This approach aims to design an optimal feedback gain vector K such that the feedback law

$$\theta_c(k)=-K.x(k) \tag{9}$$

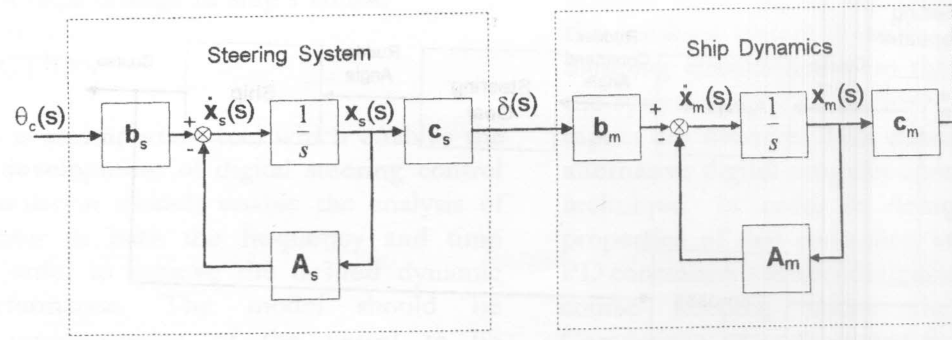


Figure 2. SISO representation of ship dynamics and steering system.

involves the minimization of a quadratic performance index of the type

$$J=\sum_{k=0}^{\infty}[x^T(k).Q.x(k)+\lambda.\theta_c^2(k)] \quad (10)$$

subjected to the initial condition x(0) = 0, where the symmetric non-negative definite matrix Q and the positive scalar weighing factor λ take into account the relative weights to be assigned to the different output variables as well as to the control input $\theta_c(k)$. The feedback vector K is thus given by

$$\mathbf{K}=(\lambda+h^T.H.h)^{-1}h^T.H.\phi \quad (11)$$

where the positive definite real, symmetric constant matrix H is the steady state solution to the associated discrete matrix Riccati equation

$$H=Q+\phi^T[H-H\,h(\lambda+h^T H\,h)^{-1}\,h^T H]\phi \quad (12)$$

The final form of the digital control system, with a desired input variable $\psi_d$ is shown in Figure (3).

## THE PID AND PD CONTROLLERS

To demonstrate how well the proposed optimal digital controller works, a PID controller will be also designed for comparison. A simple discrete PID controller can be written as:

$$\theta_c(k)=K_p.e(k)+K_I\sum_{i=1}^{k}e(i-1)+K_D[e(k)-e(k-1)] \quad (13)$$

where

$$e(k) = \Delta\psi(k) = \psi d(k) - \psi(k)$$

and $r_0 = K_P + K_D$
$r_1 = -K_p + K_I - 2.K_D$
$r_2 = K_D$.

From equation (13) we obtain the difference equation

$$\theta_c(k)-\theta_c(k-1)=r_0.e(k)+r_1.e(k-1)+r_2.e(k-2)$$

which can be reformulated easily in state space form [14] as:

$$q_c(k+1)=A_c.q_c(k)+b_c.e(k)$$
$$\theta_c(k)=c_c.q_c(k)+d_c.e(k) \quad (14)$$

where

$$A_c=\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}, b_c=\begin{bmatrix}0\\ 1\end{bmatrix}, \quad (15)$$
$$c_c=[r_2 \quad (r_0+r_1)], d_c=r_0.$$

Consider the closed loop control system, Figure (4), comprised of the model process with the PID controller located in the forward path with unity feedback and a desired input variable $\psi_d(k)$.
Both the process, equations (7), and the PID controller, equations (14), can be easily introduced in one set of state space equations [11], whereas the output signal $\psi(k)$ can be derived directly with respect to the desired signal $\psi_d(k)$.

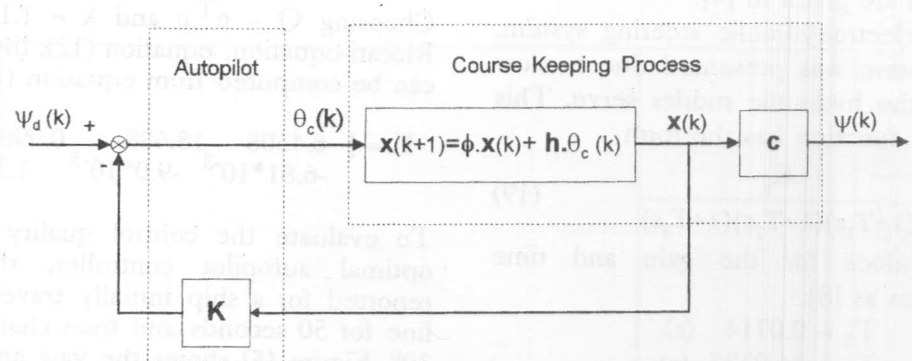


Figure 3. Ship control system with digital LQR controller.

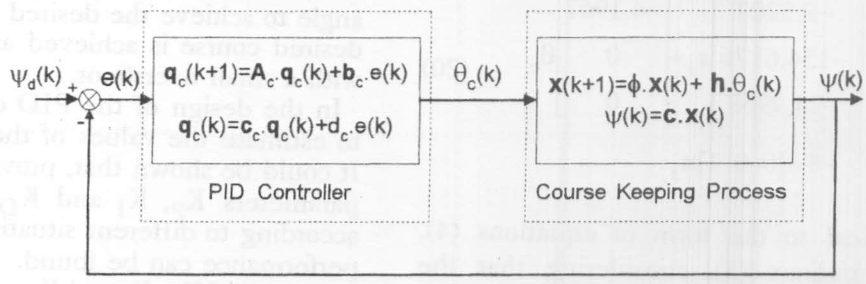


Figure 4. Steering control system with digital PID controller.

The over all state space equations will be

$$q(k+1)=A_r.q(k)+b_r.\psi_d(k)$$
$$\psi(k)=c_r.q(k) \tag{16}$$

where,

$$A_r=\begin{bmatrix} A_c & -b_c.c \\ h.c_c & \phi-h.d_c.c \end{bmatrix},$$
$$b_r=[b_c \quad h.d_c]^T, \tag{17}$$
$$c_r=[0 \quad c],$$

and $q(k) = [q_c(k) \mid x(k)]^T$.

For designing a PD controller, the parameter $K_D$ will be set equal zero.

Once the controller parameters values are identified, the response of the course angle $\psi(k)$ and the rudder angle deflection $\delta(k)$ can be calculated.

## NUMERICAL APPLICATION

Consider a large container ship with the following parameters:

| | | |
|---|---|---|
| Length LBP | = 170.6 | m |
| Breadth B | = 24.4 | m |
| Draft d | = 10.4 | m |
| Displacement Δ | = 20,000 | tdw |
| Nominal speed $u_0$ | =22.6 | knots |
| Nominal rpm $n_0$ | = 116.0 | rpm |
| Propeller diam. D = | 6.2 | m |

According to the assumption of constant forward speed and neglecting the nonlinear terms, the linear model of dynamical equations of motion represented in state variable form is

$$\dot{x}_m=\begin{bmatrix} -.0331 & -.0348 & 0 \\ -.0165 & -.0579 & 0 \\ 0 & 1 & 0 \end{bmatrix} x_m + \begin{bmatrix} 1.5*10^{-3} \\ -3.7*10^{-3} \\ 0 \end{bmatrix}\delta \tag{18}$$

$$\psi=[0\ 0\ 1]x_m$$

where $x_m = [\ v\ r\ \psi\ ]^T$.

The parameters for ship steering dynamics given in this steering model are given in [4].

Concerning the electrohydraulic steering system, the third order system was presented due to slow rate saturation in the hydraulic rudder servo. This third order transfer function has the form

$$\frac{\delta(s)}{\theta_c(s)} = \frac{K_s}{(1+T_1 s)(1+T_2 s)(1+T_3 s)} \tag{19}$$

The numerical values for the gain and time constants are chosen as [8]:

$K_s = -0.4408$ $\quad T_1 = 0.0714$ (s)

$T_2 = 0.0915$ (s) $\quad T_3 = 16.0772$ (s).

The transfer function stated in equation (19) can be rewritten in state space form as

$$\dot{x}_s = \begin{bmatrix} 0 & 0 & -9.5207 \\ 1 & 0 & -154.6176 \\ 0 & 1 & -24.9968 \end{bmatrix} x_s + \begin{bmatrix} -4.1967 \\ 0 \\ 0 \end{bmatrix} \theta_c \tag{20}$$

$$\delta = [0 \;\; 0 \;\; 1] x_s$$

which are identical to the form of equations (4). According to equations (5), considering that the heading angle $\psi$ is the output variable and the rudder command angle $\theta_c$ is the control input, equations (18) and (20) can be combined in one state space form as

$$\dot{x} = \begin{bmatrix} -.0331 & -.0348 & 0 & 0 & 0 & 1.5*10^{-3} \\ -.0165 & -.0579 & 0 & 0 & 0 & -3.7*10^{-3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -9.5207 \\ 0 & 0 & 0 & 1 & 0 & -154.6176 \\ 0 & 0 & 0 & 0 & 1 & -24.9968 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4.1967 \\ 0 \\ 0 \end{bmatrix} \theta_c$$

$$\psi = [0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0] \, x$$

Since the container ship dynamics are relatively slow, a sampling time of 1 second has been chosen. Referring to equations (7), the transition matrix $\phi$ can be calculated as:

$$\phi = \begin{bmatrix} .9677 & -3.33*10^{-2} & 0 & 8.2*10^{-6} & 9.7*10^{-6} & -1.3*10^{-7} \\ -1.58*10^{-2} & .944 & 0 & -1.9*10^{-5} & -2.2*10^{-5} & 2.5*10^{-6} \\ -8.01*10^{-3} & .9717 & 1 & -8.4*10^{-6} & -1.9*10^{-5} & -2.2*10^{-5} \\ 0 & 0 & 0 & .9493 & -5.9*10^{-2} & 3.62*10^{-3} \\ 0 & 0 & 0 & .1546 & -9.54*10^{-3} & -2.6*10^{-4} \\ 0 & 0 & 0 & 6.2*10^{-5} & -3.8*10^{-4} & -3.6*10^{-5} \end{bmatrix}$$

whereas the control vector h will be

$$h = [\, -1.5*10^{-5} \quad 3.5*10^{-5} \quad 1.0*10^{-5} \quad -4.1051 \quad -.5848 \quad -2.24*10^{-2} \,]^T$$

Choosing $Q = c^T.c$ and $\lambda = 1.1$ and solving the Riccati-equation, equation (12), the feedback gain K can be computed from equation (11)

$$K = [\, -6.4308 \quad 18.688 \quad 0.9982 \quad -6.81*10^{-3} \quad -9.0*10^{-5} \quad 1.3*10^{-6} \,]$$

To evaluate the control quality of the proposed optimal autopilot controller, the simulation is reported for a ship initially travelling in a straight line for 50 seconds and then changing the heading 30°. Figure (5) shows the yaw angle $\psi$, the rate of yaw r and the rudder angle $\delta$ versus time. It should be noted that the rate of yaw is very small with a maximum value of 0.4 (deg/s). The maximum rudder angle to achieve the desired course is about 7°. The desired course is achieved after about 300 seconds, with a small overshoot.

In the design of the PID controller, it is not easy to estimate the values of the controller parameters. It could be shown that, provided the PID controller parameters $K_P$, $K_I$ and $K_D$ are carefully adjusted according to different situations, then an acceptable performance can be found.

A range of $K_P$, $K_I$ and $K_D$ values were studied with the PID controller, $K_P \in [0,1]$, $K_I \in [0,0.01]$ and $K_D \in [-2,0]$, and the results presented correspond to the best values of all different combinations studied. The results for some of the estimated PID and PD controllers are presented in Figure (6) and (7). For the PID controller, Figure (6), it is noted that $K_I$ controls the degree of overshoot, and the results presented correspond to the best pair of $K_P$ and $K_D$ values of all the different combinations studied. For this reason, it is suggested to design a PD controller, where $K_I$ is absent, with different values of $K_P$ as shown in Figure (7). It can be noted that decreasing $K_P$ values will increase the delay time and the settling time of course keeping response. Whereas in what concerns the rudder angle, it can be stated that the changing of $K_I$ or $K_P$ values has an insignificant effect on the behavior of the rudder angle.

Finally, comparing the results for the three simulations described, using the LQR controller, the PID controller and PD controller,which are presented in Figure (5), (6) and (7), it will be noted that there is effectively no overshoot when using the LQR controller. Also, for the LQR controller, the ship responds more quickly to achieve the zero steady state error than that for PID and PD controllers. Hence the superiority of the LQR controller is demonstrated.

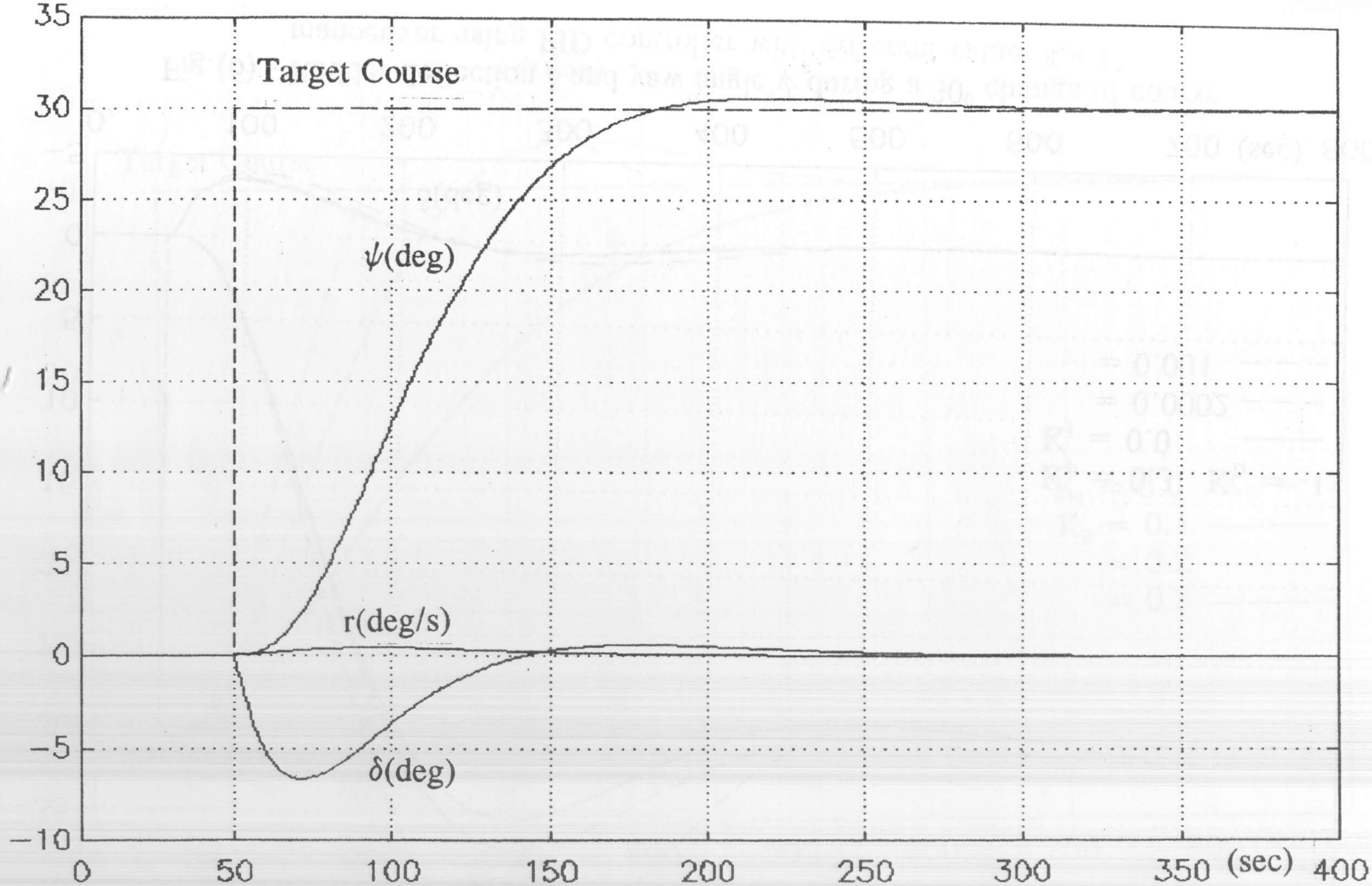


Fig.(5) - Rudder deflection $\delta$, yaw angle $\psi$, and yaw rate r during a $30^0$ change of course manoeuver using LQR controller

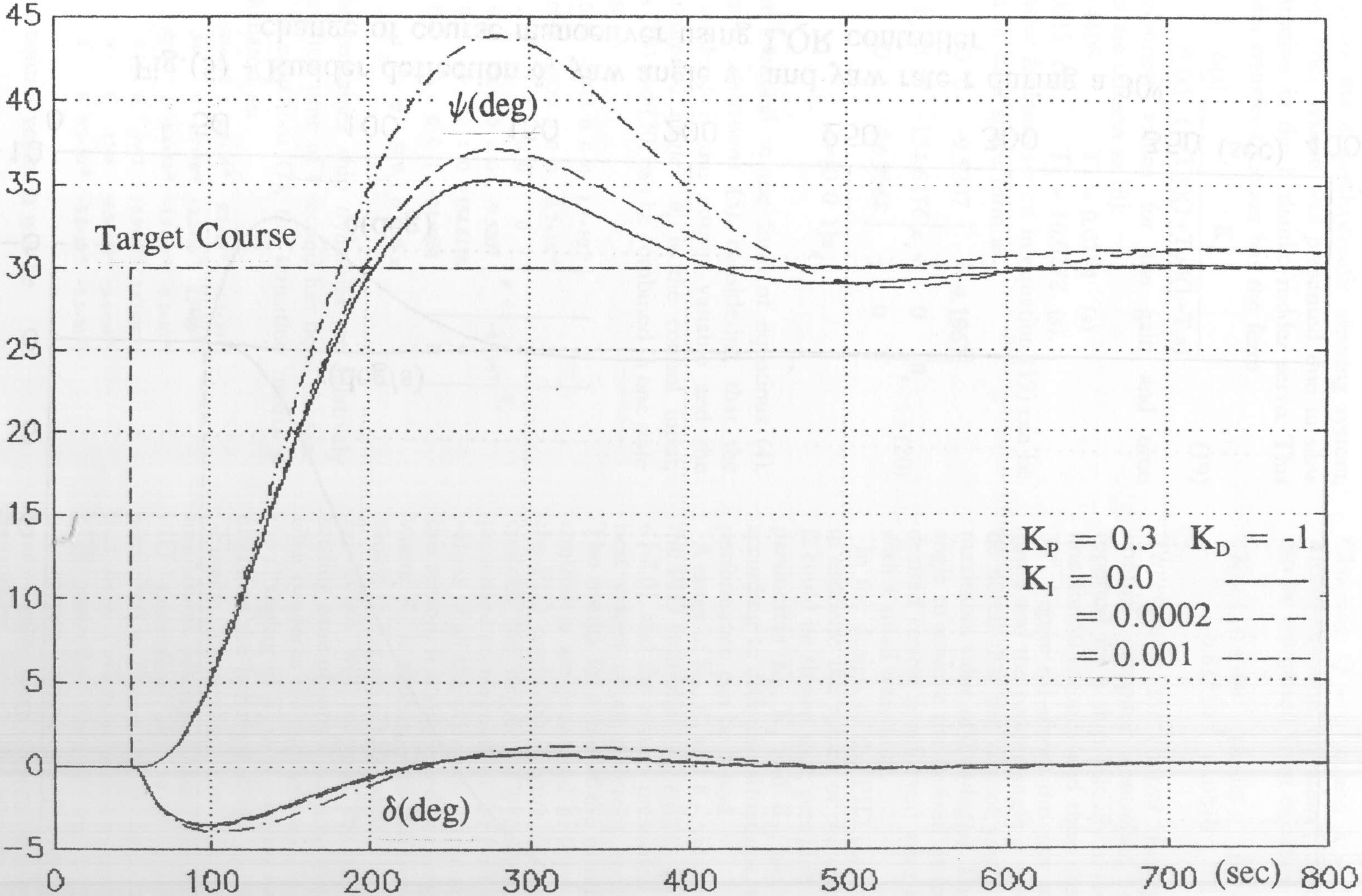


Fig.(6) - Rudder deflection $\delta$ and yaw angle $\psi$ during a $30^0$ change of course manoeuver using PID controller with different values for $K_I$

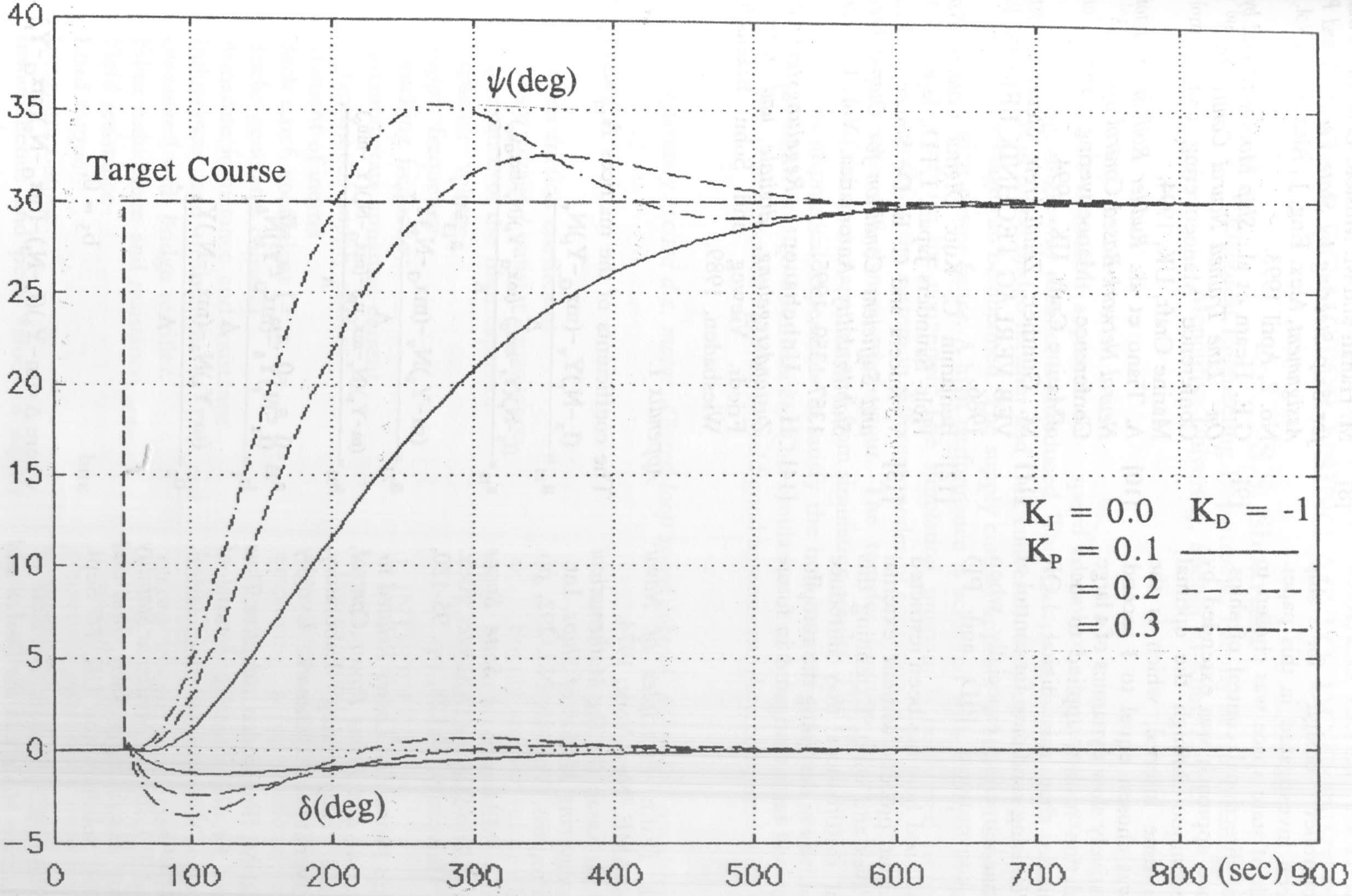


Fig.(7) - Rudder deflection $\delta$ and yaw angle $\psi$ during a $30^0$ change of course manoeuver using PD controller with different values for $K_P$ and $K_D$

## CONCLUSION

An autopilot controller design for ship manoeuvering has been investigated in this paper. The modern approach of state space was chosen to study the problem of directional control of ships. The digital state space approach was extended by considering LQR technique through the optimal state feedback. The time interval, which is the sampling duration, was chosen equal to 1 second. This is due to the relatively slow dynamics of a large container ship. Simulation results, applied to such container ship, indicate that this digital LQR controller type is a promising candidate for control of ship heading and manoeuvering especially when compared to the conventional PID and PD controllers.

The ship model discussed here has been restricted to a single input-output process without external disturbances from waves and wind. Extending this model with external disturbances may introduce some additional difficulties in designing the autopilot controller. This factor will be investigated in future work.

## REFERENCES


[1] J.P. Comstock (Ed.): *Principles of Naval Architecture*, SNAME, New York, 1974.

[2] A. Ogawa and H. Kasai: *On The Mathematical Model of Manoeuvering Motion of Ships*, Int. Shipbuilding Progress, Vol. 25, No. 292, pp. 306- 319, 1978.

[3] A. El-Iraki and M.Hanafi: *A State Space Representation of Ship Steering Dynamics*, Alex. Eng. J., Alex. University, Vol.28, pp. 95-120, 1989.

[4] M. Blanke: *Ship Propulsion Losses Related to Automatic Steering and Prime Power Control*, Thesis, Control Engineering Laboratory, Technical University of Denmark, Lyngby 1981.

[5] M. Fukino and M. Tomizuka: *Adaptive Time Optimal Control for Ship Steering*, Journal of Dynamic Systems, Measurement and Control, Vol. 108, Sept. 1986.

[6] T.I. Fossen and T. Lauvdal: *Nonlinear Stability Analysis of Ship Autopilots in Sway, Roll and Yaw*, MCMC'94, Southampton UK, 7-9 Sept. 1994.

[7] D.A. Bennett and J.C. Johnson: *The Role of Validated Simulation Models in the Design and Development of Digital Control Systems for Ship Steering*, MCMC'94, Southampton UK, 7-9 Sept. 1994.

[8] M. Hanafi and M. Mosleh: *Controller's Design for Ship Steering by State Feedback and Pole Assignment*, Alex. Eng. J., Alex. Univ., Vol. 32, No. 2, April 1993.

[9] G.E. Hearn et al.: *Ship Motion Control by an On- Line Trained Neural Controller*, 3rd Int. Conference Manoeuvering and Control of Marine Craft, UK, 1994.

[10] A. Tiano et al.: *Rudder Roll Stabilization by Neural Network-Based Control Systems*, 3rd Int. Conference Manoeuvering and Control ofMarine Craft, UK, 1994.

[11] M. Günther: *Zeitdiskrete Steuerungssysteme*, VEB VERLAG TECHNIK BERLIN, Berlin 1986.

[12] Benjamin C. Kuo: *Digital Control Systems*, Holt- Saunders Japan, LTD., Japan 1981.

[13] V. Kucera and C. E. De Souza: *A Necessary and Sufficient Condition for Output Feedback Stabilizability*, Automatica, Vol.31, No.9, pp. 1357-1359, 1995.

[14] H. Unbehauen: *Regelungstechnik II, Zustandsregelung, digitale und nichtlineare*, Friedr. Vieweg und Sohn Braunschweig/ Wiesbaden, 1989.


*Appendix 1:*

The coefficients of the matrices $A_m$ and $b_m$ are:

$$a_{11}=\frac{(I_z-N_{\dot r})Y_v-(mx_G-Y_{\dot r})N_v}{\Lambda},$$

$$a_{12}=\frac{(I_z-N_{\dot r})(Y_r-mu_0)-(mx_G-Y_{\dot r})(N_r-mx_Gu_0)}{\Lambda},$$

$$a_{13}=0 ,$$

$$a_{21}=\frac{(m-Y_{\dot v})N_v-(mx_G-N_{\dot v})Y_v}{\Lambda},$$

$$a_{22}=\frac{(m-Y_{\dot v})(N_r-mx_Gu_0)-(mx_G-N_{\dot v})(Y_r-mu_0)}{\Lambda},$$

$$a_{23}=0 ,a_{31}=0 ,a_{32}=1 ,a_{33}=0 ,$$

$$b_1=\frac{(I_z-N_{\dot r})Y_\delta-(mx_G-Y_{\dot r})N_\delta}{\Lambda},$$

$$b_2=\frac{(m-Y_{\dot v})N_\delta-(mx_G-N_{\dot v})Y_\delta}{\Lambda},$$

and $b_3 = 0,$

where $\Lambda=(m-Y_{\dot v})(I_z-N_{\dot r})-(mx_G-N_{\dot v})(mx_G-Y_{\dot r})$ .