OVERTOPPING EARTH-DAM FAILURE

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ABSTRACT

Despite modern advances in technology, dam failures continue to occur. About forty percent of earth dams have been failed because of overtopping. In the present study new models are developed for overtopping dam failures. The proposed models are derived depending on hydrodynamic principle (Broad crested-weir) and erosion equation. Breach erosion is assumed nonlinear function of the outflow mean velocity. The model of cubic erosion function and rectangular cross section is derived and tested by using the available data which was collected by Singh [4]. Also the nonlinear erosion with fourth higher degree ,for both rectangular and trapezoidal cross sections, are derived. The rectangular cross section gives more accurate results than any other model .

Keywords: Overtopping, Dam failure, Earth dam, Breached dam, Dam erosion, Failure time , failure discharge.

Notation

A_b breach water area cross section;
A_i numerical coefficients (i = 1 and 2);
A_s surface area of the reservoir;
b bottom width of breach;
H water elevation above reference datum ;
H_o initial water elevation ;
H_{tf} the hydraulic head at the instant that erosion is terminated;
h hydraulic head;
h_o initial hydraulic head ;
I the inflow discharge into the reservoir ;
Q outflow discharge from spillway and powerhouse ;
Q_b outflow discharge through breach ;
$Q_{b\ max}$ maximum outflow discharge through breach (at failure time) ;
R correlation coefficient;
S breach side slope ;
t time ;
t_f time of failure ;
t_o initial time ;
U outflow velocity ;
Z breach bottom elevation above reference datum ;
Z_f final breach bottom elevation;
Z_o initial breach bottom elevation ;
α_1 discharge coefficient ;
α_2 erosivity coefficient ;
β_1 discharge exponent ; and ;
β_2 erosivity exponent.

INTRODUCTION :

Some of embankment dams which are expected to overtop during probable maximum floods may be failed. Failure of dam is time-dependent phenomenon, multiphase (water-soil), and the process involved during failure are very dynamic and complicated.

Singh and Scarlatos [4] listed the hydraulic models of failures which were developed before 1988. A list of these models with their special features is given in *Table (1).* They also developed an analytical model for simulation of earth dam breach erosion. They derived solutions for rectangular, triangular , and trapezoidal-shaped breaches for either linear or quadratic functions only. They [4] obtained a data for 52 historical dam-failure cases from three sources. By using the historical data, they tested their models for special cases.

Tabl (1): Mathematical models for dam breach erosion.

Model and year (1)	Hydrody-namics (2)	Sediment transport (3)	Solution method (4)	Breach morphology (5)	Parameters (6)	Other features (7)
Cristofano (1966)	Broad-crested weir flow	Empirical formula	Manual iterative	Constant breach width	Angle of response, others	None
Harris and Wagner (1967); BRDAM (Brown and Rogers (1977)	Broad-crested weir flow	Schoklitsch bed-load formula	Numerical solution	Parabolic breach shape	Breach ediments	None
Fread 1977)	Broad-crested weir flow	Linear d erosion	Numerical iterative	Rectangular, triangular, trapezoidal	Breach dimensions ,others	Tailwater effect
Lou (1981); Poce and Tsivoglou. (1981)	Full mic system	Empirical Meyer-Peter and Mueller	Priessman-n's finite differences	Regime type relation	Critical shear stress, sediment	Tailwater effect
Breach (Fread 1984, 1985)	Broad-crested weir flow	Meyer-Peter and Mueller formula Smart formula	Numerical iterative	Rectangular, triangular, trapezoidal	Critical shear, sediment	Tailwater dry slope stability
BEED (Singh and Scarlatos 1985)	Broad-crested weir flow	Einstein-Brown formula	Numerical iterative	Rectangular, trapezoidal	sediment, others	Tailwater saturated slope stability

Worman A.[5] mentioned that Haward and McLane (1988) derived an expression for the critical shear stress in which both seepage and surface flow are taken into account. In 1989, Powledge [2,3] discussed model and prototype research ,which have been conducted in the United States and Great Britain, to evaluate how embankments for dams perform when they subjected to overtopping flow. Also they presented a summary of historical case of overtopping events at dam and levee embankments. In 1993, Worman A.[5]derived an expression for the critical flow condition ,which causes an incipient erosion, considering the seepage force and the effect of the dynamic pressure distribution on buoyancy.

In the present study, analytical solutions are derived for cubic function of rectangular breach and fourth degree for both rectangular and trapezoidal breaches.

MATHEMATICAL MODELS:

The performance of overtopping embankment dam is influenced by the water flowing breach ,the reservoir-volume balance equation, and the rate of erodibility relationship. The general accepted equation to determine the flow over and through the breach is the broad crested weir flow; Figure (1):

$$Q_b = \alpha_1 (H-Z)^{\beta_1} . A_b \tag{1}$$

in which,

Q_b = Outflow discharge ;
α_1 and β_1 = Empirical coefficients.
H = Water elevation from a reference datum ;
Z = Breach bottom elevation from a reference datum; and;
A_b = Breach water area cross section.

The water-volume balance equation can be written as :

$$(A_s)\frac{dH}{dt} = I - Q_b - Q \tag{2}$$

in which,

A_s = Surface area within the reservoir;
I = The inflow discharge into the reservoir ;
Q = from crest overtopping ; spillway ; and;
t = Time.

The additional equation is the erosion rate as a function of flow velocity ; i.e.,

$$\frac{dZ}{dt} = -\alpha_2 (U)^{\beta_2} \tag{3}$$

in which,

α_2 and β_2 = Empirical coefficients ; and ;
U = Outflow velocity.

Assumptions:

The following assumptions are considered:

(1) the difference between I and Q in Eq. (2) is of much less order of magnitude than Q_b.
(2) the value of A_s is independent of H (i.e.,prismatic reservoir)
(3) the proper initial conditions are :

$$H = H_o \text{ and } Z = Z_o \text{ at } t = t_o \tag{4}$$

According to the above assumptions the two basic equations 2 and 3 written as:

$$\frac{dH}{dt} = -\frac{\alpha_1}{A_s}(H-Z)^{\beta_1} . A_b \tag{5}$$

and

$$\frac{dZ}{dt} = -\alpha_2 [\alpha_1 (H-Z)^{\beta_1}]^{\beta_2} \tag{6}$$

The main objective of this paper is to develop analytical solutions for nonlinear erosion ($\beta_2 = 3$ and 4) for rectangular breach and $\beta_2 = 4$ for trapezoidal cross section.

(I) Rectangular section

Depending on the value of β_2= 3 and 4 and based on the above equations and assumptions, using Ref.[1], analytical solutions were developed for rectangular cross-section. The rectangular breach has constant width b and enlarges only in the vertical direction as shown Figure(1)

$$A_b = b\,(H - Z) \qquad (7)$$

(1) **Nonlinear erosion: β_2 = 3**

Combining Eqs. (5) and (7) and dividing by Eq. (6) one obtains for β_1 =1/2

$$\frac{dH}{dZ} = \frac{b}{\alpha_1^2\,\alpha_2 A_s}$$

$$\frac{dh}{dZ} = \frac{b}{\alpha_1^2 \alpha_2 A_s} - 1$$

The solution of the above equation with the initial conditions is:

$$h = \left(\frac{b}{\alpha_1^2 \alpha_2 A_s} - 1\right)(Z - Z_o) + h_o \qquad (8)$$

and to get the relation between h and t the following relation is considered:

$$\frac{dh}{dt} = \frac{dH}{dt} - \frac{dZ}{dt}$$

The solution of the above equation according to the initial conditions is:

$$h = \left[\frac{\alpha_1^3 \alpha_2 t}{2}\left(\frac{b}{\alpha_1^2 \alpha_2 A_s} - 1\right) + \frac{1}{\sqrt{h_o}}\right]^{-2} \qquad (9)$$

Nonlinear erosion :β_2= 4

The following equations are developed considering β_2 = 4 :

$$[h - h_o] + 2A_1(\sqrt{h} - \sqrt{h_o}) + 2A_1^2 \, Ln\left[\frac{A_1 - \sqrt{h}}{A_1 - \sqrt{h_o}}\right] = Z_o - Z \qquad (10)$$

and

$$\left[\frac{A_1}{\sqrt{h}} - \frac{A_1}{\sqrt{h_o}}\right] + Ln\left[\left(\frac{\sqrt{h} - A_1}{\sqrt{h}}\right)\left(\frac{\sqrt{h_o}}{\sqrt{h_o} - A_1}\right)\right] = \frac{\alpha_1^4 \alpha_2}{2} A_1^3 t \qquad (11)$$

where;

$$A_1 = \frac{b}{\alpha_1^3 \alpha_2 A_s}$$

(II) Trapezoidal section :

Depending on the value of β_2 = 4 and based on the above equations and assumptions, analytical solutions were developed for trapezoidal cross section. The trapezoidal breach has constant width b and enlarges as shown in Figure(1)

$$A_b = b(H - Z) + S(H - Z)^2$$

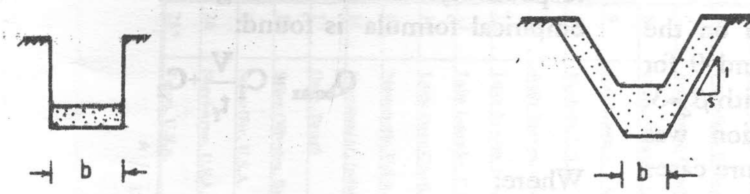
Nonlinear erosion : β_2 = 4

After simplification, the following equations are developed for trapezoidal breach considering β_2 =4:

$$2A_3(\sqrt{h}\,Ln\,a + \sqrt{h_o}\,Ln\,a_o) - \left(\frac{1 + 2A_2A_3}{a_1}\right) Ln\left[\frac{(a_1 + c).(a_1 + c_o)}{(a_1 - c).(a_1 - c_o)}\right] = \frac{(Z - Z_o)}{2} A_3^2 \qquad (12)$$

and

$$Ln\left[\frac{h a_o}{a h_o}\right] - 2A_2\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{h_o}}\right) + \left(\frac{1 - 2A_2A_3}{2A_3^2 a_1}\right) Ln\left[\frac{(a_1 - c_o).(a_1 + c)}{(a_1 + c_o).(a_1 + c)}\right] = \alpha_1^4 \alpha_2 A_2 t \qquad (13)$$



Erosive Patterns of Various Breach Shapes: (a)Rectangle; (b)Trapezoidal

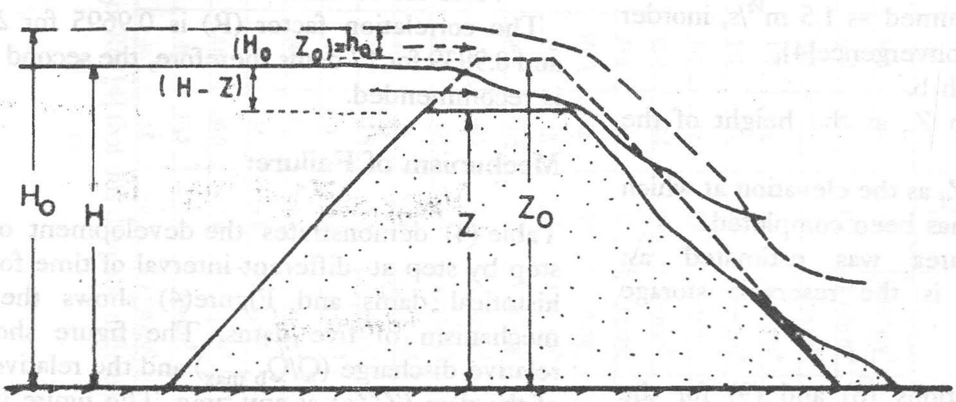


Figure 1. Schematic sketch of earth dam and failure mechanism.

in which:

$$A_2=\frac{b}{\alpha_1^3\alpha_2 A_s},\; A_3=\frac{S}{\alpha_1^3\alpha_2 A_s}$$

$$a=A_2-\sqrt{h}+A_3h$$

$$a_o=A_2-\sqrt{h_o}+A_3h_o$$

$$a_1=\sqrt{1-4A_2A_3}$$

$$c=1-2A_3\sqrt{h}$$

$$c_o=1-2A_3\sqrt{h_o}$$

Depletion of Reservoir Water after Termination of Erosion.

When the erosion process has been completed ($Z=0$ or $Z=$ constant) ,Eq.(5) can be written as

$$\frac{dH}{dt}=-\alpha_1\frac{b}{A_s}H^{3/2}$$

The solution of the above equation is

$$H_t=\frac{4}{\left[\frac{\alpha_1 b(t-t_f)}{A_s}+\frac{2}{(H_{tf})^{1/2}}\right]^2} \qquad (14)$$

where

H_{tf} is the hydraulic head at the instant that erosion is terminated;

H_t is the hhead at any time;and;

t_f is the time at which the erosion process has been completed.

It is noticed that we can also get Eq.(14) by put a_2 in Eq.(9) equal to zero. The initial head in this case will be H_{tf} and the initial time is $t-t_f$.

RESULTS AND ANALYSIS :

The analysis of earth-dams failure based on the above assumption gives the equations 8 and 9 for rectangular breach with nonlinear erosion with β_2=3. The performance of the analytical solution was evaluated using data from historical dam-failure cases Ref. [4] and showed in Table (2).

The input data :

(1) coefficientα_1 was assumed as 1.5 $m^{1/2}/s$, inorder to take of the flow convergence[4],
(2) terminal breach width b.
(3) initial crest elevation Z_o as the height of the dam.
(4) final crest elevation Z_f as the elevation at which the erosion process has been completed.
(5) reservoir surface area was estimated as: A_s=V/H_o, where V is the reservoir storage volume.

As we have two equations (8) and (9) for the model, the two unknown quantities α_2 and H_o had to be estimated. In Table (2) the simulated discharge and time of failure is given for 21 historical cases. From this table it can be seen that the overall performance of nonlinear new model (with β_2 =3) is near closed to the observed values.
A detailed testing of the model using Eq. (8) and (9) is done for the failure of Teton dam at the Teton River in Idaho, which was failed in June 5, 1976, to make a comparison between the derived new model and the suggested models which was derived by Singh and Scarlatos (1988) Figure (2). The input data for simulation are provided in Ref.[4]. From Figure (2) it can be noticed that the proposed model is more accurate and simulate to the observed than Singh's model.

Another comparative study is carried out for the time of failure and the maximum discharge in Singh's models and the proposed model, Table (3). The table shows that the proposed model results is more closer to the observed data than Singh's results.

A relationship between the quantity of storage volume (V), time of failure (t_f) and the maximum discharge ($Q_{b\ max}$) ,from the historical data, for 21 dams and 17 dams are shown in Figures (3-a&b), respectively. From the figures, the following empirical formula is found:

$$Q_{bmax} = C_1 \frac{V}{t_f} + C_2 \qquad (15)$$

Where:
C_1 is a constant and equals 2.75 for 21 dams and 2.0 for 17 dams, and
C_2 is a constant and equals -1427 for 21 dams and 91.08 for 17 dams with discharge units.

The correlation factor (R) is 0.9695 for 21 dams and 0.9939 for 17 dams therefore, the second relation is recommended.

Mechanism of Failure:

Table (4) demonstrates the development of failure step by step at different interval of time for the 21 historical dams and Figure(4) shows the failure mechanism of five dams. The figure shows the relative discharge ($Q/Q_{b\ max}$) and the relative height of the dam (Z/Zo) at any time. The figure indicates that the failure behavior is varied for each dam. This because the dimensions and construction materials are not the same for the studied cases.

SUMMARY AND CONCLUSIONS:

An analytical model for dam failure for both rectangular and trapezoidal breaches have been developed for the simulation of earth dam process. The model of rectangular breach for nonlinear erosion with β_2 =3 is tested and compared with the previous models. The following conclusions are obtained:

(1) The rectangular breach model (with β_2 = 3.0) is able to stimulate the maximum outflow discharge on the failure time than any other model.
(2) The dams failed with difference initial hydraulic head ,which affect on the performance of the model specially on the failure time.
(3) The derived model can be used to expect the maximum allowable overtopping head, the maximum outflow discharge, and the development of dam profile, during the failure.

Table(2) Check of rectangular breach model.

Dam No.	Name and country of dam	year built/ failed	Average breach breadth (b) (m)	Dam Height (Z_o) (m)	Height at end of breaching (Z_f) (m)	Water - surface area of reservoir (A_s) (m^2)	Initial water - surface height (H_o) (m)	Maximum height of water ($H - Z_f$) (m)	Erosivity coefficient α_2	Peak outflow discharge $Q_{b\max}$ (m^3 / sec) calculated Eq.(8)	Peak outflow discharge observed	Time of failures (sec.) calculated Eq.(9)	Time of failures (sec.) observed	Initial head (H_o - Z_o) = h_o (m)
1	A pishapa, U.S.A.	1920/1933	86.5	34	3.5	6.617×10^5	35.22	14.0	0.0001	6.8×10^3	6.85×10^3	9025	9000	1.22
2	Baldwin Hills, U.S.A.	1951/1963	16.5	49	21.5	2.245×10^4	49.17	12.6	0.000598	1.11×10^3	1.10×10^3	4680	4680	0.17
3	Break Neck Run,	1877/1902	30.5	7	0.0	7.0×10^3	7.122	0.34	0.002	9.2	9.2	10810	10800	0.12
4	Buffalo Creek, U.S.A.	1972/1972	125	14	0.0	4.357×10^4	14.28	3.9	0.00172	1.44×10^3	1.42×10^3	1840	1800	0.28
5	Euclides de Cummha,	1958/1977	131	53	0.0	2.566×10^5	54.18	3.0	0.000235	1.024×10^3	1.02×10^3	25160	26280	1.18
6	Frankfurt, Germany	1975/1977	6.9	10	0.0	3.5×10^4	10.84	3.9	0.000126	7.9x10	7.9 x 10	9015	9000	0.84
7	Frenchman Creek,	1952/1952	60.4	12.5	0.0	1.68×10^6	13.6	6.2	0.000027	1.4×10^3	1.41×10^3	29680		1.10
8	Goose Creek, U.S.A.	1903/1916	26.4	6	1.9	1.768×10^6	7.6	5.5	0.000126	5.08×10^2	5.65×10^2	1805	1800	1.60
9	Hatchtown, U.S.A.	1908/1914	160.2	19	0.0	7.79×10^5	20.95	4.2	0.000104	2.1×10^3	2.1×10^3	10870	10800	1.95
10	Hatfield, U.S.A.	1908/1911	91.5	6.8	0.0	1.809×10^6	7.22	5.9	0.000115	1.96×10^3	3.4×10^3	7245	7200	0.42
11	Kelly Barnes, U.S.A.	1948/1977	26.5	11.5	0.0	4.391×10^4	12.11	6.6	0.00056	6.74×10^2	6.8×10^2	1810	1800	0.61
12	Lake Avalon, U.S.A.	1894/1904	137	14.5	0.0	5.354×10^5	15.56	5.0	0.000156	2.32×10^3	2.32×10^3	7260	7200	1.06
13	Lake Latonka, U.S.A.	1965/1966	33.5	13	0.0	1.223×10^5	13.96	3.2	0.000147	2.86×10^2	2.9×10^2	10820	10800	0.96
14	Little Deer Creek,	1962/1963	23	26	4.6	6.654×10^4	32.44	11.4	0.0002	1.33×10^3	1.33×10^3	1250	1200	6.44
15	Mammoth, U.S.A.	1916/1917	9.2	21.3	0.0	6.388×10^5	26.0	18.9	0.0000191	1.13×10^3	2.52×10^3	10780	10800	4.70
16	Nanaksagar, India.	1962/1967	46	16	0.0	1.313×10^7	21.65	15.1	0.0000038	4.05×10^3	9.7×10^3	43150	43200	5.65
17	Oros, Brazil	1960/1960	200	35.5	0.0	1.831×10^7	36.38	11.4	0.0000069	1.155×10^4	1.15×10^4	223050		0.88
18	Salles Oliveira, Brazil	1966/1977	168	35	0.0	7.4×10^5	37.6	9.3	0.000125	7.2×10^3	7.2×10^3	7205	7200	2.60
19	Schaeffer, U.S.A.	----/1921	210	30.5	3.0	1.285×10^5	31.85	5.9	0.00087	4.5×10^3	4.5×10^3	1850	1800	1.35
20	Sherburne, U.S.A.	1892/1905	46	10.5	0.0	4.0×10^3	11.45	5.8	0.0095	9.64×10^2	9.6×10^2	80		0.95
21*	Teton, U.S.A.	1972/1976	46	93	14.0	3.288×10^6	95.07	64.5	0.0000297	3.6×10^4	6.6×10^4	14400	14400	2.07

* It is appear from Ref.[4] that this dam had different recorded values of terminal breach weadth and time of failure.

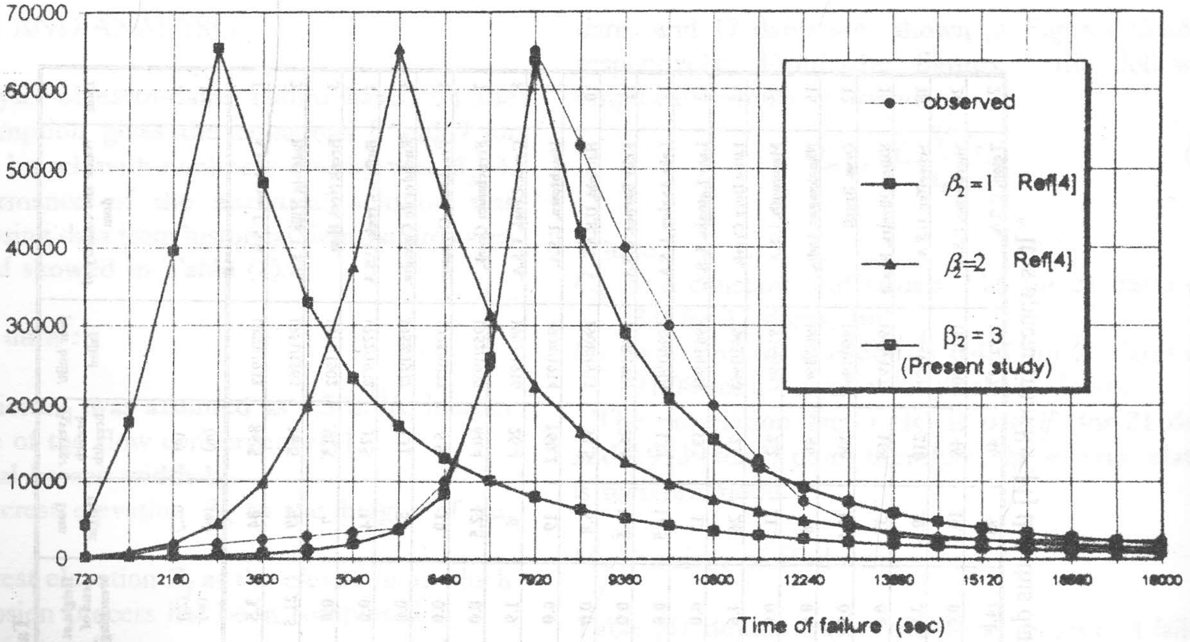


Figure 2. Simulated and observed [4] outflow discharge during failure of Teton dam.

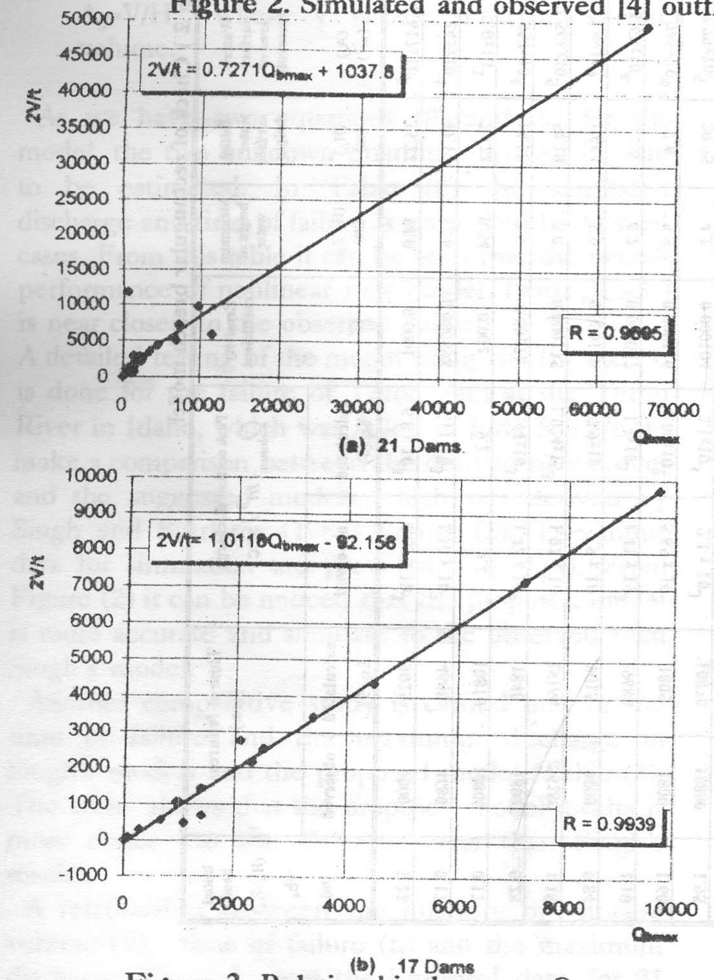


Figure 3. Relationship between $Q_{b\ max}$ and 2V/t. (observed [4]).

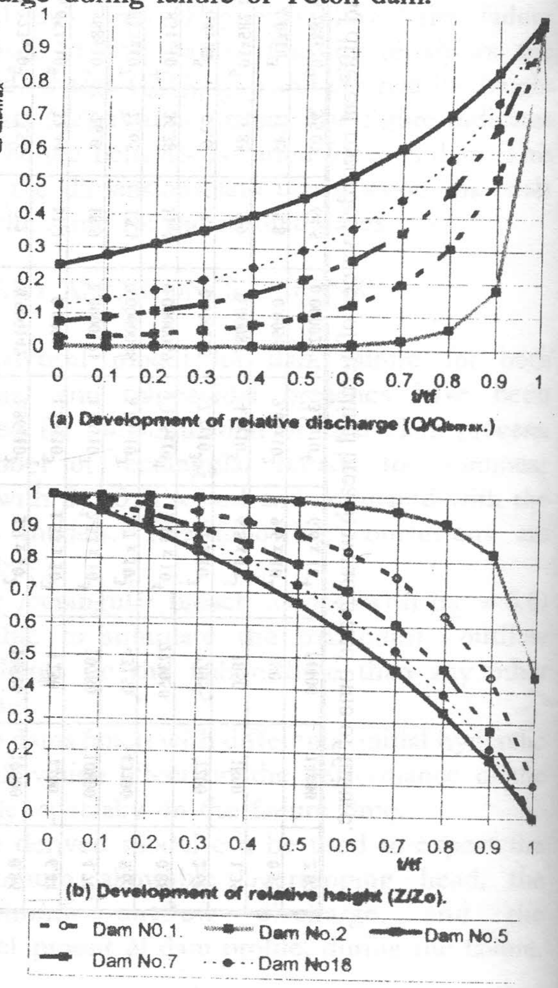


Figure 4. Development of dam failure.

Table (3) : Comparison between proposed model and Ref.[4].

Dam number. c	Erosivity coefficient α_2			Simulated maximum discharge				failure time	
	Linear	Non linear			Linear	Non linear			
	β_2 =1.0c	β_2 = 2.0c	β_2 = 3.0	Observed	β_2 = 1.0c	β_2 = 2.0c	β_2 =3.0	Recorded	β_2 = 3.0
1	0.0020	0.00040	0.0001000	$6.85x10^3$	$6.35x10^3$	$6.90x10^3$	$6.800x10^3$	9000	9025
2	0.0070	0.00095	0.0005980	$1.10x10^3$	$6.75x10^2$	$4.00x10^2$	$1.110x10^3$	4680	4680
3	0.0010	-	0.0020000	0.92x10	0.45x10	-b	0.9200x10	10800	10810
4	0.0085	-	0.0017200	$1.42x10^3$	$1.10x10^3$	-b	$1.420x10^3$	1800	1840
5	0.0014	0.00080	0.0002350	$1.02x10^3$	$1.05x10^3$	$6.10x10^3$a	$1.020x10^3$	26280	25160
6	0.0010	0.00080	0.0001260	7.90x10	9.20x10	1.40x10a	7.900x10	9000	9015
7	-	-	0.0000270	$1.41x10^3$	-	-	$1.410x10^3$	-	29680
8	0.0013	0.00060	0.0001260	$5.65x10^2$	$3.22x10^2$	$2.51x10^2$	$5.650x10^2$	1800	1805
9	0.0008	0.00025	0.0001040	$2.10x10^3$	$2.20x10^3$	$2.40x10^3$	$2.100x10^3$	10800	10870
10	0.0020	0.00065	0.0001150	$3.40x10^3$	$1.70x10^3$	$1.50x10^3$	$3.400x10^3$	7200	7245
11	0.0050	0.00080	0.0005600	$6.80x10^2$	$5.40x10^2$	$2.67x10^2$	$6.740x10^2$	1800	1810
12	-	-	0.0001560	$2.32x10^3$	-	-	$2.320x10^3$	7200	7260
13	0.0010	0.00050	0.0001470	$2.90x10^2$	$3.50x10^2$	$5.80x10^2$a	$2.860x10^2$	10800	10820
14	0.0090	0.00095	0.0002000	$1.33x10^3$	$1.50x10^3$	$1.20x10^3$	$1.330x10^3$	1188	1250
15	0.0050	0.00085	0.0000191	$2.52x10^3$	$1.20x10^2$	$1.20x10^2$	1.1300x10	10800	10780
16	0.0003	0.00015	0.0000038	$9.70x10^3$	$3.10x10^3$	$2.80x10^3$	$4.050x10^3$	4320	43150
17	-	-	0.0000069	$1.15x10^4$	-	-	$1.155x10^4$	-	223050
18	0.0020	0.00035	0.0001250	$7.20x10^3$	$7.30x10^3$	$6.10x10^3$	$7.200x10^3$	7200	7205
19	0.0080	0.00210	0.0008700	$4.50x10^3$	$4.40x10^3$	$5.80x10^3$a	$4.500x10^3$	1800	1850
20	-	-	0.0095000	$9.60x10^2$	-	-	$9.640x10^2$	-	80
21	-	-	0.0000297	$6.60x10^4$	-	-	$3.600x10^4$	14400	14400

a : The model was able to simulate the maximum outflow discharge but in much less failure time.
b : The model was not able to simulate either the maximum outflow discharge or the failure time.
c : Data from Ref. [4].

Table(4). Dam failure development.

t/t_f		Dam number																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0.0	Q/Q_{bmax}	0.03	0.00	0.21	0.02	0.25	0.10	0.07	0.16	0.31	0.02	0.03	0.10	0.16	0.42	0.12	0.23	0.02	0.15	0.11	0.07	0.00
	Z/Z_o	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	Q/Q_{bmax}	0.03	0.00	0.24	0.03	0.28	0.12	0.09	0.17	0.34	0.02	0.04	0.12	0.19	0.46	0.15	0.26	0.03	0.17	0.13	0.08	0.00
	Z/Z_o	0.99	1.00	0.95	0.99	0.95	0.97	0.97	0.97	0.95	0.99	0.98	0.97	0.96	0.95	0.96	0.95	0.99	0.96	0.97	0.98	0.99
0.2	Q/Q_{bmax}	0.04	0.00	0.27	0.03	0.31	0.15	0.11	0.20	0.38	0.03	0.04	0.14	0.22	0.49	0.17	0.29	0.03	0.20	0.15	0.10	0.01
	Z/Z_o	0.97	1.00	0.90	0.97	0.89	0.93	0.94	0.94	0.88	0.97	0.96	0.93	0.91	0.89	0.92	0.90	0.97	0.92	0.93	0.95	0.98
0.3	Q/Q_{bmax}	0.05	0.00	0.31	0.04	0.35	0.18	0.13	0.25	0.42	0.04	0.06	0.17	0.27	0.54	0.20	0.33	0.04	0.23	0.18	0.12	0.01
	Z/Z_o	0.95	0.99	0.84	0.95	0.83	0.88	0.90	0.90	0.81	0.95	0.94	0.88	0.85	0.82	0.87	0.83	0.95	0.86	0.89	0.91	0.97
0.4	Q/Q_{bmax}	0.07	0.01	0.36	0.06	0.40	0.23	0.16	0.29	0.47	0.05	0.07	0.22	0.32	0.58	0.24	0.38	0.06	0.28	0.22	0.14	0.01
	Z/Z_o	0.92	0.99	0.77	0.92	0.75	0.82	0.85	0.86	0.73	0.92	0.91	0.82	0.78	0.75	0.81	0.76	0.92	0.80	0.84	0.87	0.96
0.5	Q/Q_{bmax}	0.09	0.01	0.42	0.08	0.46	0.29	0.21	0.34	0.53	0.08	0.10	0.27	0.36	0.63	0.30	0.44	0.08	0.33	0.27	0.18	0.02
	Z/Z_o	0.88	0.98	0.69	0.89	0.67	0.74	0.79	0.81	0.64	0.89	0.86	0.75	0.69	0.68	0.74	0.68	0.88	0.73	0.78	0.81	0.94
0.6	Q/Q_{bmax}	0.13	0.02	0.49	0.11	0.53	0.38	0.27	0.41	0.60	0.11	0.14	0.35	0.43	0.69	0.36	0.51	0.11	0.40	0.34	0.23	0.03
	Z/Z_o	0.83	0.97	0.59	0.83	0.57	0.64	0.71	0.75	0.54	0.83	0.80	0.65	0.58	0.59	0.65	0.58	0.82	0.64	0.70	0.75	0.91
0.7	Q/Q_{bmax}	0.20	0.03	0.58	0.17	0.61	0.50	0.35	0.51	0.67	0.17	0.21	0.47	0.52	0.75	0.45	0.59	0.18	0.49	0.43	0.30	0.05
	Z/Z_o	0.75	0.96	0.48	0.75	0.46	0.51	0.61	0.67	0.44	0.75	0.72	0.53	0.45	0.50	0.55	0.47	0.74	0.53	0.60	0.64	0.85
0.8	Q/Q_{bmax}	0.31	0.06	0.69	0.27	0.71	0.69	0.48	0.63	0.77	0.27	0.32	0.64	0.63	0.82	0.58	0.70	0.29	0.61	0.55	0.40	0.10
	Z/Z_o	0.64	0.92	0.35	0.63	0.33	0.33	0.47	0.58	0.31	0.63	0.58	0.36	0.29	0.40	0.41	0.34	0.61	0.39	0.48	0.51	0.75
0.9	Q/Q_{bmax}	0.52	0.18	0.82	0.49	0.84	0.98	0.68	0.80	0.87	0.48	0.53	0.90	0.80	0.91	0.75	0.83	0.50	0.77	0.73	0.60	0.26
	Z/Z_o	0.45	0.83	0.19	0.42	0.18	0.08	0.28	0.46	0.16	0.41	0.37	0.13	0.07	0.30	0.23	0.19	0.39	0.22	0.32	0.34	0.50
1.0	Q/Q_{bmax}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Z/Z_o	0.10	0.44	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.10	0.00	0.00

(4) The relation between the volume of storage, the time of failure, and the maximum discharge can be estimated from the following empirical form:

$$Q_{bmmax} = 2\frac{V}{t_f} + 91.08 \qquad (16)$$

(5) More researches toward erosivity coefficient α_2 and the breach breadth b should be considered in future studies.

REFERENCES

[1] I.S. Gradshteyn and I. M. Ruzik, "Table of Integrals, Series and products". Academic Press, New York, N.Y., 1983.

[2] G.R. Powledge D.C. Ralston P. Miller, Chen Y. Hai, P.E. Clopper and D.M. Temple, "Mechanics of Overflow Erosion Embankments.I: Research Activities, " Journal of Hydraulics Division ASCE, vol. 115, No. HY 8, Paper No. 23762., 1989.

[3] G.R. Powledge D.C. Ralston P. Miller, Chen Y. Hai, P.E. Clopper and D.M. Temple, "Mechanics of Over flow Erosion Embankments. II: Hydraulic and Design Considerationsties ,"Journal of Hydraulics Division ASCE, vol.115,No.HY8, Paper No. 23763, 1989.

[4] V.P. Singh and P.D. Scarlatos, "Analysis of Gradual Earth-Dam Failure". Journal of Hydraulic Engineering, ASCE, vol. 14, No. 1., 1988.

[5] A. Worman, "Seepage-Induced Mass Wasting in Coarse Soil Slopes",Journal of Hydraulic Division ASCE, vol. 19, No. 10, Paper No. 3469, 1993.