DETERMINATION OF STRAIN RELAXATION IN A PLATE USING CRACK METHOD

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ABSTRACT

An investigation of strain relaxation in a plate subjected to combined tension and bending stresses are carried out . The basic concept adopted here in this investigation is to apply known stresses on a plate and then release a certain value of these stresses by making two single edge cracks. Known stresses are considered as being residual stresses. The released strain on the relieved portion was measured using strain gages. A comparison between the results obtained using two single edge cracks method and two double edge cracks method is presented for the case of a plate under tension. The effect of crack depth and its location for measuring strain relaxation is investigated under tensile loading conditions for three different materials. A good agreement between theoretical and experimental results is addressed.

Keywords: strain relaxation, residual stress, finite elements, crack

Nomenclature

A	Area of the element.
C_{ijkl}	Constitutive (strain-stress) matrix.
E_{ijkl}	Constitutive (stress-strain) matrix.
T_i, {T}	Surface traction.
U_i, {U}	Matrix of the displacement field in the element.
π_D, π_c	Total potential and complementary energy functionals.
σ_{ij}	Stress vector.
ϵ_{ij}	Strain tensor.
v_i	The component of unit vector normal to the boundary.

INTRODUCTION

The majority of the various phases of manufacturing processes of structural parts causes residual stresses. Prediction of the magnitude of residual stresses has been under focus for considerable attention during the past several decades [1-3]. A little attention was paid to predict the behavior of highly stressed cracked structures. A new calibration method is proposed in [4] using finite element analysis to determine the correlation coefficient. It is observed that the variation of the strains measured on the surface for each incremental change in hole depth is caused by two aspects. One is due to the residual stresses on the surface and the other is due to the change of the hole geometry. The strains on the surface near a loaded crack for cuts of different width to depth ratios are compared by [5]. Numerical results were presented for normal stress distributions which vary with distance from the surface as power functions of zero to third order. Their approach can apply to any surface traction on the faces of the cut and satisfies the stress free condition on the rest of the boundary. In many practical applications rectangular plates found frequently to be subjected to combined loading, tension and bending. In this paper strain relaxation has been investigated for a tensile plate using the two double edge cracks method. The effect of crack depth and crack's location in measuring strain relaxation is considered. Strain relaxation for a plate having two single edge cracks is obtained under applied combined tension and bending stresses. Actual strain relaxation of the plate can also be measured using four strain gages. Design chart for different engineering materials to determine the actual applied stress in a tensile loaded plate has been suggested. Results showed generally a good agreement between numerical and experimental estimates of the strain relaxation.

THEORETICAL ANALYSIS

An application of Tong et al [6] model, hybrid stress model, has been employed to formulate the crack tip super-element stiffness matrix. The regular finite element displacement model was used in all other remaining elements. The finite element mesh for single edge and double edge crack are given in Figures (1a), (2a) for only a quarter plate due to geometric symmetry as shown in Figures (1b), (2b). An isoparametric quadrilateral 4-node element type was used as a regular element; while 5-node super-element was used to represent the crack tip region. For crack depths of 12, 10, 8, 6, 4, 2 mm, the total number of elements was found to be in each case 71, 71, 62, 62, 63, 54 with total number of degrees of freedom 178, 178, 158, 158, 162, 142 respectively. In the finite element analysis, tension forces were equally distributed as concentrated loads at the nodal edge points. For the case of bending load, the equivalent nodal forces were calculated using force and moment equilibrium, as given in the appendix. Stress and strain distributions have been calculated using a combination of two different finite element models. The first one is the conventional finite element functional in terms of displacements and strains as in the following

$$\pi_p = \int_v 0.5 E_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \, dv - \int s_\sigma T_i u_i \, ds \qquad (1)$$

Where the volume integral represents the work of internal strain energy while the variation of the surface integral represents the external work done by the specified surface traction T_i during displacements u_i. Then by choosing a continuous function for the displacement field (shape function), the compatibility conditions will be enforced and the equilibrium conditions will be the automatic consequence of the stationary conditions. This conventional finite element displacement model is well explained with FORTRAN programming in Hinton's book [7]. The authors picked up the necessary subroutines from the original programing list, which was given in [8], and then joined them together with the main program to solve this particular plate stress problem.

Some modification in that FORTRAN finite element computer program were needed in order to be able to run it on the IBM PC instead of main frame computer system, and get exactly the same answer for the same worked example given in Ref. [7].

The second finite element model is based on the modified complementary energy principle (hybrid stress) which is known as the two field principles. One for the assumed stress field within the element and other for the independent assumed displacement on its boundary. This modified complementary energy functional can be written as

$$\pi_c = \int_{\partial A} \{T\}^T \{U\} \, ds - 0.5 \int_{\partial A} \{t\}^T \{U\} \, ds \qquad (2)$$

where,

$$T_i = \sigma_{ij} \, v_j \qquad \text{on } \partial A$$

$$0.5 \, (U_{i,j} + U_{j,i}) = C_{ijkl} \, \sigma_{kl} \qquad \text{in A} \qquad (3)$$

This second model is used in the element located at the crack tip and is called the stress hybrid singular super element. The concept of that super element was first developed by Tong et al [6]. This stress hybrid singular super element was checked as a single stiffness subroutine with the results obtained in Ref. [6]. After results confirmation, this super stiffness matrix was properly joined to the other regular stiffness matrix to form a global stiffness matrix in a subroutine called STIFPS. This program was compiled using the Microsoft FORTRAN Optimizing Compiler [9] installed on IBM PC. Stress and strain relaxation have been determined using these models for the plate under both tensile loading only and combined tensile with bending loading conditions.

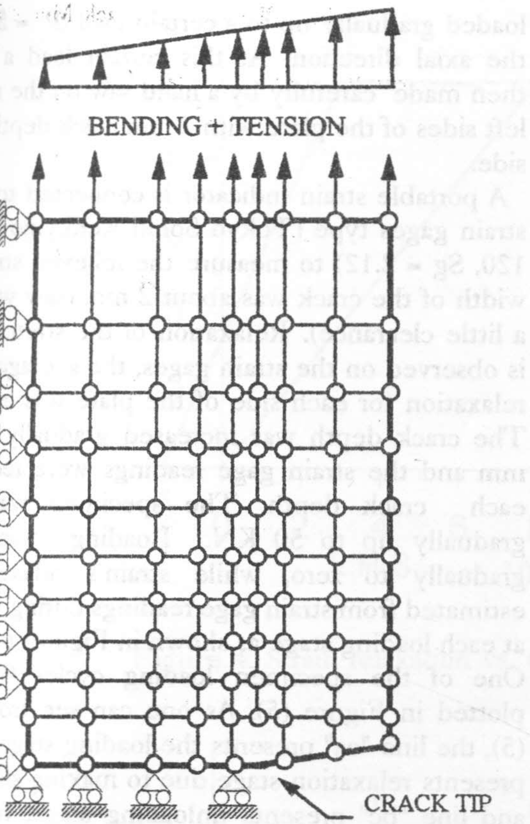

Figure 1a. Mesh for one quarter specimen.

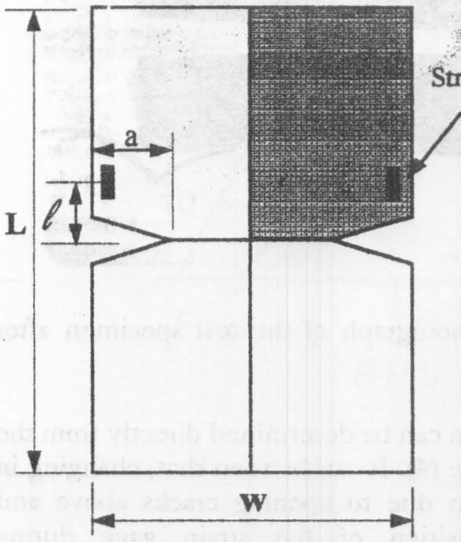

Figure 1b. Specimen with single edge crack.

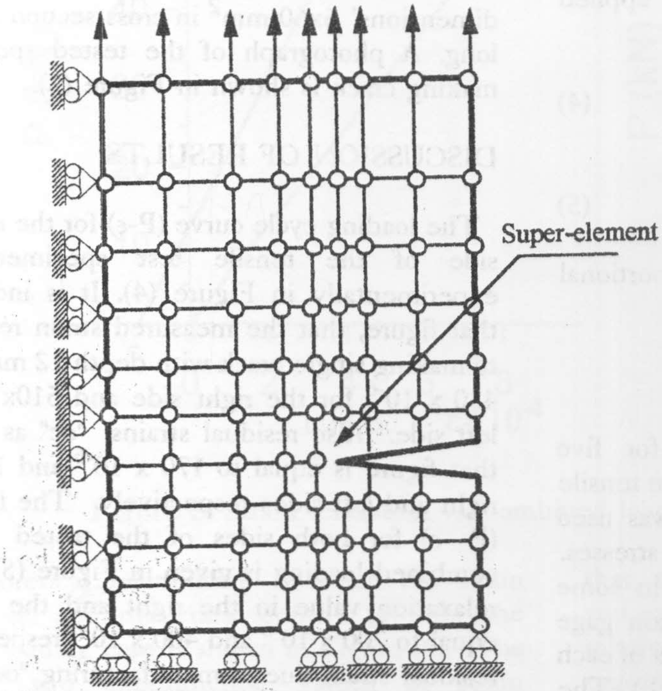

Figure 2a. Mesh for one quarter specimen.

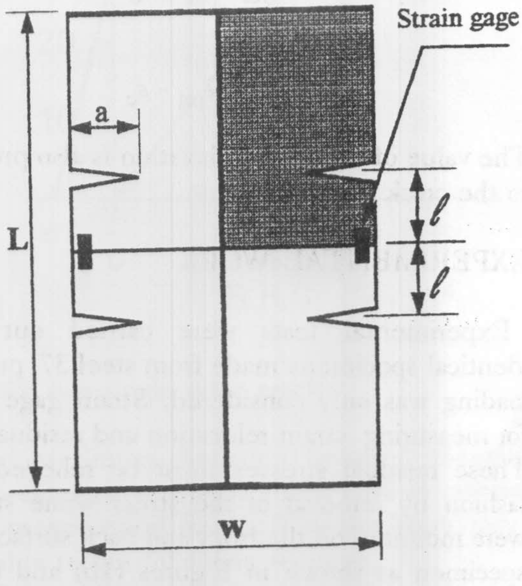

Figure 2b. Specimen with double edge cracks

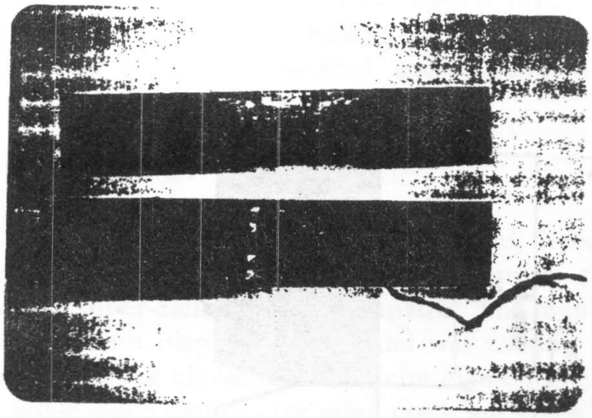

Figure 3. A photograph of the test specimen after making cracks.

Residual strain can be determined directly from the curves in Figure (4). It can be seen that, changing in strain relaxation due to opening cracks above and below the position of the strain gage during application of the load is to be a function of both residual and applied stresses. Separation of strains caused by these two stress components can be obtained as shown in Figure (4), where the strain relaxation is and the strain caused by the applied stress is given by

$$\epsilon_{rel} = \epsilon_a - \epsilon_b \quad (4)$$

$$\Delta\epsilon = \epsilon_{rel} - \epsilon_c \quad (5)$$

The value of the strain relaxation is also proportional to the crack depth.

EXPERIMENTAL WORK

Experimental tests were carried out for five identical specimens made from steel 37, pure tensile loading was only considered. Strain gage was used for measuring strain relaxation and residual stresses. These residual stresses must be relieved in some fashion by removal of the stress while strain gage were mounted on the front and back surfaces of each specimen as shown in Figures (1b) and (2b). The specimens were mounted gently on the universal tensile testing machine (VEB type ZDM), and loaded gradually up to a certain load (P = 50KN) in the axial direction. At this certain load a crack is then made carefully by a hand saw on the right and left sides of the plate with 2 mm crack depth at each side.

A portable strain indicator is connected to the four strain gages type (Tokyo Sokki Kenkyujo Fla-3-11-120, Sg = 2.12) to measure the relieved strain. The width of the crack was about 2 mm (saw width plus a little clearance). Relaxation of the stretched plate is observed on the strain gages, the average of strain relaxation for each side of the plate was calculated. The crack depth was increased gradually up to 12 mm and the strain gage readings were recorded at each crack depth. The specimen was loaded gradually up to 50 KN. Loading was reduced gradually to zero, while strain's relaxation was estimated from strain gage readings during unloading at each loading stage as shown in Figures (4) and (5). One of the specimen loading cycles (P-ϵ) was plotted in Figure (5). As one can see from Figure (5), the line "oa" presents the loading stage, line "ab" presents relaxation stage due to making single crack and line "bc" presents unloading stage, finally line "co" presents the residual strain [4]. A procedure tests were carried out on standard specimens having dimensions' 6x60 mm^2 in cross section and 600 mm long. A photograph of the tested specimen after making crack is shown in Figure (3).

DISCUSSION OF RESULTS

The loading cycle curve (P-ϵ) for the right and left side of the tensile test specimen is given experimentally in Figure (4). It is indicated from that figure, that the measured strain relaxation due to making single crack with depth 12 mm is equal to 430 x 10^{-6} for the right side and 510x 10^{-6} for the left side. Also residual strains "oc" as shown from that figure is equal to 170 x 10^{-6} and 150x 10^{-6} for right and left sides respectively. The loading cycle (P- ϵ) for both sides of the tested plate under combined loading is given in Figure (5). The strain relaxation value in the right and the left sides is equal to 700 x 10^{-6} and 400 x 10^{-6} respectively. The residual strain due to manufacturing "oc" is equal to 160 x 10^{-6} and 140 x 10^{-6} for both sides.

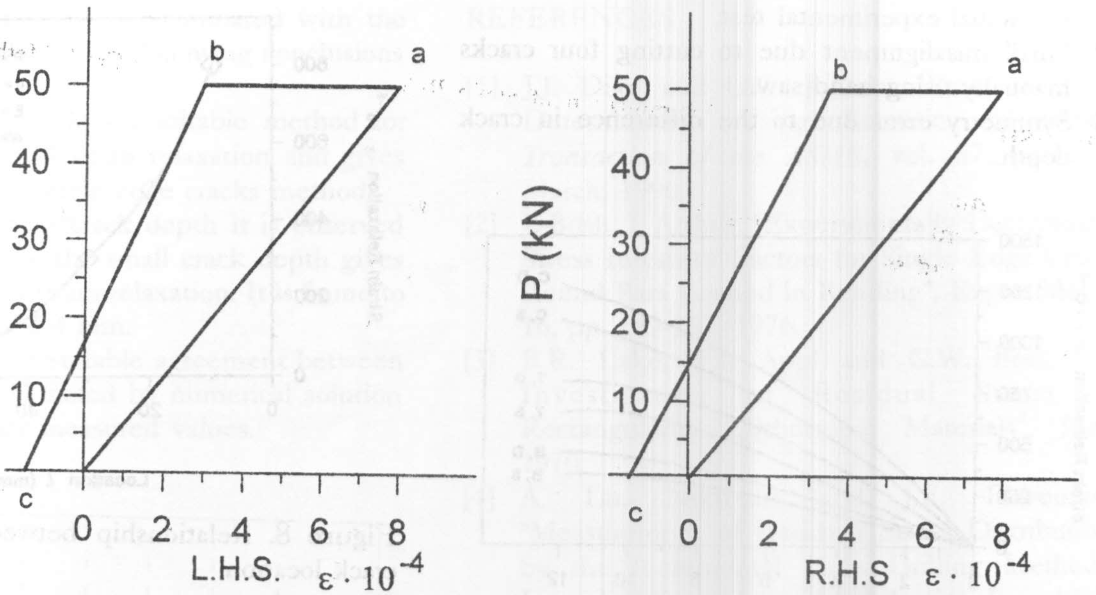

Figure 4. Strain relaxation vs. tensile load for two double edge cracks.

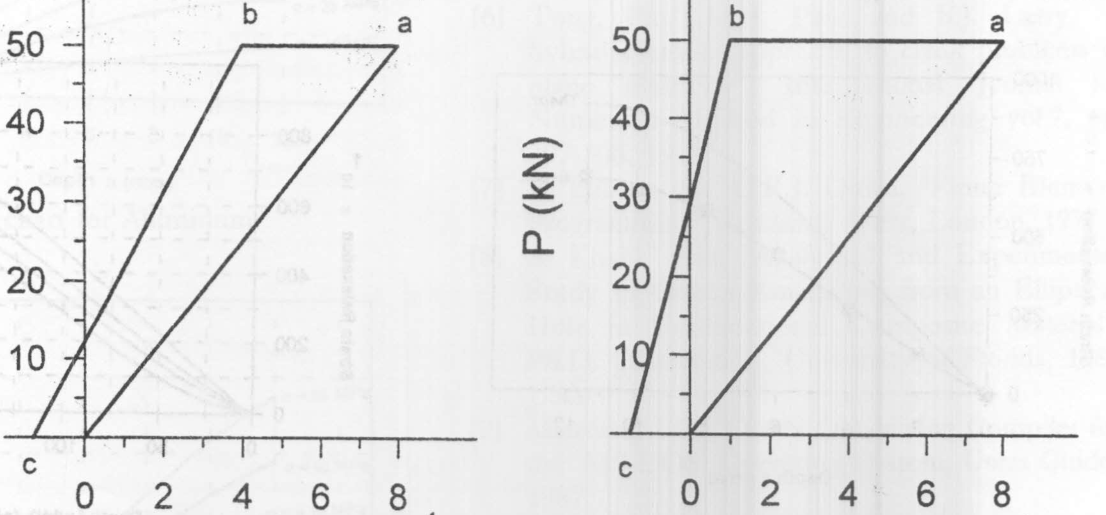

Figure 5. Strain relaxation vs. combined load for two single edge cracks.

The theoretical relationship between the strain relaxation (ϵ) and the crack depth (a) for the combined loading case is shown in Figure (6). The theoretical and experimental values of strain relaxation with respect to the crack depth are shown in Figure (7), for the tensile loading case. It is found that, the maximum deviation between theoretical and experimental results is about 20% as estimated relative to theoretical results. This deviation may be caused due to the following reasons:

1- In finite element solution, the crack tip is considered to be sharp while it is not that case in the actual experimental test.

2- Small misalignment due to cutting four cracks manually using hand saw.

3- Symmetry error due to the difference in crack depth.

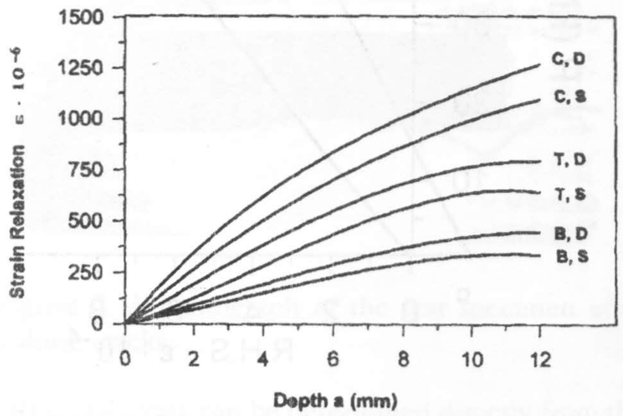

Figure 6. Variation strain relaxation with crack depth under combined load T: Tension S: Single crack. B: Bending. D: Double crack. C: Combined tension and bending.

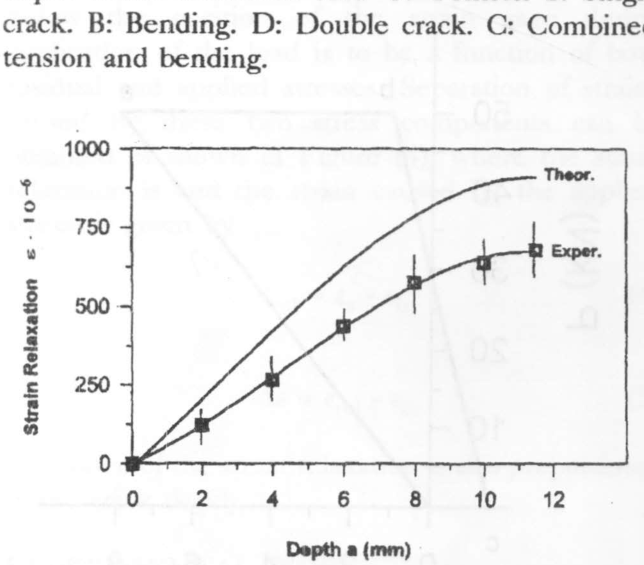

Figure 7. Theoretical and experimental strain vs. crack depth.

The effect of crack location (ℓ) to the relaxed strain has been investigated theoretically. The results are shown in Figure (8), in which it is clear that the strain relaxation decreases as the crack location increases, where (ℓ) is the distance between the center line of the crack and the center line of the strain gage.

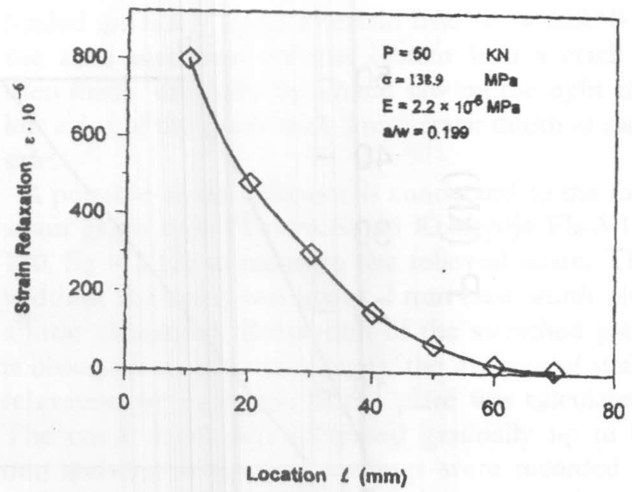

Figure 8. Relationship between relaxed strain & crack location.

Strain relaxation, crack depth to plate width ratio (a/w) and actual applied load are presented in a design charts given in Figures (9), (10), (11) for steel 37, Aluminum and Brass. These charts can be used to predict actual applied load in the loaded plate.

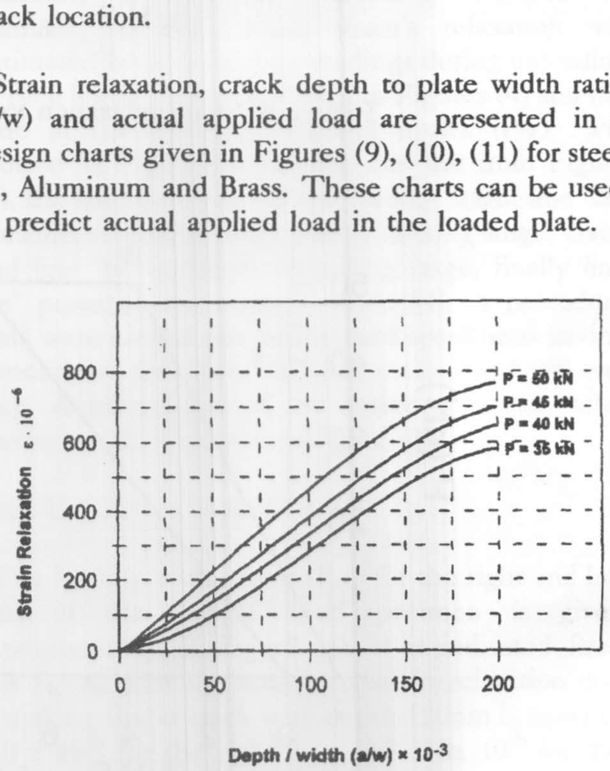

Figure 9. Design chart for steel.

CONCLUSIONS

The strain relaxation has been investigated for two double edge cracks in a plate under tensile stress. Strain relaxation is determined for a plate subjected to combined load. Method of two single edge cracks and two double edge cracks were used in the experimental investigation and compared with the results of F.E. Method. The following conclusions were observed:

1- The single edge crack is a suitable method for the determination of strain relaxation and gives less damage than double edge cracks method.
2- Within a variation in crack depth it is observed experimentally that, the small crack depth gives reasonable value of strain relaxation. It is found to be in the range of 2-4 mm.
3- In general there is a suitable agreement between strain relaxation predicted by numerical solution and experimentally measured values.

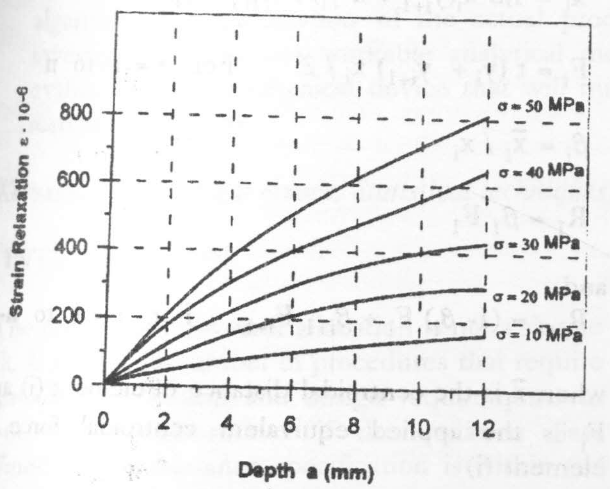

Figure 10. Design chart for Aluminum.

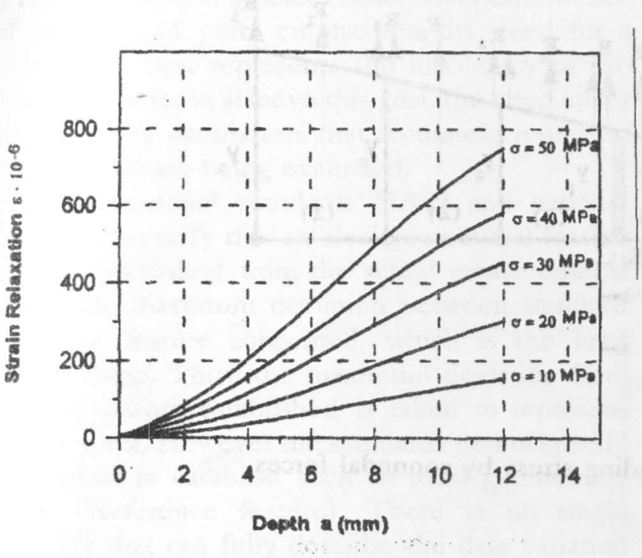

Figure 11. Design chart for Brass.

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Appendix

Representation of bending stress by equivalent nodal forces:

The applied bending stress can be represented by concentrated equivalent nodal forces for each loaded element (i), as in Figure (12), by using the following two equations:

1. The force equilibrium

$$R_i^{(i)} + R_{i+1}^{(i)} = \int_{\ell_{i+1}}^{\ell_i} \sigma^{(i)}\, t\, dx$$

2. The moment equilibrium

$$R_{i(i)}\, \ell_i + R_{i+1}^{(i)}\, \ell_{i+1} = \int_{\ell_{i+1}}^{\ell_i} \sigma^{(i)}\, t\, x^{(i)}\, dx \quad \text{for } \ell_{i+1} < x^{(i)} < \ell_i$$

Where t is the thickness of the plate, $\sigma^{(i)}$ is the bending stress at element (i) and $R_{i+1}^{(i)}$, $R_i^{(i)}$ represent the left and right equivalent nodal forces for element number (i).
By using some trigonometric relations one can easily obtain the following expressions:

$$y_1 = \sigma_{max} = 6\, M_{max} / tw^2$$

$$y_2 = 2y_1\, (w / 2 - x_1) / w$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$y_i = 2y_1 [\, (w / 2 - x_1) - x_{i-1}\,] / w$$ For $i = 3$ to n

Where M_{max} is the maximum bending moment, n is the number of loaded elements and w is the width of the plate. From Fig. 12, one can find the following quantities:

$$\bar{x}_i = 1/3\, x_i (y_{i+1} + 2\, y_i) / (y_{i+1} + y_i)$$

$$F_i = t\, (y_i + y_{i+1})\, x_i / 2 \qquad \text{For } i = 1 \text{ to } n$$

$$\beta_i = \bar{x}_i / x_i$$

$$R_1 = \beta_1\, F_1$$

and

$$R_{i+1} = (1 - \beta_i)\, F_i + \beta_{i+1}\, F_{i+1} \qquad \text{For } i = 1 \text{ to } n\text{-}1$$

where $\bar{x}_i$ is the centroidal distance of element (i) and F_i is the applied equivalent centroidal force at element (i)

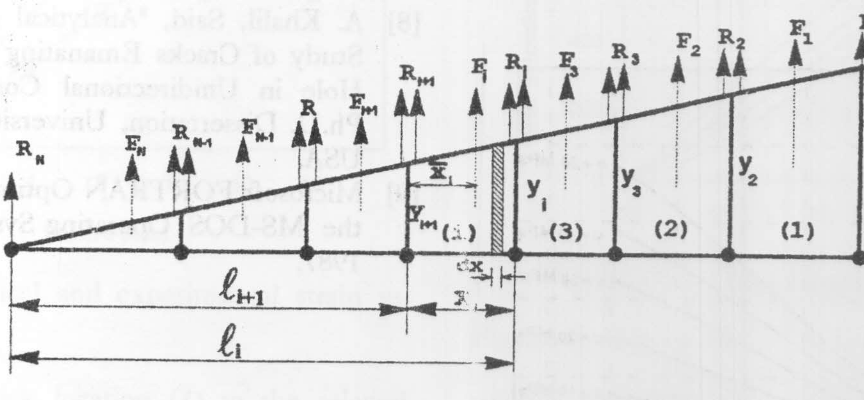

Figure 12. Representation of bending stress by connodal forces.