HYBRID SOLUTION FOR AXIAL PILES IN LAYERED SUPPORTING MEDIA

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ABSTRACT

The article presents a hybrid analytical/numerical method to predict the load-settlement behavior of axially loaded piles in layered supporting media. Unlike other numerical methods, e. g. finite element, neither interpolation over the element, nor matrix inversion are involved in the analysis. The method accounts for the variation of pile and soil properties with depth. Field equation is solved exactly in each layer, and compatibility between layers are satisfied. The method treats various type of pile tip supporting condition, and its applicable to non linear soil response. The method is superior in its mathematical simplicity, accuracy, applicability, and computer running time. The relationship between the pile movement and the soil resistance "skin friction" is introduced using the subgrade reaction or the t-z method. The correlation between pile tip movement and soil reaction is incorporated via the p-z method. Soil reaction, axial load and displacements along the pile are obtained using one single formula. The share of the load transmitted to the bearing stratum, the displacement at the tip and at the top of the pile are evaluated through the same formula. A new pile function classification based on pile top displacement is presented.

Key Words: Hybrid Methods, Piles, Layered Media, Nonlinear soil response.

1. INTRODUCTION

In broad categories, we may classify the methods developed, and used to predict pile load - settlement relationship to ; load transfer methods, methods based on theory of elasticity, and numerical methods such as, finite element, and finite difference. The first methods Coyle et al (1966), Coyle et al (1967) and Seed et al (1957) used measured pile resistance, and pile movement relationships, through an iterative process till convergence. The second methods Banerjee et al (1978), Butterfield et al (1971) and Randolph et al (1978) employ, with some mathematical complexity, elasticity solution that utilizes Mindlin solution of surface loading in semi infinite media. Numerical solutions, Desai et al (1977), Meyer et al (1986), Valliappan et al, (1974) and Zienkiewick (1971) with their unlimited application suffer from computer requirements in memory and time.

The method developed in this article utilizes, and combines pile field equation, with (t-z), and (p-z) relationships Coyle et al (1966), Kiousis et al (1987) and Kraft et al (1981), to furnish a solution ready for numerical treatment.

2. PILE EQUATION OF MOTION

Select a coordinate system as such the positive direction of the vertical axis z is directed upward. The differential equation for equilibrium of a vertical pile segment coinciding with the vertical axis, and subjected to axial load at its top and embedded in a soil mass with tangential subgrade reaction k, is given by

$$k\,u + \frac{\partial N}{\partial z} = 0 \qquad (1)$$

Where,

z distance along the pile
u axial displacement of the pile,
N axial force, and

k force per unit length of the pile needed to produce unit displacement. The negative k indicates negative soil resistance to pile movement. The axial force is related to the axial displacement, and pile properties by the equation,

$$N = -EA_p \frac{\partial u}{\partial z} \tag{2}$$

Where, EA_p is the axial rigidity of the pile. Combining Equations (1) and (2) yields

$$\frac{\partial^2 u}{\partial^2 z} - \lambda^2 u = 0 \tag{3}$$

where $\lambda^2 = \frac{k}{EA_p}$

For negative k, λ is pure imaginary, and its real for positive k.

The general solution of (3) is

$$u = A e^{\lambda z} + B e^{-\lambda z} \tag{4}$$

where A and B are constants of integration. For positive k, A and B are real. Whilest for negative k, they are complex conjugate.

3. LAYERED SOIL

For the case of layered supporting media, consider n layers with thicknesses $L_1, L_2, \ldots, L_n$ with L_1 the thickness of the most upper layer. The corresponding subgrade reactions are $k_1, k_2, \ldots, k_n$, respectively. The axial rigidity of the pile in layer i is EA_{pi}. The subgrade reaction "spring constant" for the underlaying stratum is K_s, (defined as the force required to produce unit displacement at the pile tip).
Each layer has a local coordinate system with the origin at the bottom of the layer and the positive direction of the vertical axis z_i is directed upward.
The behavior of the pile in each layer is governed by Eq. (3), with a general solution similar to (4), specified for layer i as

$$u_i = A_i e^{\lambda_i z_i} + B_i e^{-\lambda_i z_i} \tag{5}$$

where

A_i and B_i constants of integration corresponding to the i^{th} layer,

z_i local coordinate of the i^{th} layer, and

$\lambda_i^2 = \frac{k}{EA_{pi}}$

The relationship relates the 2n constants, A_1 through B_n, need to be established by enforcing displacement and force compatabilities between layers. The displacement continuity at the interface between any two adjacent layers, e. g. i and i + 1, requires that

$$A_i + B_i = A_{i+1} e^{\lambda_{i+1} L_{i+1}} + B_{i+1} e^{-\lambda_{i+1} L_{i+1}} \tag{6}$$

Although, the displacements are measured in terms of local coordinates z_i, the global continuity is satisfied by (6).
Enforcing force compatibility at layers interface yields

$$A_i - B_i = \alpha_{i+1} [A_{i+1} e^{\lambda_{i+1} L_{i+1}} - B_{i+1} e^{-\lambda_{i+1} L_{i+1}}] \tag{7}$$

with

$$\alpha_i = \frac{E_{i+1} A_{pi+1} \lambda_{i+1}}{E_i A_{pi} \lambda_i} \tag{8}$$

which possesses either real or imaginary values.
In a matrix form Eqs. (6) and (7) can be expressed as

$$\begin{bmatrix} A_i \\ B_i \end{bmatrix} = [C_i] \begin{bmatrix} A_{i+1} \\ B_{i+1} \end{bmatrix} \tag{9}$$

Where the transfer matrix $[C_i]$ is given by

$$[C_i] = 0.5 \begin{bmatrix} (1+\alpha_{i+1}) e^{\lambda_{i+1} L_{i+1}} & (1-\alpha_{i+1}) e^{-\lambda_{i+1} L_{i+1}} \\ (1-\alpha_{i+1}) e^{\lambda_{i+1} L_{i+1}} & (1+\alpha_{i+1}) e^{-\lambda_{i+1} L_{i+1}} \end{bmatrix} \tag{10}$$

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From (9), it can be concluded that

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = [C_G] \begin{bmatrix} A_n \\ B_n \end{bmatrix} \quad (11)$$

Where the global transfer matrix is

$$[C_G] = [C_2]\,[C_3]\,.....[C_{N-1}]\,[C_N] \quad (12)$$

4. BOUNDARY CONDITIONS

To find A_i and B_i ($i = 1, 2, \ldots, n$), the constants of integration, first consider the equilibrium at the pile head. The axial force in the pile is given by (2), from which, the axial force N_o at the pile head is expressed as

$$N_o = EA_{p1}\,\lambda_1\,[A_1\,e^{\lambda_1 L_1} - B_1\,e^{-\lambda_1 L_1}] \quad (13)$$

At the pile tip, three various conditions may exist. Namely; pile is supported on compressible bearing stratum, or rigid bearing stratum, and the condition of a fully floating pile. Equilibrium of the first condition requires that the force developed at the pile tip is equal to the reaction of the underlaying stratum, i. e.

$$K_s\,(A_n + B_n) = EA_{pn}\,\lambda_n\,(A_n - B_n) \quad (14\text{-a})$$

Where K_s is the subgrade reaction of the bearing stratum.

The existence of a rigid bearing stratum leads to zero displacement of the pile tip, and accordingly,

$$A_n + B_n = 0 \quad (14\text{-b})$$

Finally, a "fully" floating pile means zero force at the pile tip, which necessitates

$$A_n - B_n = 0 \quad (14\text{-c})$$

The coefficients A_1, B_1, A_n, and B_n are evaluated by using Eqs. (11) and (13) with either one of the boundary conditions defined by Eq. (14) according to the pile tip conditions. These values are then introduced in Eq. (9), to generate the rest of the coefficients A_i and B_i.

Once integration constants are known, all necessary physical values are obtainable via Eq. (5) and its consecutive derivatives.

5. NONLINEAR SOIL RESPONSE

The case of soil non-linear response, and a possible soil yielding could be treated according to the following steps :

1. For each soil layer define its force-displacement relationship, (t-z) curve, similar to the one shown in Figure (2). Additional relationship (p-z) curve is required for the bearing stratum, e. g. Figure (3). One (t-z) curve is used for soil of constant properties with depth, i. e. one soil layer.
2. Each force-displacment curve is divided into segments, as shown in the typical Figure (1). The slope of any line segment represents the secant modulus of subgrade reaction at displacement level projected on the horizontal axis. The segments horizontal components Δu_i may be taken arbitrary and need not to be equal.
3. For all layers, we evaluate the subgrade reaction coefficients $K_{1,i}$, where the first subscript refers to displacement level, and the second subscript denotes layer number.

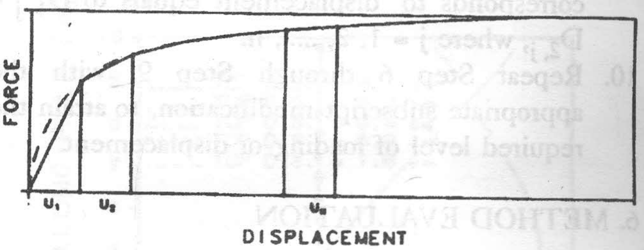

Figure 1. General (t-z) or (p-z) curve.

4. For an applied displacement $U_o = 1$ at pile head, the coefficients A_I through B_n are calculated, using the procedure described in sections 3, and 4.
5. Find the displacement at the mid point of each layer, corresponding to $U_o = 1$. From these values, find the displacement U_o that causes the mid point displacement in one layer, e. g. layer (of the highest displacement), to reach Δu_i,

and consequently evaluate the corresponding load at the pile head. We define the load calculated in step 5 as $N_{o,\ 1}$, and the displacements at mid points, associated with it by $D_{1,\ i}$, where the first subscript refers to stage of loading and the second one refers to layer number. For the load $N_{o,\ 1}$, the second subscript refers to stage of loading and the first one, always o, indicates the pile head position.

6. Find the coefficients A_I through B_n for pile head load of $N_{o,1}$. With the exception that, for layer i, defined in step 5, the subgrade reaction corresponds to the second displacement level, is used.
7. Evaluate the displacemtns along the pile length, where the displacement at the mid point of each layer, e. g. layer j, is given as the sum of the displacement $D_{k,.j}$, where k varies from 1 to the present loading stage. The displacements in this stage are calculated using coefficients obtained in step 6.
8. Find the smallest incremental force $N_{o,\ 2}$ that causes the displacement at mid point of any layer, except for layer i mentioned in step 5, to reach the displacement level Δu_1 <u>OR</u> $N_{o,\ 2}$ that causes the displacement level in layer i to reach the displacement level $\Delta u_1 + \Delta u_2$.
9. Evaluate the total applied force $N_{o,\ 1} + N_{o,\ 2}$ corresponds to displacement equals to $D_{1,\ j} + D_{2,\ j}$, where j = 1, 2, ..., n.
10. Repeat Step 6 through Step 9, with the appropriate subscript modification, to attain the required level of loading or displacement.

6. METHOD EVALUATION

Figures (2), and (3) show the (t-z), and the (p-z) curves, Kiousis et al (1987), used to substantiate the proposed method. The pile is a thin wall steel tube of outside diameter D = 27.3 cm, thickness t = .927 cm, and elastic modulus E = 210 GPa. Detail of soil profile is found the same reference, where curve of one layer is considered. Figure (4) shows load - pile head displacement relationship developed by the present method. 20 elements / layers (of the same force-dsplacement curve) were used in the analysis, Figure (4b). Identical results were obtained using 10 elements, Figure (4a). This is not surpersing, since the exact soultion of field equation is utilized over each layer.

Its interesting to note that, a unique definition for the pile function based on pile top displacement value is demonstrated in Figure (4). In the current case study, the pile resists higher load through friction, as long as its top displacement is less than 0.33 cm. For higher displacement, end bearing resistance contributes more than friction to pile capacity. Obviously, this transit displacement value is unique for current case study. Figure (5) shows the displacement along pile length for various values of pile top displacement. Figure (6) depicts the force developed in the pile, and the force resisted by friction along the pile length. The distribution of the skin friction forces for pile top displacement of 0.20 cm corresponds to peak resistance zone shown in Figure (2). For pile top displacement of 0.80 cm, and 1.00 cm, the skin friction is constant with depth, which simulates the behavior of the residual resistance zone shown in Figure (2).

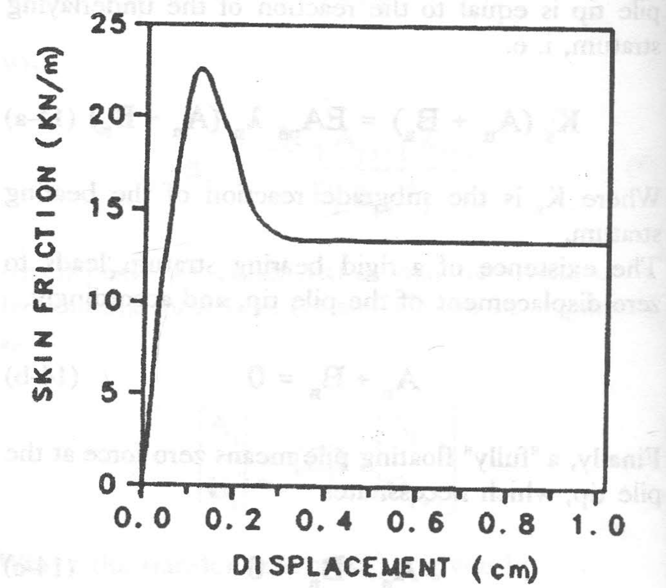

Figure 2. (t-z) curve.

The authors are not aware of any analytical / field results available in the literature for solid section piles, with non-linear force - displacement curves similar to Figure (2), for comparison purpose.

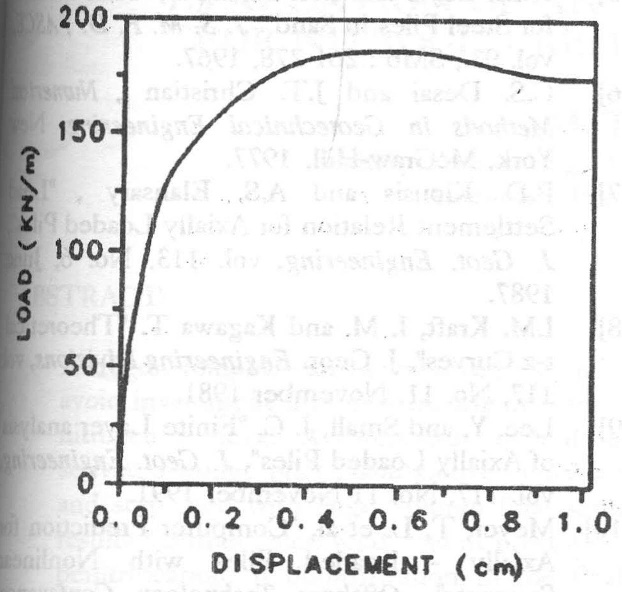

Figure 3. (p-z) curve.

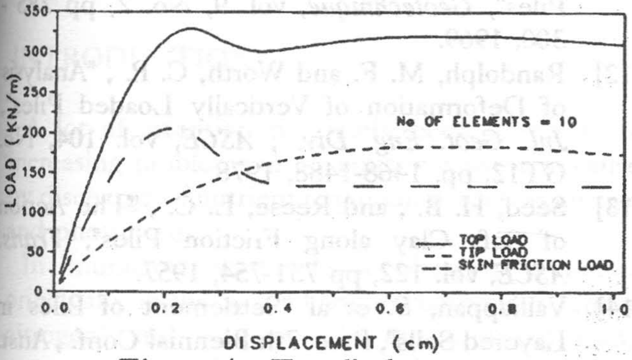

Figure 4a. Top displacement.

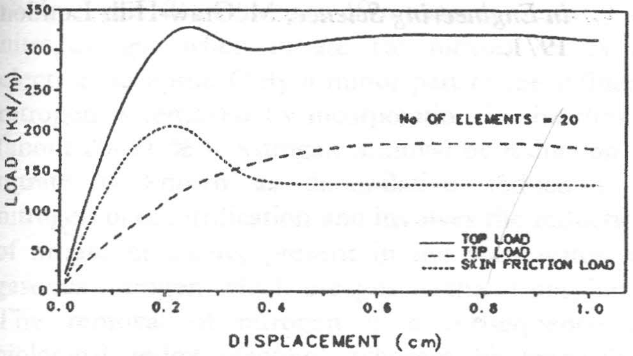

Figure 4b. Tio displacement.

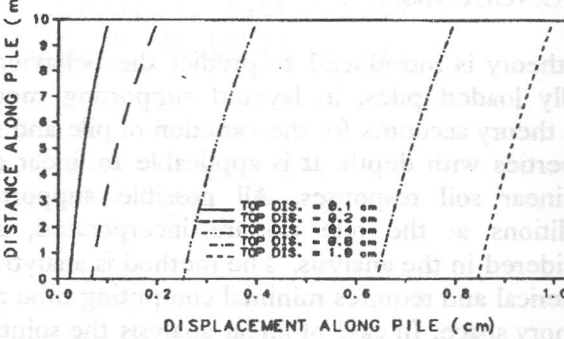

Figure 5. Displacement along pile length.

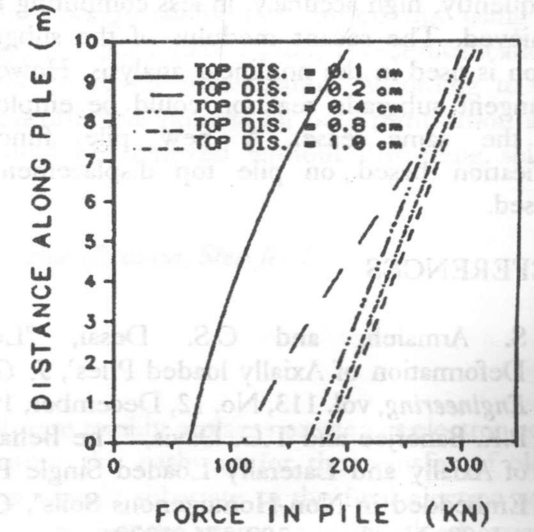

Figure 6a. Variation of axial force along pile length.

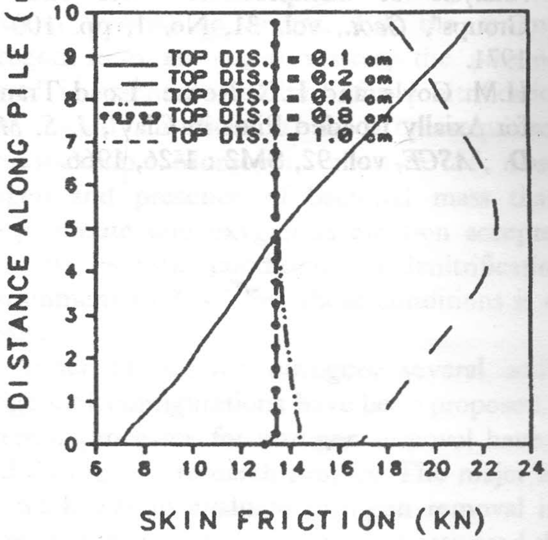

Figure 6b. Variation of skin friction force along pile length.

7. CONCLUSION

A theory is introduced to predict the behavior of axially loaded piles, in layered supporting media. The theory accounts for the variation of pile and soil properties with depth. It is applicable to linear and nonlinear soil responses. All possible supporting conditions at the pile tip are incorporated, and considered in the analysis. The method is analytical/ numerical and requires minimal computing time and memory space. In case of linear analysis the solution is straight forward, while its incremented, and not iterative for non--linear case analysis. And consequently, high accuracy, in less computing time is achieved. The secant modulus of the subgrade reaction is used in the nonlinear analysis. However, the tangent subgrade reaction could be employed with the same ease. A new pile function classification based on pile top displacement is proposed.

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