EFFECT OF ADDITIONAL PARAMETERS ON ACCURACY OF AERIAL TRIANGULATION BY BUNDLE BLOCK ADJUSTMENT

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ABSTRACT

The Wide-range use of analytical Photogrammetry is emphasized. Aerial triangulation using bundle block adjustment is introduced as a rigorous and flexible method of block adjustment. The computation with selft-calibration by additional Parameters as a means to overcome the effect of the systematic image errors is shown and investigated.

Keywords: Areial triangulation, Boundle Block adjustment , Additional parameters, Systematic image errors, Self-Calibration.

1. INTRODUCTION

The most general and efficient methods of aerotriangulation applied toady are block-triangulation with independent models and the fully analytical method of bundle block triangulation. Although such rigorous methods require a high effort in programming as well as in computing, their application is extended continuously.

Nowadays, the availability of general programs for block adjustment with additional parameters and with several program modules to ensure flexible handing, and the recently gained knowledge about reliability problems, provide the basis for exploiring completely the accuracy potentials of modern photogrametry and especially its analytical approach. As a major result of this, a photogrammetric accuracy level is available. This accuracy level carries analytical photogrammetry into new fields of application, such as network densfication, cadastral surveying and other subjects dealing with the provision of coordinates to ground points in rural and urban areas. It is advisable to utilize this accuracy potential to obtain more economic tools for point determination purposes.

This paper is mainly concerned with the study of the additional parameters and their effect on the accuracy of aerial triangulation using bundle block adjustment.

The simultaneous procedure offers a rigorous approach whereby aerial triangulation is performed in one step. The desired parameters are adjusted as a result of one simultaneous, least square adjustment of the (n) photos in the block by a direct iterative method, using linear form of the collinearity condition equation or the non-linear form respectively [12].

The evaluation application and potential of bundle adjustment has been fully discussed by Brown [8]. Only a brief description of the procedure is presented here.

This procedure offers a new approach whereby the desired parameters are adjusted as result of one simultaneous least square solution of all the photographs by an iterative method. The iteration is involved because of the fact that the associated condition equations are non-linear. However, one can also form non-linear "normal equations", in which case the solution can be considered as "direct". From a theoretical point of view, such a simultaneous procedure should provide the most accurate results [11].

The fundamental requirements in this procedure are the estimates of the exterior orientation parameters. Furthermore, depending on the specific approach taken, the estimates for coordinates of all pass point may also be needed, Thus , a simultaneous procedure should include a feasible method of obtaining the necessary estimated (approximate) values initially [4].

A radical departure from standard practices came with the development of the simultaneous or bundle procedure in analytical photogrammetry which is to be discussed in this section. In the simultaneous method, the basic unit is the pair of coordinates X and Y of an image on the photograph. Using these coordinates (along with some camera constants and the ground position of several of the images), ground coordinates of intermediate points and estimates of the camera's orientation are derived from a simultaneous adjustment. This method differs from the sequential adjustment and independent models in that the solution leads directly to the final coordinates in a single solution and does not treat the "absolute" or "relative" orientions separately. As a result, the solution and associated error propagation are more rigorous in that certain correlation are not ignored [1]

2. AERIAL TRIANGULATION USING ANALYTICAL METHOD

Moreover analytical aerotriangulation is the most example intensive and successful development of photoyrammetry. It tends to be more accurate than analogue on semianalytical, largely because analytical techniques can more effectively eliminate systematic errors such as film shrinkage, atmospheric refraction and camera lens distortion [11].

Analytical photogrammetry deals with the solution of problems by mathematical computations using measurements which were done on the photographs as input data. In general, a mathematical model is constructed to represent relations between points in the object space and their corresponding images on the photographs. The principle of perspective and protective geometry is inherent in this. In the conventional use, the contained projection is "Central", the perspective center of the camera is less being the projection center [5].

The analytical photogrammetrist have always tried to justify their efforts on the basis that they should be able to obtain more accurate results in less able time than in the instrumental approaches. There is probably no special application over which more discussions and efforts have been expended with less results than spatial triangulation. This is due to both the strength and weakness of the procedure under the state of the art. Its weakness lies in the fact that it frightens away a great majority of the practitioners and involves considerable cost, and causes the reluctance to make use of it. The strength of analytical phorogrammetry, however, lies beyond this and justifies its continuous application and growth [12].

The analytical method is based on using of automatic precise devices and computers for a comparatively short period of time. During the past two decades a rigid theory of construction and adjustment of strip and block photogrammetric networks achieved very high level, highly precise comparators and monocular comparators with automatic recording of results of image measurements were created, technology of different methods of phototriangulation and approaches for elimination effects of systematic errors were worked out in detail. These achievement enables one to increase the accuracy and the efficiency of analytical phototriangulation in comparison to another method [11].

3. PRACTICAL SYSTEM OF ANALYTICAL METHOD

The working system involves, broadly speaking, the following:

- Object (terrain, infra structures, highway interchange etc.)
- Sensing tool (camera or other sensor),
- Environment (atmosphere, soil, water, etc.)
- Data acquisition tool (instrument or comparator)
- Data processing mechanisms (computer, accessories and the mathematical models)
- The human worker.

Each offers working limitations and contributes towards errors of various nature. The comprehensive knowledge of such limitations and error contributions is essential for an efficient job [12].

4. The MATHEMATICAL MODELS

Usually, a photogrammetric camera can be defined geometrically by introducing the concept of a lens (considered a point) as the center of projection along with an image point in the focal plane. The general photogrammetric problem can then be expressed mathematically as the central projection of a three-dimensional object space onto a two-dimensional image space. A graphic depiction is shown in Figure (1) where the image plane is shown as a dispositive, that is, falling between the lens and object space. In the figure, the center projection has the coordinates X_L, Y_L and Z_L and the camera records the image (Xa Ya and Za) of an object point with coordinates XA, YA and ZA all relative to an arbitrary Cartesian coordinate system in object space.

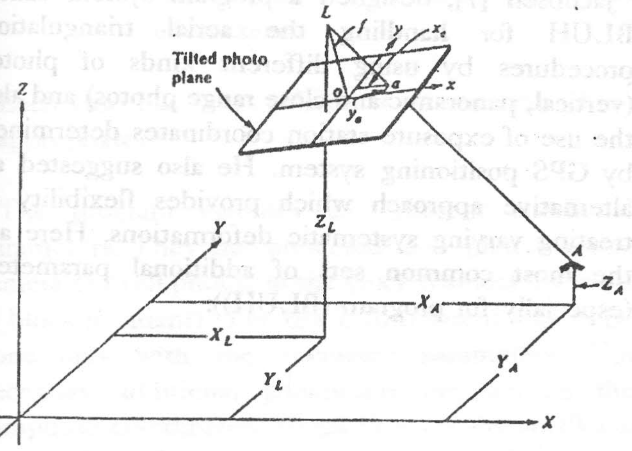


Figure 1. The collinearity condition.

A second Cartesian coordinate system is used to define the image coordinates (x and y) in the focal plane of the camera. The problem, therefore, in the simultaneous adjustment, is to determine the geometric relationship that exists between the object-space coordinate system and the image-plane coordinate system. The photo coordinates are assumed to be translated from the fiducial system to the principal point by applying small known translations in the x and y direction [1].

The relationship between image and object space as a function of three angles is given by [11]:

$$x_a = -f\left[\frac{m_{11}(X_A-X_L)m_{12}(Y_A-Y_L)m_{13}(Z_A-Z_L)}{m_{31}(X_A-X_L)m_{32}(Y_A-Y_Lm_{33}(Z_A-Z_L)}\right]$$

$$y_a = -f\left[\frac{m_{21}(X_A-X_L)m_{22}(Y_A-Y_L)m_{23}(Z_A-Z_L)}{m_{31}(X_A-X_L)m_{32}(Y_A-Y_Lm_{33}(Z_A-Z_L)}\right]$$

5. ADJUSTMENT WITH ADDITIONAL PARAMETERS

The simplicity of the basic concept of the simultaneous adjustment (e.g., the problem is developed from the two collinearity condition equations only), coupled with many ingenious concepts and algorithms for efficient commutation and "bookkeeping" that have been developed through the years, provides a system that can be expanded to include solutions that would normally be prohibitive. The concept of the banded-bordered system of normal equations was presented at the ISP Commission III Symposium in Stuttgart, Germany in 1974 by Brown [2] and further expounded upon by Brown (1975). Bascially, Brown extends the concept of the banded reduced normal equation system of a block, along with its solution using "recursive partitioning" to include a border that accommodates parameters that are common to large subsets of the original observation equations. The elegance of the approach allows for the inclusion of these additional parameters with a great saving in computional time over that encountered with a "brute force" conventional adjustment. Brown suggests that the system can be applied to "self-calibration" of the cameras used in a block when using auxiliary sensors, in satellite applications of photogrammetry where orbital constrainst are to be imposed, and in the form of "added parameters" in empirical error models which account for detectible systematic errors in the plate residuals from a block adjustment. The error models can be general polynomials that regard the systematic error to be common to all photographs or to be present in a sub-set of all the photographs. In addition, the method allows for the general simultaneous adjustment to incorporate the concurrent adjustment of the geodetic network used as the bases of control for the block or for the inclusion of equal elevation constraints as may be encountered by the measurement of point on the shore line of lakes of unknown elevation. All in all, the method presents an attractive extension of the basic simultaneous adjustment [3].

6. SELF CALIBRATION

This technique consists in modifying the mathematical model expressing the actual physical situation. This is applied by simply adding a correction, say d, to each of the coordinates that are measured. In the bundle method dx and dy are added to the x and y photograph coordinates, while in the model method dx, dy and dz are added to the X,Y and z model coordinates including those of perspective centers [6].

Corrections d's are function of several unknown parameters which are to be determined simultaneously with the original unknowns of the problem. The bundle and model method required naturally substantial computation, and hence not much is added by solving for the additional parameters [10].

Generally speaking, The correction terms may be derived into two types. The first type models the effect of different expected sources of errors, such as tangential lens distortion and non-perpendicularity of the comparator axes. The disadvantages of this approach is the possible occurrence of correlations between these added parameters and themselves, and between them and the original unknowns of the problem [10].

1- $x' = x - y.P1$	$y' = y - x.\ P1$
2- $x' = x - y.P2$	$y' = y + x.\ P2$
3- $x' = x - x \cos 2b.\ P2$	$y' = y - y.\ \cos 2b.\ P3$
4- $x' = x - x \sin 2b.\ P4$	$y' = y - y.\ \sin 2b.\ P4$
5- $x' = x - x \cos b.\ P5$	$y' = y - y.\ conb.\ P5$
6- $x' = x - x\ sinb.\ P6$	$y' = y - y.\ sinb.\ P6$
7- $x' = x + y.r.\ cosb.\ P7$	$y' = y - x.r.\ cosb.\ P7$
8- $x' = x + y.r.\ sinb.\ P8$	$y' = y - x.r.\ sinb.\ P8$
9- $x' = x - x.(r^2 - 16384).\ P9$	$y' = y - y(r^2 - 16384).P9$
10- $x' = x - x.\sin(r.0.049087).P10$	$y' = y - y \sin(r.0.049087).P10$
11- $x' = x - x.\sin(r.0.098174).P11$	$y' = y - y \sin(r.0.098174).P11$
12- $x' = x - x.\ \sin 4b.\ P12$	$y' = y - y.\ \sin 4\ b.\ P12$
13- $x' = x + x.P13$	$y' = y + y.\ P13$
14- $x' = x + P14$	$y' = y$
15- $x' = x$	$y' = y = P15$
16- $x' = x + x.\ tgps.P16$	$y' = y + y.\ tgps.P16$
17- $x' = x + tgps.\ P17$	$y' = y$
18- $x' = x$	$y' = y + tgps.P18$
19- $x' = x - (y/f - x/r^2).P19$	$y' = y - (y/f - y/(c^2+y^2)).\ P19$
20- $x' = x - \arctan y/x.\ P20$	$y' = y$
21- $x' = x - \sin (y/3200x.\ P20$	$y' = y$
22- $x' = x$	$y' = y \sin (y/300).\ P22$
23- $x' = x - \sin (y/150).\ P20$	$y' = y$

The second type of correction terms is a general polynomial which represents the combined effect of systematic errors irrespective of individual sources. A proper choice of such terms can avoid the problem of correlation.

The additional parameters may be assumed identical for all photographs in a strip or a block, especially when just one stable camera is used in the flight missions [10].

7. TYPES OF ADDITIONAL PARAMETERS

Jacobsen [7], designed a program system called BLUH for handling the aerial triangulation procedures by using different kinds of photos (vertical, panoramic and close range photos) and also the use of exposure station coordinates determined by GPS positioning system. He also suggested an alternative approach which provides flexibility of treating varying systematic deformations. Here are the most common sets of additional parameters (especially for program (BLUH)):

where: $r^2 = x^2 + y^2$ arctan b = y/x

- x', y' :The corrected photocoordinates (nearly free from systematic error).
- x, y : The actual refined photocoordinates.
- f : The calibrated focal length for the camera.
- $P_1, P_2, \ldots, P_{22}$: The coefficient obtained empirically for compensating the specific different cases and programs, namely: 1. Different sets of photo deformation. 2.Compensation for GPS elements. 3. Compensation for inner and outer orientations.

Each case has been refered to in the text where appropriate.

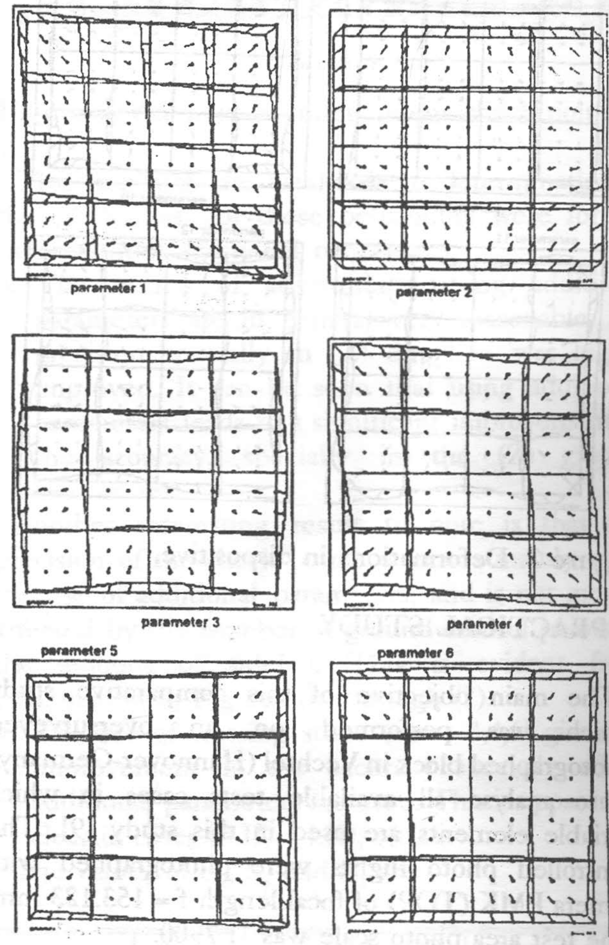
The program includes 23 different additional parameters. They are connected to all photos of one camera or to all photos in the block (camera invariant or block invariant). The block adjustment should be done only with the necessary parameters. Not necessary additional parameters can worsen the computed coordinates. Sigma O is usually smaller if the number of parameters is larger, but the size of sigma O (σ_O) is not a criteria for the possibility of the mathematical determination [7].

Three different statistical tests (correlation, total correlation student tests are used for checking the necessity of any single additional parameter. So at first the standard set of parameters (1-12) should be included in the adjustment [7].

The parameter 13 is corresponding to the focal length, 13 and 14 are changing the location of the principal point. They cannot be determined with usual aerial photos if the area is not mountainous. Parameter 16 is changing the focal length depending upon the GPS time, a drift of the kinematic GPS coordinates can be compensated by this. 17 and 18 are changing of the principal point as function of the GPS time. These additional parameters can cause numerical problems if the block is not stable enough. Parameter 19 to 23 are special parameters for panoramic photos. Parameter 19 is correcting the focal length -it should be used together with 13. Parameter 20 is respecting the moving projection center, 21-23 are able to fit irregular effects of the dynamic scanning [7].

8. ILLUSTRATION OF ADDITIONAL PARAMETERS

The conventional set of additional parameters 1 through 12 which are included in the program are due to the patterns of deformation illustrated in Figure (2) [9].



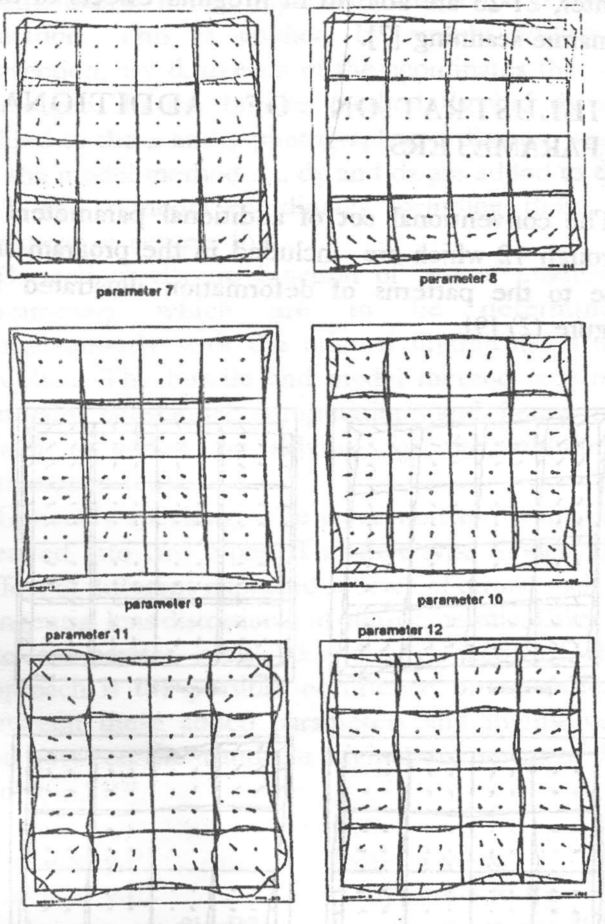


Figure 2. Deformations in dispositive.

9. PRACTICAL STUDY

The main objective of this comparative study which was performed on an oversurveyed photographed block in Vechtel (Hannover-Germany) is to analyse all available tests cases in which variable elements are used in this study [9]. The controlled photo flights were photographed with camera PMK (TOP) of focal length f = 153.123 mm. The test area photo scale was 1: 7900.

The number of cases in the test was 6.

The different alternative technical data for each case can be summarized as follows:

1. Number of strip (E-W) direction.
2. Number of crossing strip (N-S) direction
3. Number of vertical control points.
4. Number of horizontal control points.
5. Number of photos.

The bundle block adjustments for each case was computed at first without additional parameters and secondly with additional parameters. A summery of cases for test blocks is given in Table (1).

Table 1. Summary of cases for block Vechtel -Hannover (Germany).

Additional parameters (Case No).	No. of Strips E-W	No. of Crossing Strips N-S	% Side Lap	Vertical Control Points	Horizontal Control Points
Without (1-I)	5	-	20	55	53
With (1-II)	5	-	20	55	53
Without (2-I)	5	-	20	22	10
With (2-II)	5	-	20	22	10
Without (3-I)	5	5	20	22	10
With (3-II)	5	5	20	22	10
Without (4-I)	5	5	20	13	10
With (4-II)	5	5	20	13	10
Without (5-I)	9	-	60	13	10
With (5-II)	9	-	60	13	10
Without (6-I)	9	5	60	13	10
With (6-II)	9	5	60	13	10

10. *Results and Analysis*

The following effects were taken into consideration:

1. The effect of side overlap.
2. The effect of number of horizontal control points.
3. The effect of number of vertical control points.
4. The effect of presence of additional parameters.
5. The effect of presence of crossing (Tie) strip.

Various factor tests were run. The main series of tests (cases) concerned the use of multiple strip and crossing strips. Auxiliary topics of interest included the use of additional parameters in each case.

Tweleve different cases were used. This comparative study is essentially aimed to evaluate the effect of each element on the accuracy of computed ground coordinates. Tables (2) and (3) show the collected data.

1. The generally good photogrammetric measuring quality is reflected in the magnitudes of the standard image coordinate errors (σ_o) which range from 4.56 to 6.50 μm.
2. The mean squared check point errors (S_x,S_y,S_z) were used to determine which element in the alternative different cases has a significant effect on the accuracy of the adjustment.

Table 2. Accuracy of Independent Check Points Vechtel-Hannover (Germany).

Additional parameters (Case No).		S_x (cm)	S_y (cm)	Sr =	S_z (cm)
Without	(1-I)	4.7	4.8	6.72	22.20
With	(1-II)	2.9	3.7	4.70	6.200
Without	(2-I)	6.4	8.2	10.4	30.60
With	(2-II)	4.4	6.2	7.60	10.10
Without	(3-I)	5.4	5.8	7.92	59.90
With	(3-II)	4.9	5.5	7.36	10.10
Without	(4-I)	5.3	5.7	7.78	79.60
With	(4-II)	4.7	5.6	7.31	10.70
Without	(5-I)	6.4	8.1	10.3	79.20
With	(5-II)	3.8	7.2	8.14	8.600
Without	(6-I)	5.5	6.3	8.36	111.4
With	(6-II)	4.5	6.0	7.50	9.600

Table 3. Results from applying the bundle block adjustments can be summarized and analyzed as follows.

Additional parameters (Case No).		Accuracy of Photo Coordinates (σ_o) (Microns)	S_x Relative (cm)	S_y Relative (cm)	S_z Relative (cm)
Without	(1-I)	5.87	3.0	3.3	12.70
With	(1-II)	4.56	2.5	2.9	5.100
Without	(2-I)	5.90	3.1	3.4	14.60
With	(2-II)	4.68	2.6	2.9	5.600
Without	(3-I)	6.40	2.4	2.4	19.10
With	(3-II)	5.30	1.9	1.9	4.100
Without	(4-I)	6.37	2.4	2.3	22.70
With	(4-II)	5.32	1.9	1.9	4.300
Without	(5-I)	6.25	2.2	2.2	24.40
With	(5-II)	4.95	1.6	1.9	3.340
Without	(6-I)	6.50	3.8	4.9	33.10
With	(6-II)	5.32	3.5	4.0	5.200

The accuracy of independent check points are given in Table (2) and Figure (3). It can be seen that the accuracy in (Z) direction is very poor relative to the accuracies for both X and Y directions. For example the value of (S_z) reaches form (22.2 cm) to (111.4) cm whereas (S_x) and S_y) range from (4.8) to (6.3) cm.

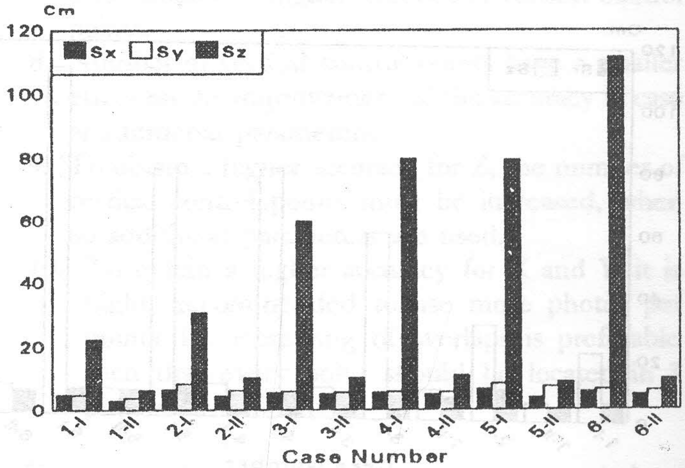


Figure 3. Accuracy of independent check points.

This indicates the weakness in determination of the (Z) values. All these behaviours were for the cases without additional parameters.

3. The results of adjustments using additional parameters are in general very reasonable, the precision specially in (Z) direction was highly improved. It can be seen that using additional parameters leads to a significant improvement in the accuracy, specially in the (Z) ground coordinates.

Another interesting result to note is that the precision of the check points depends mainly upon the use of additional parameters, and is not greatly affected by the number of ground control points or the amount of sidelap. This is evident from examination of Figures (4) through (7).

4. The use of crossing strips (cases 3,4,6) gives an improvement in the accuracy specially when additional parameters are used. This is apparently clear in cases (3.II). (4.II), (6.II).

The values of S_x, S_y and S_z are:

	Case 3.II	Case 4.II	Case 6.II
S_x (cm)	4.9	4.7	4.5
S_y (cm)	5.5	5.6	6.0
S_z (cm)	10.1	10.7	9.6

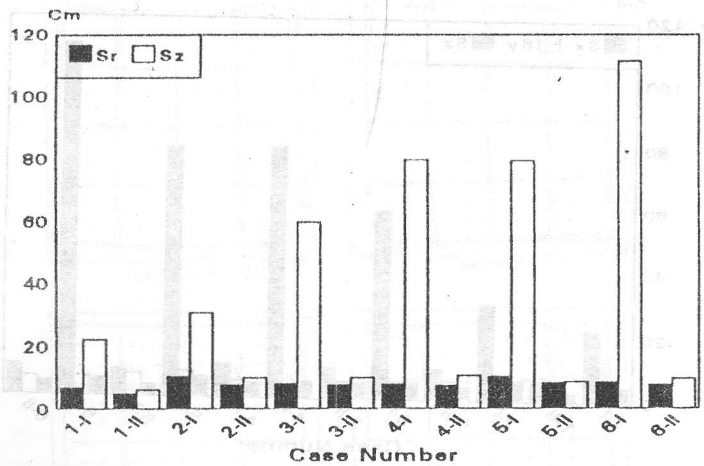


Figure 4. Accuracy of Planimetric positions and elevation position.

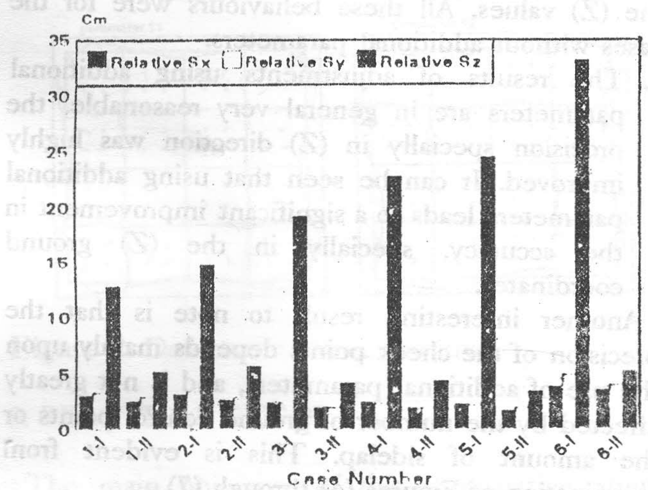


Figure 5. Relative standard deviation.

Therefore, the conclusion seems justified that the combination of additional parameters with crossing strips has a highly good potential for improving the attainable accuracy of the bundle block adjustment.

5. Based on the normed covariance, the relative standard deviation S_{xrel}, S_{yrel}, and S_{zrel} for the neighbored points are computed for evaluating the accuracy of coordinate differences of distances. For cases without additional parameters the value of both $S_{xrel.}$ and $S_{yrel.}$ are seemingly the same and range from 2.2 cm to 4.9 cm., in the same time $S_{zrel.}$ is more or less high and ranges from 12.7 cm to 33.1 cm. By using additional parameters these values are reduced by an amount of 20% to 60% which represent a significant difference.

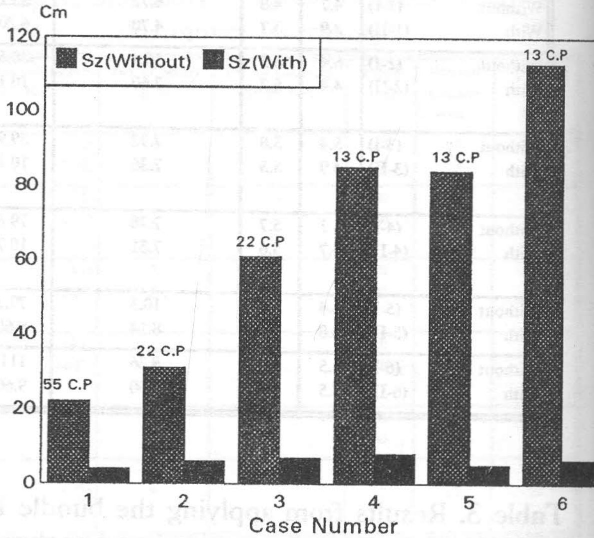


Figure 6. Relation between the No. of vertical control position and the accuracy of determining (Z) coordinates.

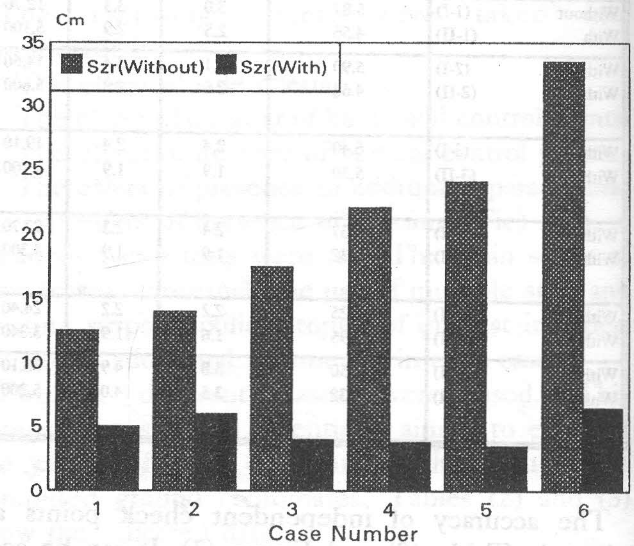


Figure 7. The effect of using additional parameters in accuracy of relative (Sz).

11. CONCLUSIONS

In has been recognized that, the wide-range use of analytical photogrammetry with additional parameters compared to analogue photogrammetry becomes true and is a must. This is justified by availability of hard and soft ware, as well as the expected high accuracy from the analytical photogrammetry. Moreover, all kinds of errors can be mathematically modeled and connected together.

Based on the results in this paper many important conclusions concerning bundle adjustment can be withdrawn:

1. The bundle block adjustment is considered to be one of the most rigorous methods of block adjustment with a high degree of accuracy and flexibility in aerial triangulation.
2. In bundle block adjustment, the computation with self calibration by additional parameters leads to the most accurate results because more than 90% of systematic error are eliminated.
3. The proper selection of the type and number of additional parameters in bundle adjustment is very important in order to compensate for the systematic errors as far as possible. This is because the bundle block adjustment is more sensitive against systematic errors than any other adjustment.
4. The characteristics of systematic errors in navigation data in combined bundle block, especially their separability from systematic errors in image points and in ground control points, should become one of the topics of further research activities.
5. To overcome the systematic error in bundle block adjustment, the use of additional parameters is a must. The main function of additional parameters is the determination of systematic errors in bundle block adjustment, so a high improvement in accuracy will be attainable.
6. For mapping purposes the most economic solution is the block of 20% side lap with one or two crossing strip in order to improve the accuracy in Z-direction. The main function of crossing strips is to stabilize the block of photos and thus the number of control points in the whole control system can be reduced.
7. For precise determination of objects, refinements should be applied on photo-coordinates, sufficient overlap and sufficient geometry for the whole system for stabilizing the height by using either cross strips or a higher number of vertical control points.
8. Number of vertical control points have a smaller effect on the improvement of the accuracy in case of additional parameters.
9. To obtain a higher accuracy for Z, the number of vertical control points must be increased, when no additional parameters are used.
10. To obtain a higher accuracy for X and Y, it is highly recommended to use more photos per points, i.e. increasing of overlaps is preferable such that every point should be located in 3 photos as a minimum.

To sum up the following practical recommendations should be given:

1. The use of limited number of control points with additional parameters provides a reasonable improvement in the attained accuracy of the ground coordinates, especially for Z direction.
2. Well-distributed control points with additional parameters is a must in case of 20% side laps without any cross strips.
3. With 60% side lap or crossing strips the control points distribution can be free all over the whole block.
4. Crossing strips aid the accomplishment of bundle block adjustment with a limited number of control points.

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