A MODEL FOR DETERMINING THE EFFECTIVE THERMAL CONDUCTIVITY OF PARTICLE BEDS

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ABSTRACT

Due to the numerous applications of packed particle beds, the thermal performance has been studied, both theoretically and experimentally. A model has been developed in order to calculate the effective thermal conductivity of a single size particle bed based on a two-dimensional heat transfer analysis of a unit cell. The results of the model were found to be in good agreement with the experimental data. A comparison with some of the available theoretical models showed that the developed model produced good results.

Keywords: Particle beds, effective thermal conductivity, Heat conduction.

Nomenclature

α	thermal accommodation coefficient of the fill gas
ϵ	Particle bed porosity
γ	C_p/C_v
λ	mean free path of the gas molecules (m)
A	cross sectional area (m^2)
$k_{s \text{ or } g}(T)$	thermal conductivity of solid or gas, respectively, at temperature T and atmospheric pressure (W/mK)
P	gas pressure (Pa).
Pr	Prandtl number
q	heat flux (W)
$\dot{q}$	volumetric heat source strength (W/m^3)
r	radius (m)
T	gas temperatire (K)
t_j	jump distance, distance filled with gas between the spheres at the point of calculations (m)

Subscripts

o	value at standard temperature and pressure
g	gas
s	solid
uc	unit cell

INTRODUCTION

The thermal performance of particle beds has been a subject of interest, from both a theoretical as well as an experimental point of view, due to its wide applications. Packed beds have been proposed in fusion reactors for solid breeder ceramic and multiplier materials in the blanket [1-4]. It has been suggested that a packed bed could be used as a mean of controlling the blanket temperature via changing the cover gas composition and/or pressure [5]. Packed beds of $(U,Pu)O_2$ have also been considered as a possible type of fuel for fast breeder reactors [6]. Knowledge of the effective bed thermal conductivity is essential in the design and analysis of such applications. Therefore, several theoretical models have been developed in order to calculate the effective thermal conductivity. However, most models, especially those based on analytical solution of the heat conduction equation are limited to certain conditions. For example, in the Hall and Martin model [7], it is assumed that heat flow is linear in the bed. In reality, heat flow lines tend to bend towards points of contact, hence increasing the conductivity [8]. This assumption will produce high errors in beds with a high ratio of solid/gas thermal conductivity , that is especially found in fusion applications, due to the significant amount of heat flow through contact areas in such beds. Accordingly, the model was modified by introducing an effective contact area parameter, as suggested by Dalle-Donne and Sordon [9]. Furthermore, an empirical multiplier of 1.2 was added for better agreement with the data [8]. Such experimental factor renders the model useless outside the experimental ranges. The Okazaki, Yamasaki, Gotoh and Toei model [10] examines the probabilities of sphere packing, the number of contacts per sphere and heat flow per contact. However, the model did not account for Smoluchowski effect (decrease of heat flow at very low gas pressures when the mean free path becomes finite) or the contact area [10]. The Schlunder, Zehner and Bauer model [11] is an empirical set of equations that includes all of the relevant parameters. Although this model tends to produce good results, it can not be applied outside the range of their experimental results.

In this paper a model was developed in order to calculate the effective thermal conductivity of a single size particle packed bed based on a two-dimensional heat transfer analysis of a unit cell. Results from the model were compared with experimental data from UCLA [12,13] and Imura-Takagoshi [11] experiments. The results of the model were also compared with other theoretical models. Two theoretical models using different approaches were selected. The first model was developed by Ades [14] based on solving the heat conduction equation in a 2-D unit cell using finite difference grid with 20 radial and 26 axial nodes. The model was then applied to calculate the thermal conductivity of sphere-pac UO_2 nuclear fuel. The second model was developed by Gorbis [15] based on dividing the unit cell into a number of zones, each with a distinct thermal resistance then calculating the equivalent thermal resistance of the cell.

MODEL DESCRIPTION

The evaluation of the effective thermal conductivity of the bed is based on the assumption that the conductivity of the bed corresponds to that of a 2-D cylindrical unit cell [14] which is repeated throughout the bed. Figure (1) shows the unit cell that is used in this model which was developed by Ades [14]. The radius of the unit cell is equal to

$$r_{uc} = r_s + a \tag{1}$$

where (a) is a parameter used to reproduce the porosity of the bed. Parameter (a) depends on the sphere radius and porosity. It is calculated from

$$a = \sqrt{\frac{2r_s^2}{3(1-\varepsilon)}} - r_s \tag{2}$$

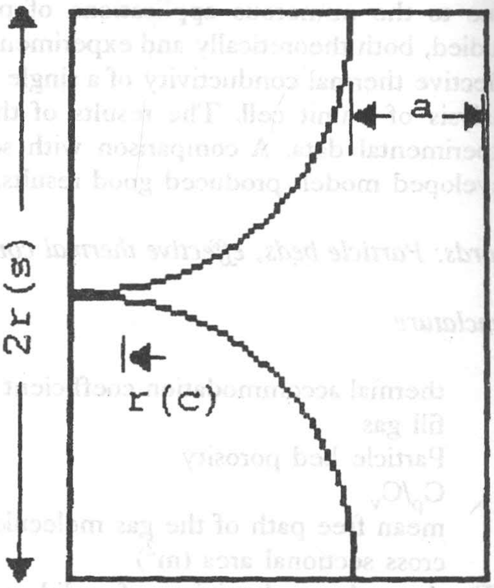

Figure 1. A unit cell for a single size particle bed [14].

The height of the cell is equal to one sphere diameter.

Since the gas is stagnant, no convection will take place through the gas. Assuming that the radiation heat transfer between spheres is negligible, the steady state 2-D heat conduction equation in cylindrical coordinates can be applied to the unit cell

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = -\dot{q} \tag{3}$$

where k is the thermal conductivity of either the solid or the gas, depending on the region in which the equation is applied, and $\dot{q}$ is the volumetric heat generation source strength.

The equation is solved using the finite element method with the following boundary conditions [14]

$$T = T_1 \quad \text{at } z = 0 \tag{4}$$

$$T = T_2 \quad \text{at } z = 2r_s \tag{5}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0 \tag{6}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = r_s + a \tag{7}$$

The effective thermal conductivity of the bed is calculated using the heat flux, q_{uc}, passing through the unit cell

$$k_{eff} = \frac{q_{uc}(2r_s)}{A_{uc}(T_1 - T_2)} \tag{8}$$

where A_{uc} is the unit cell cross sectional area.

The thermal conductivity of the solid spheres is a function of temperature and material of the solid in the cell. The thermal conductivity of the gas is calculated according to Knudsen number at its position in the bed [16,17]

$$k_g = \frac{k_g(T)}{1 + \sigma} \tag{9}$$

where

$$\sigma = 4\,\frac{2-\alpha}{\alpha}\,\frac{\gamma}{\gamma+1}\,\frac{1}{Pr}\,\frac{\lambda_o}{t_j}\,\frac{P_o}{P}\,\frac{T}{273} \tag{10}$$

METHOD OF SOLUTION

The method of finite element in two dimensions r-z was used for solving the heat conduction differential equation. Triangular elements had been used in the model. The material of any element is determined by the position of the coordinate center of that element. The element dimensions depend on the position of the element in the unit cell. Large elements were used throughout the solid while smaller elements were used inside the gas. The smallest elements were used in the separation distance between the two spheres where the conductivity is minimum due to Smoluchowski effect.

It has been noticed that the calculated unit cell effective thermal conductivity is sensitive to the element size in the Knudsen domain. The size of elements in such region was reduced by decreasing the element size and increasing the number of elements in this region till the reduction in size did not affect the numeric results of the output effective thermal conductivity. The model has the capability of using up to 10640 elements with 5467 nodes. The use of larger mesh would increase the contact area between the spheres, leading to a fictitious increase in the value of k_{eff}.

Temperature differences ($T_1 - T_2$) of 20-25°C have been used. Appropriate temperature dependent thermal conductivities have been used for both the solid and the gas in the unit cell. An iteration procedure is developed to account for the dependence of the physical parameters on the temperature.

MODEL VALIDATION

1. *Comparison to Experimental Results*

In order to establish the model's accuracy, it was compared with different experimental results. Data from University of California, Los Angeles [12,13] were mainly used, since these data correspond to ratios of k_s/k_g closest to the ratios applicable in fusion reactors. In addition, data from Imura-Takagoshi experiment were also selected [11]. Calculations were done for Al/He ($k_s/k_g = 1500$), Al/N_2 ($k_s/k_g = 10{,}000$), and Cu/N_2 ($k_s/k_g = 16{,}500$) over pressures ranging from 0.05 atm to 3 atm and compared to the experimental results for cases with different average particle diameters (from 0.12mm to 2mm). Table (1) summarizes the different experimental conditions of the data used.

For the calculations the solid and gas thermal conductivities were represented by [18-20]

$$k_{Al}(T) = 237 - 0.06\,(T-300) \tag{11}$$

$$k_{Cu}(T) = 398 - 0.06\,(T-300) \tag{12}$$

$$k_{He}(T) = k_{He,o}(T_o, P_o)\left(\frac{T}{T_o}\right)^{0.668} \qquad (13)$$

$$k_{N2}(T) = k_{N_2,o}(T_o, P_o)\left(\frac{T}{T_o}\right)^{0.805} \qquad (14)$$

$$k_{UO_2}(T) = \frac{3824}{129.25+T} + 6.126x10^{-11}T^3 \qquad (15)$$

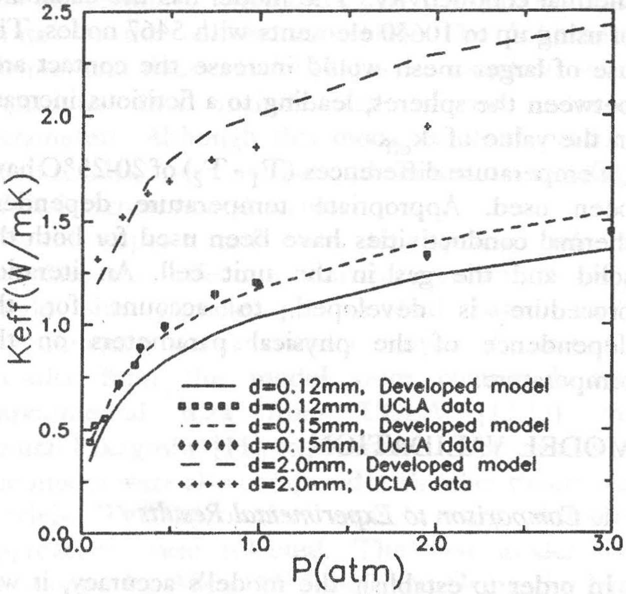

Figure 2. Comparison between the developed model and UCLA data for Al/He bed.

Figure (2) shows the comparison between the developed model and the experimental results for packed beds of Al/He. In Figure (2), the effect of changing the pressure on the effective thermal conductivity is shown for the three particle sizes. It can be noticed from the figure that increasing the gas pressure increases the bed thermal conductivity (k_{eff}). This can be explained by the fact that the thermal conductivity of the gas increases with increasing the pressure due to contraction of the Knudsen domain. The change in k_{eff} is more pronounced at low pressures as the gas thermal conductivity is highly sensitive at such pressures. In the case of small size spheres (0.12 mm), a change in the pressure from 0.1 atm to 0.3 atm (a factor of 3) changes k_{eff} by about 60%, whereas changing the pressure from 1 atm to 3 atm (again a factor of 3) changes k_{eff} by only 20%. The figure also shows that the change in k_{eff} over the whole range of pressure is smaller for the case of large aluminum particles. For the 2 mm aluminum spheres, changing the pressure from 0.09 atm to 3 atm changes k_{eff} by a factor of 2 whereas for the 0.12 mm spheres it changes k_{eff} by a factor of 3. This can be attributed to the larger fraction of solid in the later case.

Figure (3) shows the comparison between the results of the developed theoretical model and the experimental data for packed beds of Al/N_2. The effect of changing the pressure on the effective thermal conductivity is shown for two particle sizes. Again, increasing the gas pressure increases the bed thermal conductivity. However, the change in k_{eff} is much smaller in the case of Al/N_2 than in the case of Al/He (see Figure (2)). In the case of small size spheres (0.12 mm), changing the pressure from 0.1 atm to 3 atm in a bed of Al/N_2 changes k_{eff} by about 60% whereas in an Al/He bed, k_{eff} changes by more than 200% in the same pressure range. This observation also holds for the case of large particles. There is a 22% change in the effective thermal conductivity in the case of Al/N_2 2mm-spheres bed between 0.1 atm and 2 atm, while in the case of Al/He the change is 90%. This observation can be explained by the poor thermal conductivity of nitrogen which directs the heat flux to pass through the solid whose conductivity is not affected by the gas pressure. For the case of the 2mm spheres, the developed model predicts that k_{eff} is monoincreasing with pressure, while for the reported experimental data the results are inconsistent with the behavior of k_{eff} in other experiments. It goes down as pressure increases up to about 0.5 atm then goes up again. The thermal conductivity was also calculated for one case of Cu/N_2 bed and compared to experimental data by Imura and Takagoshi [11] who used much lower pressures than the UCLA experiment. The results are shown in Figure (4). In this figure the bed thermal conductivity remains constant at extremely low pressures (< 10^{-4} atm). This is due to the very high thermal resistance of the gas at these pressures, which forces almost all the heat flow to pass through the solid spheres. As the pressure increases, the gas thermal conductivity increases and its resistance decreases, leading to an increase of the heat flow through the gas.

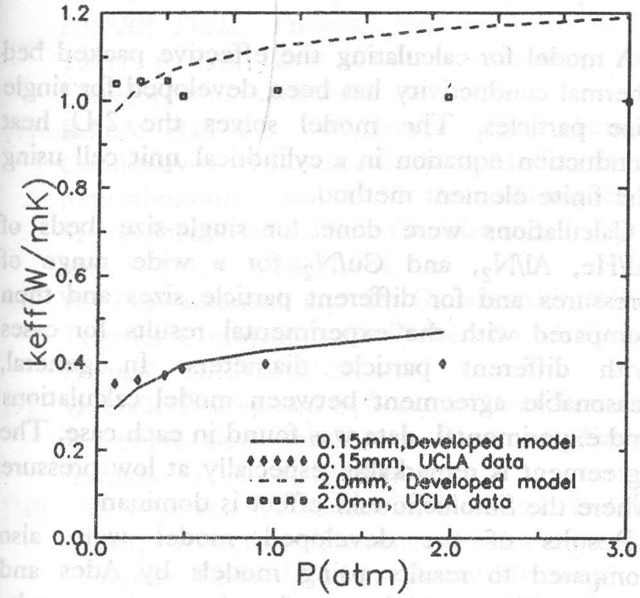

Figure 3. Comparison between the developed model and UCLA data for a bed of Al/N_2.

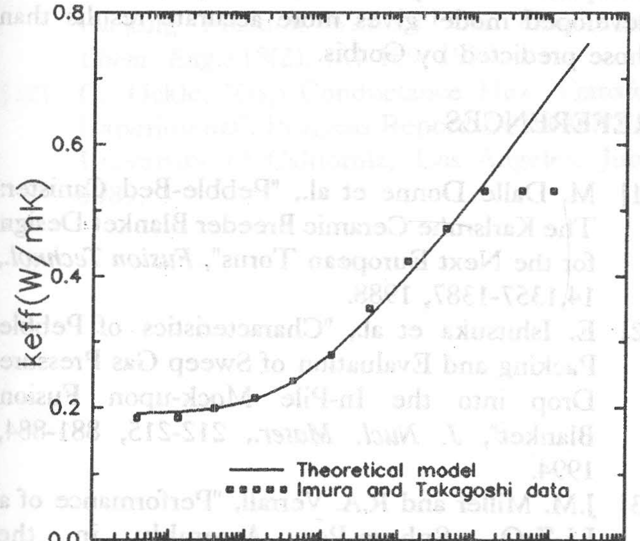

Figure 4. Comparison between the developed model and Imura-Takagoshi data for a bed of Cu/N_2.

2. Comparison to other Theoretical Models

As mentioned earlier, results of the developed model were compared to two theoretical models by Ades [14] and Gorbis [15]. Figure (5) shows the results of the developed model calculations for a UO_2/He single sized particle bed with 40mm solid spheres. Two temperatures were selected for the comparison, 500°C and 2500°C.

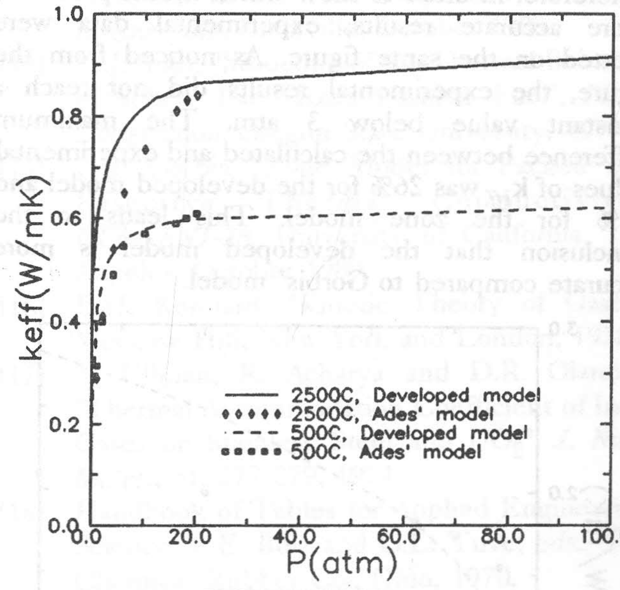

Figure 5. Comparison between the developed model and Ade's model for a bed of UO_2/He.

Figure (5) shows also the calculated results using the Ades' model for the two selected temperatures. The results were in good agreements with the developed model. The effective thermal conductivity at 2500°C was found to be higher than that at 500°C due to the increase in the thermal conductivity of helium with increasing temperature despite the drop in the thermal conductivity of the UO_2. As the helium pressure increases, the effective thermal conductivity of the bed increases with a slowing rate at higher temperatures until it reaches an almost constant value. The developed model showed that for the case of 500°C, the change in k_{eff} between 20 atm and 100 atm was less than 4%, whereas for 2500°C the change was less than 6%.

Although both results from the developed model and Ades' model are comparable, it should be noted that Ades' model is not applicable in cases of high ratios of k_s/k_g due to the used numerical scheme.

Figure (6) shows a comparison between the results of the developed model and the zone model developed by Gorbis [15] for a particle bed of Al/He. The aluminum spheres were 0.721mm in diameter and the bed porosity was 0.384. There was a significant difference in the results of both models. The zone model seemed to indicate that the effective thermal conductivity remained constant after a pressure of approximately 0.5 atm, whereas the developed model indicated that k_{eff} was still affected by the pressure increase even after 3 atm. Therefore, in order to show which model produced more accurate results, experimental data were plotted on the same figure. As noticed from the figure, the experimental results did not reach a constant value below 3 atm. The maximum difference between the calculated and experimental values of k_{eff} was 26% for the developed model and 51% for the zone model. This leads to the conclusion that the developed model is more accurate compared to Gorbis' model.

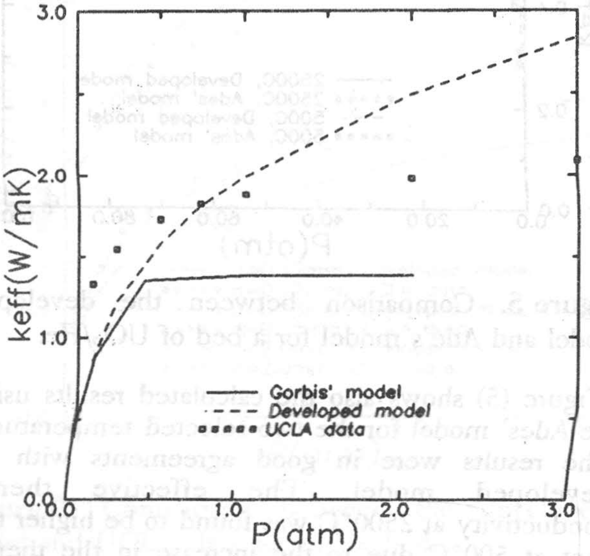

Figure 6. Comparison between the theoretical model and gorbis model for a bed of Al/He.

Table 1. Experimental data from UCLA and Imura-Takagoshi used for comparison with the developed model [11-13].

Solid	Gas	Particle size d_s (mm)	Particle bed Porosity ϵ	Average bed temperature T_{av}(K)	Pressure range (atm)
Al	He	0.12	0.4259	325	0.07-3
Al	He	0.15	0.426	340	0.06-3
Al	He	2.0	0.384	325	0.09-3
Al	N_2	0.15	0.426	340	0.1-2
Al	N_2	2.0	0.384	340	0.09-3
Cu	N_2	1.8	0.39	325	4×10^{-6}-3

CONCLUSIONS

A model for calculating the effective packed bed thermal conductivity has been developed for single size particles. The model solves the 2-D heat conduction equation in a cylindrical unit cell using the finite element method.

Calculations were done for single-size beds of Al/He, Al/N_2, and Cu/N_2 for a wide range of pressures and for different particle sizes and then compared with the experimental results for cases with different particle diameters. In general, reasonable agreement between model calculations and experimental data was found in each case. The agreement is remarkable especially at low pressure where the Smoluchowski effect is dominant

Results of the developed model were also compared to results using models by Ades and Gorbis. The model was found to give results comparable to those of Ades. However, the results differed substantially from those by Gorbis. Comparison to experimental data showed that the developed model gives more accurate results than those predicted by Gorbis.

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