TIME-COST TRADE-OFF IN REPETITIVE PROJECTS

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ABSTRACT

In this paper the modified repetitive project model MRPM for repetitive projects time-cost/ trade-off is given. The MRPM incorporates the basic concepts of line of balance LOB and critical path method CPM in a linear programming model to optimize project completion time/cost. This model is intended to determine the activity duration that would lead to finishing the project at the duration corresponding to minimum project total cost. The proposed MRPM has fewer constraints and less decision variables than the RPM model, which leads to substantial saving in computation time without sacrificing the RPM model efficiency and accuracy. The MRPM , also, comprises the indirect costs elements that are proportional to the project overall completion time in the objective function. By altering the objective function the model can be used, also, to determine the minimum project duration.

Keywords: Construction management, time-cost, repetitive projects.

Notations

CC_k	Crash cost for activity k,
CD_k	Crash duration for activity k,
CS_k	Time cost slope for activity k,
D_k	Activity k optimal duration,
I_k	Start node number of activity k,
IC	Indirect cost per unit time,
J_k	End node number of activity k,
k	Activity code number,
k'	Last activity in the network,
kk	The following activity,
m	Stage number m=1,2,...,S,
Maxj	Network end node number ,
Mini	Network start node number,
N	Number of activities in the network,
ND_k	Activity k normal duration,
NC_k	Activity k normal cost,
PD	optimal project duration,
PDmin	Crash project duration,
PDur	Target project duration,
PR_k	Production rate of activity k,
Q_k	Set of activities follows activity k,
S	Number of stages in the project,
$SB_{k,kk}$	Stage buffer,
$ST_{k(1)}$	Start time for activity k at first stage,
$ST_{k'(1)}$	Start time for the last activity,
$ST_{kk(1)}$	Start time for activity kk at first stage,
$ST_{k(s)}$	Start time for activity k at last stage,
$ST_{k'(s)}$	Start time for the last activity at last stage,
$ST_{kk(s)}$	Start time for activity kk at last stage,
$TB_{k,kk}$	Time buffer between two sequential activities,
Y_k	Time shortened for activity k to achieve the objective function

1. INTRODUCTION AND REVIEW OF LITERATURE

Projects made up of repetitive stages cover a wide range of construction spectrum. The main concern in managing these projects is to maintain constant production rate and continuity of work for working crews. Carr, R. I. et al. (1974) stated that, the *CPM* is a powerful tool for projects which meets two criteria :

1. The number of activities are commensurate with the project complexity.
2. The activities have clear dependencies which define the project progress from start to completion.

sequential activities, k, and, *kk*. The slope of each strip is the production rate, PR_k.

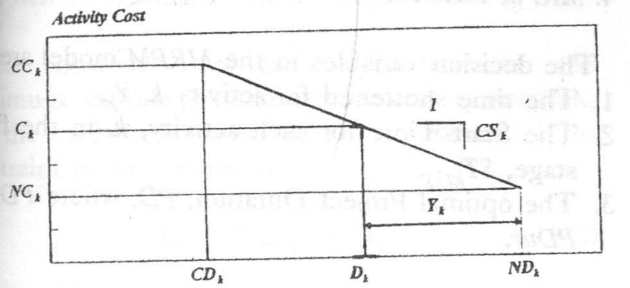

Figure 1. Typical activity time-cost linear relation.

The horizontal distance between two sequential activities is the time buffer, $TB_{k,kk}$, as shown in Figure 2. The stage buffer, $SB_{k,kk}$, is the minimum number of stages need to be kept between the start of activity, *kk*, and the start of preceding activity, *k*.

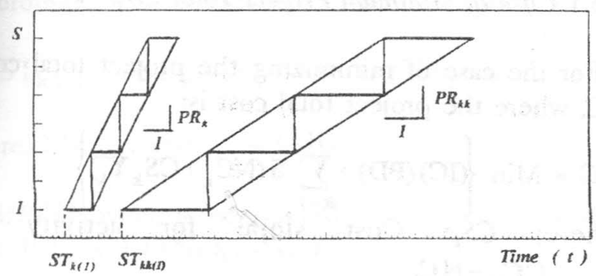

Figure 2: Two sequential activities schedule

The main objective of the *RPM* model (Reda, R. M. 1990) is to minimize the project direct cost for a feasible project duration while satisfying the following constraints:

1. Allow for a minimum necessary time buffer $TB_{k,kk}$ between two sequential activities on the same stage.
2. Allow for a minimum stage buffer $SB_{k,kk}$ between any two sequential activities at a particular time,*t*.
3. Specify a target project duration, *PDur*.

In formulating the *RPM* model the following is considered:

1. An activity on the arrow network is used to express a typical stage of the project. *LOB* is used to show the activity schedule in the project time plan for the successive stages. The work on each activity is conducted one unit at a time and the learning phenomenon is neglected.
2. Each activity *k* (where $k = 1,2,..,N$) has a time buffer, $TB_{k,kk}$, at each stage, m.
3. Any two sequential activities may have a stage buffer, $SB_{k,kk}$, of a specific number of stages between the starts of activities *k* and *kk* to meet practical and/or technological purposes. The stage buffers have to be identified by the planner for these activities.
4. For each activity, *k*, in the network, the time-cost relation is linear.
5. No idle time is allowed for employed crews, thus once a crew starts working on an activity at the first stage it will continue working with the same production rate until the completion of the last stage.
6. A constant duration is set for the same activity at all stages to maintain a constant production rate. If an activity duration needs to be changed to meet a particular feasible project duration, then an equal change must be made to the activity duration at all stages.

The *RPM* formulation is summarized as follows:

Objective Function :

$$\text{Min} \sum_{k=1}^{N} CS_k Y_k$$

Model Constraints :

$$Y_k \leq ND_k - CD_k$$

$$ST_{k'(s)} - Y_{k'} \leq PDur - ND_{k'}$$

$$ST_{kk(s)} - ST_{k(s)} + Y_k \geq TB_{k,kk} + ND_k$$

$$ST_{kk(1)} - ST_{k(1)} + Y_k \geq TB_{k,kk} + ND_k$$

$$ST_{k(s)} - ST_{kk(1)} + (S-SB_{k,kk}-1)Y_{kk} < (S-SB_{k,kk}-1)\, ND_{k,kk}$$

$$ST_{kk(1)} - ST_{k(1)} + (SB_{k,kk} + 1)\, Y_k > (SB_{k,kk} + 1)\, ND_k$$

$$ST_{k(s)} - ST_{k(1)} + (S-1)\, Y_k = (S-1)\, ND_k$$

PS: The number of decision variables (Y_k, $ST_{k(1)}$, and $ST_{k(s)}$) are 3N.

Where;

k is predecessor of kk;
k' is the last activity in the network;
CD_k Crashed duration for activity, k;
$ST_{k'(s)}$ Start time for the last activity in the network, k', at last stage;
$Y_{k'}$ Time shortened for the last activity in the network, k'; and
CC_k Crashed cost for activity, k.

The *RPM*, model has the advantage of scheduling repetitive projects activities to finish the project at the minimum possible direct cost given a target project duration, *PDur*. The *RPM* requires developing a simple network representing a typical stage of the project, and the time / cost relation for each activity.

RPM has dependent variables $ST_{k(s)}$ which increase the number of decision variables. Also, it has unnecessary constraints which will be discussed and omitted later.

2. MRPM ASSUMPTION

The *RPM* model assumptions listed above are adopted in the *MRPM*.

3. *MRPM* MODEL INPUT DATA

The input data for the *MRPM* model is composed of:

1. Number of activities in the network, N.
2. The following information are given for each activity, k: Activity nodes I_k, J_k, normal duration ND_k, crashed duration CD_k, normal cost NC_k, and crashed cost CC_k.
3. Number of stages in the project, S.
4. The target project duration *PDur* where; *PDur* is greater than the crash project duration *PDmin*.
5. Indirect cost per unit time *IC*. The indirect cost *IC* includes, only, indirect cost elements which are proportional to the completion time.
6. For any two sequential activities k and kk where $I_{kk} = J_k$.
   6.1. Time buffer, $TB_{k,kk}$.
   6.2. Stage buffer, $SB_{k,kk}$.

4. *MRPM* DECISION VARIABLES

The decision variables in the *MRPM* model are:

1. The time shortened for activity k, Y_k.
2. The Start Time for each activity, k, in the first stage, $ST_{k(1)}$.
3. The optimal Project Duration, *PD*; where $PD \leq PDur$.

PS: The number of decision variables in this model are, (2N +1).

5. OBJECTIVE FUNCTION

The objective function, Z, for the proposed *MRPM* model can be constructed either to minimize the project total cost, *TC*, or the project duration, *PD*.

5.1 *Case of Minimum Project Total Cost*

For the case of minimizing the project total cost, *TC*, where the project total cost is:

$$TC = \text{Min} \left\{ (IC)(PD) + \sum_{k=1}^{N} S(NC_k + CS_k Y_k) \right\}$$

where; CS_k: Cost slope for activity k,

$$Cs_k = \frac{CC_k - NC_k}{ND_k - CD_k}$$

As S and NC_k are constants they can be omitted from the optimization process and the total cost objective function Z is:

$$Z = \text{Min}\ (IC\ PD + \sum_{k=1}^{N} CS_k Y_k) \qquad (1a)$$

5.2 *Case of Minimum Project Duration*

In this case, the objective function can be expressed mathematically as follows;

$$Z = \text{Min PD} \qquad (1b)$$

6. MODEL CONSTRAINTS

In the *MRPM* to achieve the network logic, time buffer, stage buffer, and project duration, the following set of constraints should be fulfilled.

6.1 *Activity Duration Constraints*

This constraint sets the upper limit for the maximum possible reduction in the activity duration, Y_k, the activity needs to be shortened. This constraint is formulated as;

$$Y_k \leq ND_k - CD_k \tag{2}$$

6.2 *Project Duration Constraint*

This constraint limits the completion time for each path in the network to be less than or equal to the optimum Project Duration, *PD*. Q_k is the set of project activities, with end node J_k ; where Max_j ,is the last node in the considered network. In Figure (3) *PD*, is the optimal project duration and, $PR_{k'}$ is the production rate for the last activity, *k'*. Accordingly, this constraint can be formulated as;

$$ST_{k'(1)} + (ND_{k'} - Y_{k'}) + (S-1) / PR_{k'} \leq PD$$

where; $k' \in Q_{k'}$ and $J_{k'} = Maxj$

In *MRPM*, work is conducted one unit at a time, thus: $PR_{k'} = 1 / (ND_{k'} - Y_{k'})$.

Accordingly, the above constraint is modified to;

$$ST_{k'(1)} + S. (ND_{k'} - Y_{k'}) \leq PD$$

$$PD - ST_{k'(1)} + S.Y_{k'} \geq S.ND_{k'} \tag{3}$$

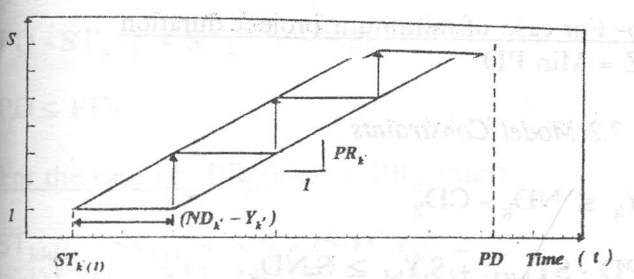

Figure 3. The completion time limit for the network paths.

6.3 *Time Buffer Constraints*

These two constraints maintain the Time Buffer, $TB_{k,kk}$, for each activity, *k*, in a typical network between the finishing time of activity, *k*,and the start of each following activity, *kk*, where $kk \in Q_k$. This time buffer requirement should be fulfilled at the first and the last stages of the activity as shown in Figures (4a) and (4b) respectively. The third constraint imposes last stage time buffer, and is formulated as;

$$ST_{k(1)} + (S-1)/PR_k + (ND_k - Y_k) \leq$$
$$ST_{kk(1)} + (S-1)/PR_{kk} - TB_{k,kk}$$

where;

$J_k = I_{kk}$; and

Q_k: The set of activities that directly follow the activity,*k*, in the network.

By substituting $PR_k = 1/ (ND_k - Y_k)$ and $PR_{kk} = 1/(ND_{kk} - Y_{kk})$. Accordingly, the above constraint takes the form,

$$ST_{kk(1)} - ST_{k(1)} + S.Y_{.k} - (S-1). Y_{kk} \geq$$
$$(S-1). (ND_k - ND_{kk}) + TB_{k,kk} + ND_k \tag{4}$$

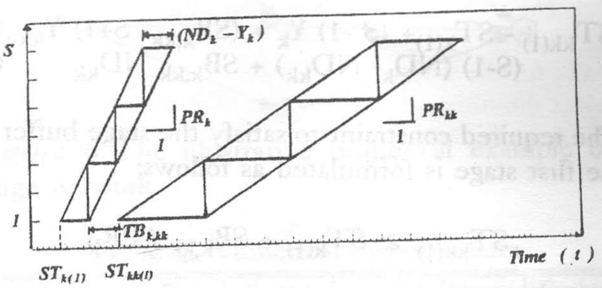

Figure 4a. Time buffer constraints in the first stage.

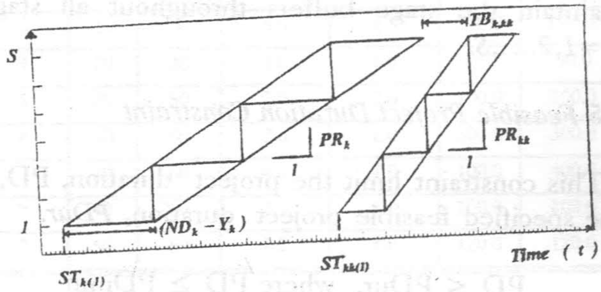

Figure 4b. Time buffer constraints in the last Stage.

The required constraint to keep the time buffer in the first stage is formulated as;

$$ST_{k(1)} + (ND_k - Y_k) \leq ST_{kk(1)} - TB_{k,kk}$$

$$ST_{kk(1)} - ST_{k(1)} + Y_k \geq TB_{k,kk} + ND_k \quad (5)$$

Since the production rate, PR_k, is maintained for each activity, *k*, and the time buffer is fulfilled at the first and last stage by the constraints 4 and 5 respectively, so the time buffer will be maintained throughout all stages.

6.4 *Stage Buffer Constraints*

As shown in Figure (5a) and Figure (5b), these two constraints specify the minimum number of stages $SB_{k,kk}$ required between the starts of two sequential activities *k* and *kk* for practical and/or technological purposes. The fifth constraint, which satisfies the stage buffer for the last stage, Figure (5a) is formulated as;

$$ST_{kk(1)} + (S - SB_{k,kk} - 1)/PR_{kk} \geq ST_{k(1)} + (S-1)/PR_k$$

$$ST_{kk(1)} - ST_{k(1)} + (S-1)\, Y_k + (SB_{k,kk} - S + 1)\, Y_{kk} \geq (S-1)\,(ND_k - ND_{kk}) + SB_{k,kk}\, ND_{kk} \quad (6)$$

The required constraint to satisfy the stage buffer in the first stage is formulated as follows;

$$ST_{kk(1)} \geq ST_{k(1)} + SB_{k,kk} / PR_k$$

$$ST_{kk(1)} - ST_{k(1)} + SB_{k,kk} \cdot Y_k \geq SB_{k,kk} \cdot ND_k \quad (7)$$

Satisfying constraints 6 and constraints 7 would maintain the stage buffers throughout all stages *m=1,2,...,S*.

6.5 *Feasible Project Duration Constraint*

This constraint limit the project duration, PD, to the specified feasible project duration, *PDur*.

$$PD \leq PDur, \text{ where } PD \geq PDmin$$

PDmin: Crashed project duration (8)

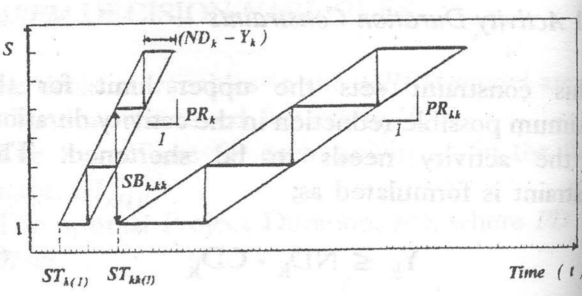

Figure 5a. Stage buffer constraints in the first stage.

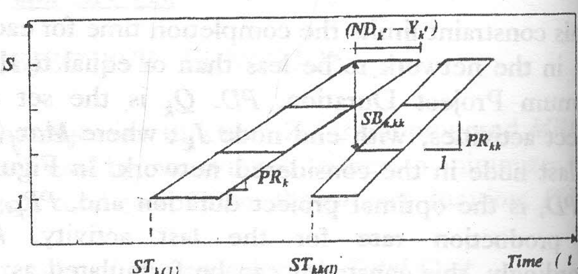

Figure 5b. Stage buffer constraints in the last stage

7. *MRPM* FORMULATION

The mathematical formulation of *MRPM* is summarized below:

7.1 *Objective Function:*

<u>a- For case of minimum project total cost</u>

$$Z = \text{Min } (IC.PD + \sum_{k=1}^{N} CS_k . Y_k)$$

<u>b- For case of minimum project duration</u>

$$Z = \text{Min PD}$$

7.2 *Model Constraints*

$$Y_k \leq ND_k - CD_k$$

$$PD - ST_{k'(1)} + S.Y_{k'} \geq S.ND_{k'}$$

$$ST_{kk(1)} - ST_{k(1)} + S.Y_k - (S-1).\, Y_{kk} \geq (S-1).\,(ND_k - ND_{kk}) + TB_{k,kk} + ND_k$$

$$ST_{kk(1)} - ST_{k(1)} + Y_k \geq TB_{k,kk} + ND_k$$

$$ST_{kk(1)} - ST_{k(1)} + (S-1)\,Y_k + (SB_{k,kk} - S + 1)\,Y_{kk} \geq (S-1)\,(ND_k - ND_{kk}) + SB_{k,kk}\,ND_{kk}$$

$$ST_{kk(1)} - ST_{k(1)} + SB_{k,kk}.\,Y_k \geq SB_{k,kk}.\,ND_k$$

$$PD \leq PDur$$

8. CONSTRAINTS REDUCTION

From Figures 1 to 5, it can be noticed that the activity strips approach each other, either at the first or the last stage. Accordingly, the constraints of the other end of strip become redundant. The proposed constraints can be released as follow:

1. For the case of:
   PR_k (max) < PR_{kk} (min) constraints 5 and 7 are eliminated.
2. For the case of:
   $PR_k(\min) \geq PR_{kk}(\max)$ constraints 4 and 6 are eliminated.

Where, the maximum and minimum production rates for an activity, k, can be calculated from the following relation:

$$PR_k(\max) = \frac{1}{CD_k} \quad (9)$$

$$PR_k(\min) = \frac{1}{ND_k} \quad (10)$$

Incorporating the above cut down of model constraints, the model constraints can be rewritten as follows;

$$Y_k \leq ND_k - CD_k$$

$$PD - ST_{k'(1)} + S.Y_{k'} \geq S.ND_{k'}$$

$$PD \leq PDur$$

<u>For the case of :</u> $PR_k(\max) < PR_{kk}(\min)$

$$ST_{kk(1)} - ST_{k(1)} + S.Y_k - (S-1).\,Y_{kk} \geq (S-1).(ND_k - ND_{kk}) + TB_{k,kk} + ND_k$$

$$ST_{kk(1)} - ST_{k(1)} + (S-1)\,Y_k + SB_{k,kk} - S+1)Y_{kk} \geq (S-1)(ND_k - ND_{kk}) + SB_{k,kk}\,ND_{kk}$$

<u>For the case of :</u> $PR_k(\min) \geq PR_{kk}(\max)$

$$ST_{kk(1)} - ST_{k(1)} + Y_k \geq TB_{k,kk} + ND_k$$

$$ST_{kk(1)} - ST_{k(1)} + SB_{k,kk}.\,Y_k \geq SB_{k,kk}.\,ND_k$$

If neither of the above two cases exists, all the seven constraints should be considered. For the simple illustrative numerical example given in Reda, R. M. (1990) the number of the the variables and constraints have been reduced from 17 to 10 and from 24 to 22, respectively, without sacrificing the model quality and efficiency. The reduction in the size of the coefficient matrix is more than 40% which will reduce dramatically the computation time. In larger problems the reduction may be more drastic.

9. NUMERICAL EXAMPLE

The nine activities network given in Figure 6 is an illustrative numerical example of the proposed *MRPM*. Table (1) displays the input data for illustrative numerical example.

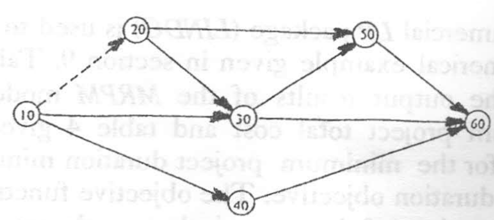

Figure 6 The illustrative numerical example one stage network.

Table 1. Numerical example data.

Activity code	Start node	End. node	Normal duration	Crashed Duration	Normal cost	Crashed cost
1	10	20	0.0	0.0	0.0	0.0
2	10	30	3.0	1.0	100.0	400.0
3	10	40	5.0	2.0	800.0	1700.0
4	20	30	3.0	1.0	10.0	90.0
5	20	50	2.0	2.0	100.0	100.0
6	30	50	2.0	2.0	300.0	300.0
7	30	60	4.0	1.0	100.0	400.0
8	40	60	2.0	2.0	100.0	100.0
9	50	60	1.0	1.0	120.0	120.0

The number of stages is *four stages*. The project indirect cost is *L.E.* 50.0 per unit time. Activity #1 has the time and stage buffers shown in Table (2) below. All other activities have no time or stage buffers. To simplify the creation of the mathematical model, a *FORTRAN* program is written to formulate the objective function and constraints and to check that the following relations are fulfilled- Hafez, S. M. (1996):

1. $ND_k \geq CD_k$, and $NC_k \leq CC_k$.
2. For $ND_k = CD_k$, then $NC_k = CC_k$.
3. For $ND_k = 0.00$,
   $CD_k = NC_k = CC_k = 0.00$ (Dummy activity)
4. $TB \geq 0.00$, and $SB \geq 1$
5. The network has only one start and one end node.

Table 2. Time and stage buffers between sequential activities.

Activity code number	Following activity code number	Time buffer	Stage buffer
4	7	1.0	2

A commercial *LP* package (*LINDO*) is used to solve the numerical example given in section 9. Table 3 gives the output results of the *MRPM* model for minimum project total cost and table 4 gives the results for the minimum project duration minimum project duration objective. The objective function(s) and constraints of the numerical example are given in Hafez, S. M. (1996).

Table 3. Minimum project total cost MRPM Output.

Activity code number	Start node number	End. node number	optimal activity duration	optimal activity cost
1	10	20	0.0	0.0
2	10	30	3.0	100.0
3	10	40	5.0	800.0
4	20	30	3.0	10.0
5	20	50	2.0	100.0
6	30	50	2.0	300.0
7	30	60	4.0	100.0
8	40	60	2.0	100.0
9	50	60	1.0	120.0

Project Duration = 22.0 Unit time
Total Direct Cost= 6520.0 L.E.
Total Indirect Cost= 1100.0 L.E.
Total Cost= 7620.0 L.E.

Table 4. Minimum project duration MRPM Output.

Activity code number	Start node number	End. node number	optimal activity duration	optimal activity cost
1	10	20	0.0	0.0
2	10	30	1.0	400.0
3	10	40	2.0	1700.0
4	20	30	1.0	90.0
5	20	50	2.0	100.0
6	30	50	2.0	300.0
7	30	60	2.0	300.0
8	40	60	2.0	100.0
9	50	60	1.0	120.0

Project Duration= 10.0 Unit time
Total Direct Cost= 12440.0 L.E.
Total Indirect Cost= 500.0 L.E.
Total Cost= 12940.0 L.E.

10. CONCLUSIONS:

The *MRPM* model has fewer constraints and decision variables and can be used more efficiently specially with large projects, where computation time and the computer capacity may limit the use of the *RPM* model. The *MRPM* model does not sacrifice the accuracy or generality of the *RPM*. The *MRPM* model introduces the indirect costs elements that are proportional to the project completion time to the objective function. This term is believed to be not only of reasonable effect on the optimization results, but also it makes the model more realistic..

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