

STABILITY OF BARRAGES WITH SUBSIDIARY GLACIS WEIRS

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ABSTRACT

To secure the stability of the existing Nile barrages on the river Nile against excess effective heads, which is an outcome from the expected longitudinal degradation, a proposed subsidiary glaciis weir is to be constructed downstream each one. The proposed weir floor has an inclined middle apron and a cutoff wall along the end horizontal part. The method of successive conformal mapping has been used to transform the studied problem into another simplified problem without cutoffs, which had already been solved analytically. A computer program is prepared to determine the successive mapping steps and calculate the seepage characteristics. The effect of some relative dimensions of the problem on seepage characteristics are studied and the results are plotted on form of curves. An illustrated example has been prepared to study the effect of a proposed downstream subsidiary glaciis weir on the stability of Nag Hammadi barrage.

Keywords: Barrages, Glaciis weirs, Gradients, Seepage, Subsidiary weirs, Uplift pressures.

Notations

B_1	total effective length of the U.S floor;		the transformed D.S floor;
B_1'	total effective length of the transformed U.S floor;	l_1	distance of D.S cutoff wall from point P;
b_1	distance of U.S cutoff wall from the toe of floor;	L_2	length of horizontal projection of middle inclined part of the D.S floor;
B_2	total horizontal projection of the D.S floor;	L_2'	length of horizontal projection of middle inclined part of the transformed D.S floor;
b_2	distance of U.S cutoff wall from the end of U.S floor;	l_2	distance of D.S cutoff wall from point H;
D	lowering depth in D.S bed;	L_3	length of the tail lower horizontal part of the D.S floor;
d	relative lowering depth in D.S bed, $d=D/B_1$;	L_3'	length of the tail lower horizontal part of the transformed D.S floor;
D_1	depth of the U.S cutoff wall;	L_f	length of the intermediate filter between the two floors;
d_1	relative depth of the U.S cutoff wall, $d_1=D_1/B_1$;	L_f'	transformed length of the intermediate filter between the two floors;
D_2	depth of the D.S cutoff wall;	l_a, l_e, \dots, l_p	relative horizontal distances of some points from the U.S cutoff wall;
d_2	relative depth of the D.S cutoff, $d_2=D_2/B_1$;	U1	net uplift pressure at any point along the U.S floor;
D.S	downstream;	U2	net uplift pressure at any point along the D.S floor;
H_1	effective differential head on U.S floor;	U.S	upstream
H_2	effective differential head on D.S floor;		
IE1	exit gradient along the intermediate filter;		
IE2	exit gradient along the D.S bed;		
k	coefficient of permeability of the soil;		
L_1	length of the U.S upper horizontal part of the D.S floor;		
L_1'	length of the U.S upper horizontal part of		

INTRODUCTION

After the construction and operation of the High Aswan dam, the sediment load in the river Nile flow has almost vanished. This has caused a longitudinal degradation along the different reaches between the existing Nile barrages. The result of such degradation is a continuous lowering in the surface water elevation D.S of each barrage. This is clearly observed downstream both of old Esna and Nag Hammadi barrages. The result of such lowering in water levels is an increase in the effective head on the barrages, which endangers their stability. The evaluation of degradation depths D.S some of Nile barrages, for some practical discharges, is investigated by El-ansary[3]. A subsidiary weir is proposed to be constructed D.S each barrage, to carry the excess head on each one. This idea was carried out by the Egyptian Engineer at the downstream of the Delta barrages in 1902 [12].

The seepage characteristics underneath two closely hydraulic structures had been studied mathematically, using conformal mapping technique, in many researches. In such studies, the D.S structure (subsidiary weir) floor was either flat and horizontal without cutoff [4], or flat and horizontal with cutoffs [2,5,8, and 11], or having inclined middle apron without cutoffs [6].

In the present study, the U.S floor, which represents the floor of the existing barrage, is assumed flat and provided with a single cutoff wall. The floor of the D.S structure, which represents the floor of the proposed subsidiary Glacis weir, has a middle inclined apron and a single cutoff wall lies at some point along the lower horizontal end part, as shown in Figure (1). Herein, the seepage characteristics are studied under the two floors with an intermediate filter between them. The successive conformal mapping method is used to transform the present problem into the problem without any cutoff walls, which was already solved by Elganainy [6].

An illustrated solved example is prepared for studying the effect of a proposed subsidiary Glacis weir D.S existing Nag Hammadi barrage on the uplift pressures under its floor and the exit gradients along the D.S bed .

THE PROBLEM AND METHOD OF SOLUTION

The problem is shown schematically in Figure (1). The total length of the U.S floor (AE), which represents the existing barrage, is assumed B1. The floor is provided by a cutoff wall (BCD) of depth D1 measured from the U.S bed and lies at distance b1 from point A. The length of the intermediate filter between the two structures (EF) is assumed Lf. The D.S structure (FGHMNOP), which represents the proposed subsidiary weir, has an inclined middle apron (GH) of length L2 and depth D, US horizontal part (FG) of length L1, and D.S horizontal lowering part (HP) of length L3. The D.S structure is also provided by a cutoff wall (MNO) of depth D2 measured from the D.S lowering bed and lies at distance l1 from point H. The two structures are founded on a homogeneous isotropic pervious soil extending to infinity in all directions.

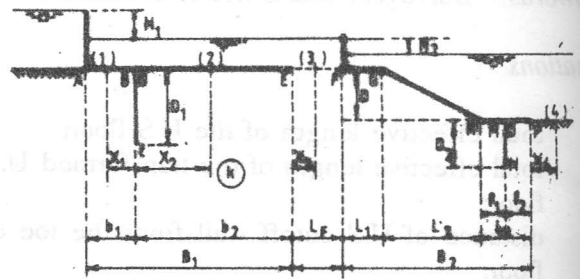


Figure 1. Schematic sketch of the problem.

The method of successive conformal mapping is one of the most accurate approximate methods for solving two dimensional problems of groundwater hydraulics. It was worked out in details by Filchakov in [7]. The method aims to transform a complex plane (y), which is provided by a slit of length d; at the origin; into another complex plane without a slit (t). The conformal mapping of (y) plane onto the (t) plane can be obtained by means of the following transformation function:

$$t = \sqrt{d^2 + y^2} \tag{1}$$

in which $y = x + iz$ and $t = s + ir$

The transformation function (1) can be used to

map any configuration, such as a pattern of streamlines representing the trajectories of the motion of water particles in flow from (y) plane onto (t) plane. If necessary, several such transformations can be made successively. By using a proper procedure we can thus prepare the ground for the application of the analytical solution to a modified and considerably simplified region. Therefore, the complicated problems of flow under hydraulic structures, which are provided with many cutoff walls, can be transformed successively onto another simplified problems.

In the present study the investigated problem, which is provided with two cutoff walls, is mapped onto another simplified problem without any cutoffs. The modified problem had been already solved analytically, using the conformal mapping technique, by Elganainy [6]. After computation of the successive transformed steps, the obtained formulae of the modified problem was used to calculate the uplift pressures underneath the two structures and exit gradients downstream both of them. Figure (2) shows the transformation steps of the studied problem. Figure (2a) represents the actual form of the problem in (y) plane with relative dimensions to the length of the U.S floor (B1), i.e $d_1 = D_1/B_1$; $d_2 = D_2/B_1$; $d = D/B_1$;etc. By using the transformation function (1), the U.S cutoff wall in (y) plane is eliminated in (y') plane as shown in Figure (2b). The origin of axes should lie at the top point of the cutoff wall before elimination. This is clearly observed in Figures (2a & c). Similarly, the D.S cutoff wall in (y'') plane is also eliminated, using the transformation function (1), as shown in Figures (2c & d). The result is a pattern of flow underneath two hydraulic structures without any cutoff walls, as shown in (z) plane of Figure (2e). Table (1) summarizes the calculations of all successive transformation steps of all key points of the problem. The modified problem dimensions B_1' ; L_f ; L_1' ; L_2' ; and L_3' can be determined from the table. The lowering depth in bed levels (D) does not change during the transformation steps and remains constant.

COMPUTATIONS AND RESULTS

This study aims to investigate the effect of the

variation in some relative dimensions of the two structures, relative effective heads and relative distance between them on the seepage characteristics. The specific values of the relative variables are: $L_f/B_1=0.25$; $H_2/H_1=0.50$; $D_1/B_1=0.20$; $b_1/B_1=0.20$; $L_1/B_1=0.10$; $L_2/B_1=0.28$; $D_2/B_1=0.10$; $L_3/B_1=0.10$; $h/L_3=0.50$; and $D/B_1=0.04$. From practical experience and last studies, the slope of the inclined middle part is taken 7 : 1, which is suitable for the river Nile bed, and kept constant for all studied cases.

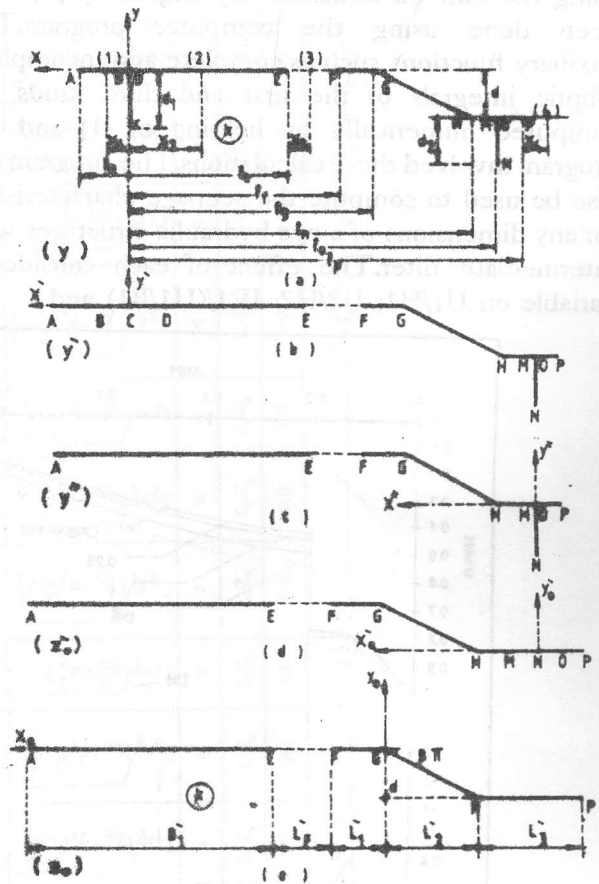


Figure 2. Successive transformation steps of the problem.

The considered variables in the study are: L_f/B_1 ; H_2/H_1 ; D_1/B_1 ; D_2/B_1 ; and D/B_1 while, the remained relative dimensions are kept constant as mentioned before. Each of the considered variables is varied (in turn) while, the others are kept

constant. The investigated seepage characteristics herein are:-

- relative net uplift pressures under the U.S floor ($U1/H1$),
- relative net uplift pressures under key points of the D.S floor ($U2/H$), points G; H; M; and O
- relative exit gradients along the intermediate filter, $IE1/(H1/B1)$, and
- relative exit gradients along the D.S bed, $IE2/(H1/B1)$.

All successive transformation steps and calculations of seepage characteristics, for the modified problem, using the derived equations by Elganainy [6] have been done using the computer program. The auxiliary functions such as complete and incomplete elliptic integrals of the first and third kinds are computed numerically by helping of [1] and the program involved these calculations. The program can also be used to compute the seepage characteristics for any dimensions of a two hydraulic structures with intermediate filter. The effect of each considered variable on $U1/H1$; $U2/H2$; $IE1/(H1/B1)$ and

$IE2/(H1/B1)$ is plotted on curves and arranged as groups.

ANALYSIS OF RESULTS

Effect of Intermediate Filter length

The results of this case is shown in Figure (3). Figure (3a) indicates that the relative net uplift pressure values along the U.S floor ($U1/H1$) have a slight increase with increases in the relative length of the intermediate filter ($Lf/B1$). The maximum excess in pressures takes place just D.S the cutoff wall, at point D. For the studied case, only about 6% of the U.S differential effective head ($H1$) is added when the value of $Lf/B1$ is increased from 0.125 to ∞ . The uplift pressure values are calculated for the case without subsidiary weirs, $Lf = \infty$, by using

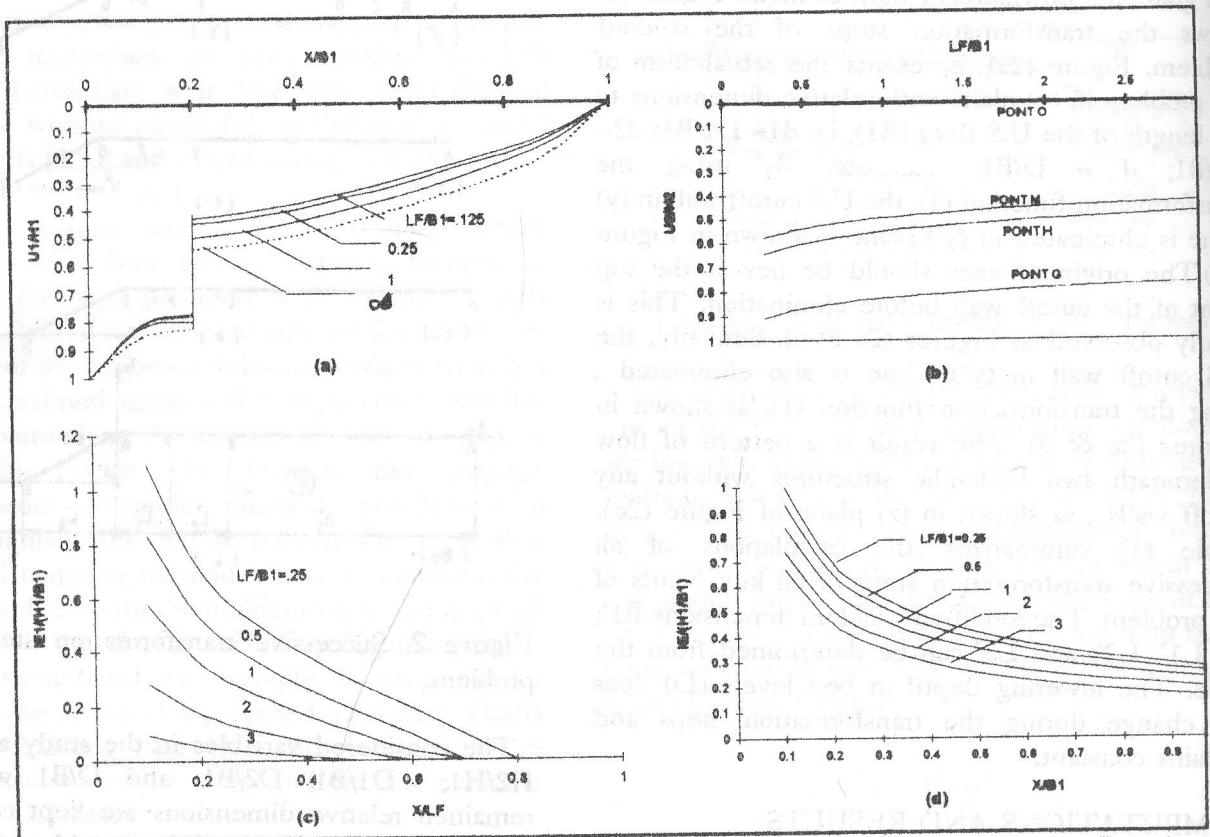
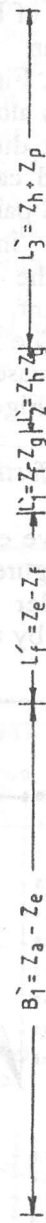


Figure 3. Effect of relative length of filter ($Lf/B1$): $H2/H1=0.5$; $D1/B1=0.2$; $b1/B1=0.2$; $L1/B1=L3/B1=0.1$; $L2/B1=0.28$; $D2/B1=0.1$; $h1/L3=0.5$; $D/B1=0.04$.

Table (1)

FIG. No.	PLANE		POINTS																
	y	x	A	1	B	C	D	2	E	3	F	G	H	M	N	O	P	4	
2-a		z	l_e	x_1	0.0	0.0	0.0	x_2	$-l_e$	$-(l_e+x_3)$	$-l_e$	$-l_e$	$-l_e$	$-l_e$	$-l_e$	$-l_e$	$-l_e$	$-(l_e+x_4)$	$-d$
2-b	y	x'	$y_a = \sqrt{l_e^2+d_1^2}$	$y_1 = \sqrt{x_1^2+d_1^2}$	d1	0.0	$y_2 = \sqrt{x_2^2+d_1^2}$	$y = \sqrt{l_e^2+d_1^2}$	$y_3 = \sqrt{-(l_e+x_3)^2+d_1^2}$	$y_f = \sqrt{l_e^2+d_1^2}$	$y_g = \sqrt{l_e^2+d_1^2}$	$y_h = \sqrt{l_e^2+d_1^2}$	$y_m = \sqrt{l_e^2+d_1^2}$	$y_n = \sqrt{l_e^2+d_1^2}$	$y_o = \sqrt{l_e^2+d_1^2}$	$y_p = \sqrt{l_e^2+d_1^2}$	$y_4 = \sqrt{l_e^2+d_1^2}$	x_4	$-d$
2-c	y'	z'	y_a+y_1	y_e+y_1	y_m+d_1	y_m	y_e+y_2	y_m-y_2	y_m-y_3	y_m-y_f	y_m-y_g	y_m-y_h	0.0	0.0	0.0	0.0	0.0	y_m-y_4	$-d$
2-d	z_1	x_0	$z_1^2 + \frac{1}{2}(l_e^2 + x_1^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(x_1^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + x_1^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + x_1^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(x_2^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + x_2^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + x_3^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$z_1^2 + \frac{1}{2}(l_e^2 + w^2) = z_1^2$	$-d$
2-e	z_0	y_0	$z_0^2 + \frac{1}{2}(l_e^2 + x_1^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(x_1^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + x_1^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + x_1^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(x_2^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + x_2^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + x_3^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$z_0^2 + \frac{1}{2}(l_e^2 + w^2) = z_0^2$	$-d$



Weaver formula, which is explained in [9]. Figure (3b) shows that the net uplift pressures at points G; H; M and O, on the D.S floor, are decreased when the length of the intermediate filter is increased. Increasing the value of L_f/B_1 from 0.125 to 3 gives an equal reduction in net pressure at the three points by about 14%. Figure (3c) indicates that the relative exit gradients along intermediate filter have an appreciable effect due to the variation of filter length. For all tested cases, a second inlet face is created along the D.S part of the filter. As expected, the relative second inlet face length, which is measured between the stagnation point ($IE_1=0.0$) and the filter end point, is increased when the filter length is increased. The relative exit gradients along the D.S bed is decreased for increasing the filter length, as shown in Figure(3-d).

Effect of Relative Effective Heads

The effect of relative effective heads (H_2/H_1) can be evaluated from Figure (4). The relative net uplift pressures acting under the U.S floor (U_1/H_1) are appreciably affected by the change in H_2/H_1 as

shown in Figure (4a). For a constant value of H_1 , the value of U_1/H_1 decreases all along the floor with an increase in the value of H_2 . As expected, the maximum variation in pressures takes place just D.S the cutoff wall, point D. Its net pressure is decreased by about 20% of H_1 when the effective heads of the two structures are equaled, $H_2/H_1=1.0$. Therefore, the net uplift pressures along the existing barrages will be decreased as the effective head of the subsidiary weir increases. Figure (4b) indicates that the relative uplift pressures at the considered key points of the D.S floor (U_2/H_2) have an appreciable decrease for the increase in H_2/H_1 . The rapidly variation in U_2/H_2 is occurred for $H_2/H_1 \leq 0.50$. The values of $U_2/H_2 \geq 1$ mean that the stagnation point, which splits the flow into opposite two directions, lies on the D.S floor. Figure (4c) indicates that the relative exit gradients along the intermediate filter decrease with increases of H_2/H_1 . For the studied case, the second inlet face comes into existence when $H_2/H_1 > 0.40$ and the filter works completely as an inlet face for $H_2/H_1 \geq 1.6$. Figure (4d) indicates that the relative exit gradients along the D.S bed are significantly increased for the increase of H_2/H_1 .

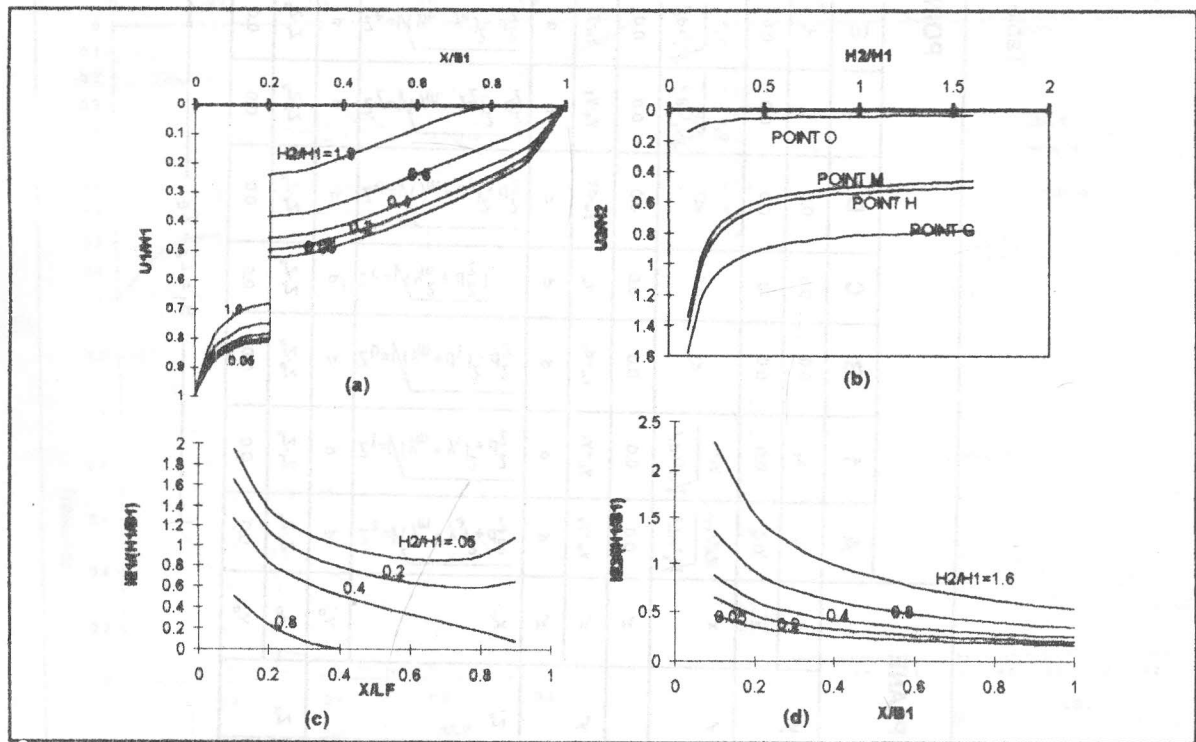


Figure 4. Effect of relative effective heads(H_2/H_1): $L_f/B_1=0.25$; $D_1/B_1=0.2$; $b_1/B_1=0.2$; $L_1/B_1=L_3/B_1=0.1$; $L_2/B_1=0.28$; $D_2/B_1=0.1$; $l_1/L_3=0.5$; $D/B_1=0.04$.

Effect of Upstream Cutoff Depth

The effect of relative U.S cutoff depth($D1/B1$) can be illustrated as shown in Figure (5). It is seen from Figure (5a) that the relative net uplift pressures along the U.S floor($U1/H1$) are significantly affected for the variation of $D1/B1$. These pressures decrease along the portion D.S the cutoff (DE) with increases in $D1/B1$, and vice-versa for the U.S portion(AB). For the studied case, a construction of a cutoff wall under the U.S floor with relative depth $D1/B1=0.5$ causes a reduction in pressure at point D by about 41% of $H1$ and increases the pressure at point B, in the other side, by about 20% of $H1$. As shown in Figure (5b), the relative net uplift pressures on the considered key points on the D.S floor have a slightly decrease for the increase of $D1/B1$. At point

G, the net pressure is decreased by only about 3.3% of $H2$ when the value of $D1/B1$ is increased from 0.0 to 0.50. Therefore, the decrease in net pressures along the D.S floor is very small compared to the decrease in pressures along the U.S floor with increases in $D1/B1$. Figure (5c) indicates that, as expected, the relative exit gradients along the intermediate filter have a significant affected for the variation of $D1/B1$. For the studied case, increasing the value of $D1/B1$ from 0.0 to 0.5 causes a reduction in gradients by about 35% all along the intermediate filter. The length of the second inlet face increases as the value of $D1/B1$ increases. The relative exit gradients along the D.S bed does not affected due to any variation of $D1/B1$, as shown in Figure (5d).

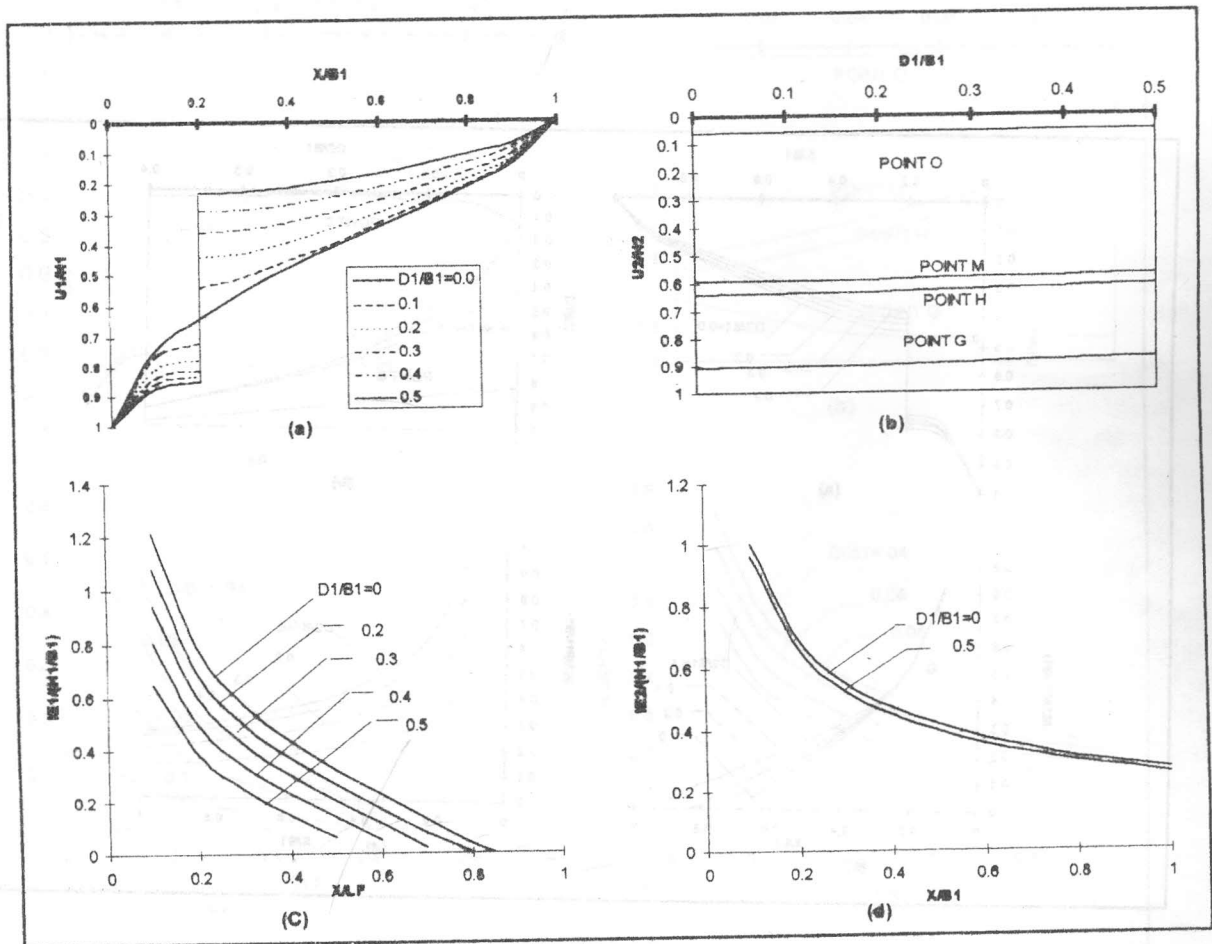


Figure 5. Effect of relative Upstream cutoff depth($D1/B1$): $Lf/B1=0.25$; $H2/H1=0.5$; $b1/B1=0.2$; $L1/B1=L3/B1=0.1$; $L2/B1=0.28$; $D2/B1=0.1$; $l1/L3=0.5$; $D/B1=0.04$.

Effect of Downstream Cutoff Depth

The effect of relative D.S cutoff wall depth ($D2/B1$) can be evaluated from Figure (6). The relative net uplift pressures along the U.S floor have a slightly decrease for increasing the value of $D2/B1$ as shown in Figure (6a). For the studied case, the maximum reduction in net pressure, which lies at point D, is about 6.7% of $H1$ for increasing $D2/B1$ from 0.0 to 0.4. As expected, the relative net uplift pressure, for any point lies upstream the cutoff on the D.S floor such as G; H and M, increases with increase of $D2/B1$, as shown in Figure (6b). For $D2/B1= 0.4$, only about 8% is dissipated U.S the

cutoff wall therefore, the floor of the weir thickness should be constant and designed for the total value of $H2$. Figure (6c) indicates that the relative exit gradients along the filter is not affected by $D2/B1$ for $X/Lf \leq 0.25$. The length of the second inlet face decreases as $D2/B1$ increases. For the studied case, the total length of the filter works as an exit face for $D2/B1 > 0.4$. Figure (6d) indicates that the relative exit gradients along the D.S bed are significantly affected due to the variation of $D2/B1$. A construction of a cutoff wall with relative depth, $D2/B1=0.4$, causes a reduction in gradients by about 50% all along the D.S bed.

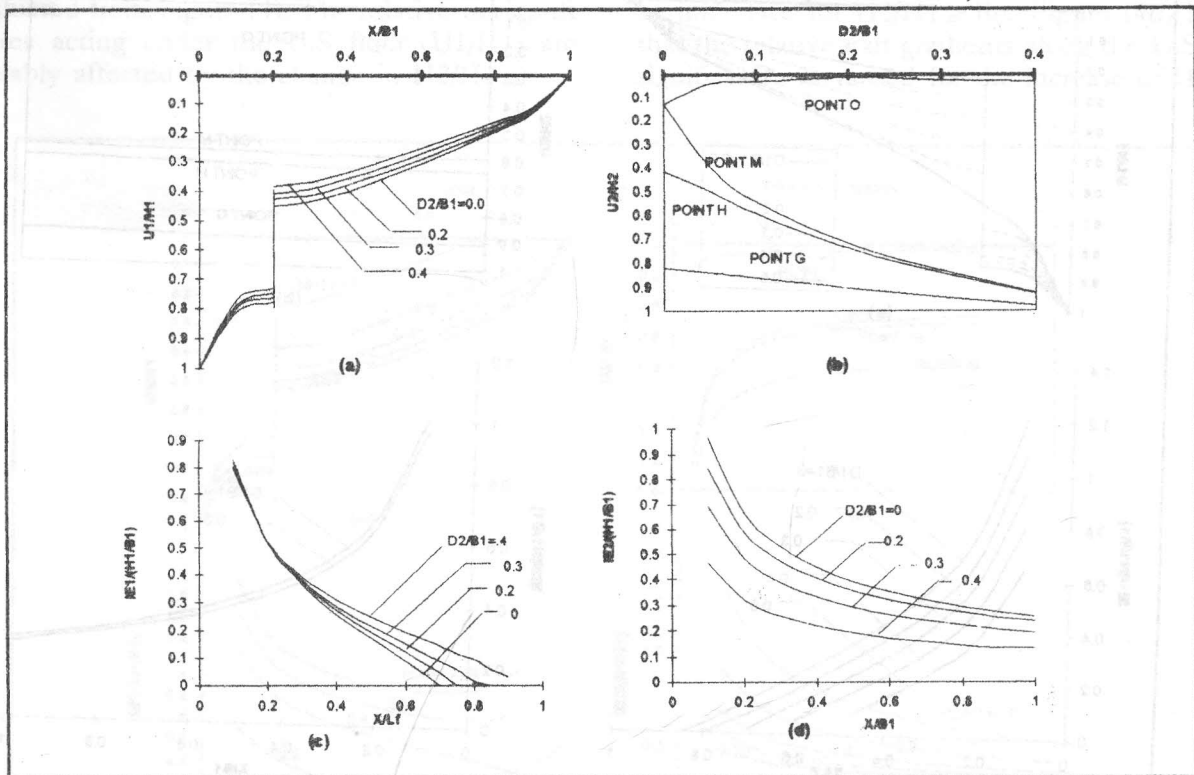


Figure 6. Effect of relative Downstream cutoff depth($D2/B1$): $Lf/B1=0.25$; $H2/H1=0.5$; $D1/B1=0.2$; $b1/B1=0.2$; $L1/B1=L3/B1=0.1$; $L2/B1=0.28$; $l1/L3=0.5$; $D/B1=0.04$.

ILLUSTRATED EXAMPLE

Nag Hammadi barrage is one of the most important barrages on the river Nile. It was constructed in 1930. The barrage has 100 vents, each 6.0m wide with piers about 2.2m thick and main piers 4.0m separating each ten vents. The barrage was designed to withstand a maximum differential head of 4.5 m. The longitudinal section of the barrage is shown in Figure (8) [10]. The barrage is subjected to a lowering in its D.S bed due to the degradation. A proposed subsidiary glacis weir is to be constructed D.S the barrage to ensure its stability against the degradation. The effect of the weir on the seepage characteristics of the barrage is studied. The effective

length of the barrage floor is taken 58.5m. The main cutoff wall, which lies at distance 13.5m from the U.S edge of the floor, is considered with depth 6.5m while, the other small sheet piles are neglected. The maximum expected lowering depth in both bed and water level D.S the barrage is assumed 3.0m. The weir is located at 30m distance from the end of barrage floor. The slope of the middle inclined apron is assumed 7:1 and the other two horizontal parts are assumed equal with length 6.0m. The maximum expected differential head is assumed 7.0m. The effective differential heads for barrage and weir are taken 4.0m and 3.0m, respectively.

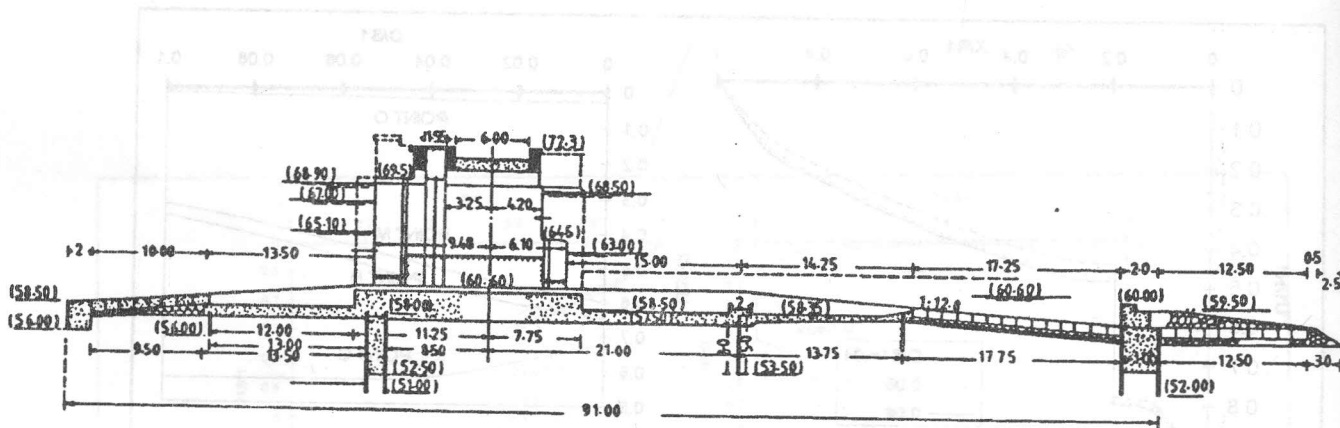


Figure 8. Longitudinal section of Nag hammadi Barrages.

Figure (9) indicates the effect of the proposed subsidiary weir on both uplift pressures under the barrage and D.S exit gradients. As shown in Figure (9a), the net uplift pressure values under the barrage floor have a significant affect due to the construction of the weir. The figure shows that the net uplift pressure values will be approximately doubled when the maximum expected lowering in D.S water level is happened, $H=7.0m$. The construction of the subsidiary weir, with effective head = 3.0m, causes an appreciable reduction in net pressures than their initial values. The weir causes a decrease in net pressure by about 20% at the point just D.S the cutoff than its initial value. Figure (9b) shows that the exit gradients along the D.S bed are decreased by 25% to 40% after construction of the weir.

CONCLUSIONS

The problem of two-dimensional seepage flow below a two successive hydraulic structures with an intermediate filter between them has been studied. The floor of the U.S structure is horizontal and provided with a single cutoff wall. The floor of the D.S structure has a middle inclined apron and a single cutoff wall, located at anywhere along the horizontal end part. This problem represents the seepage flow underneath one of the existing Nile barrages and a proposed D.S subsidiary glacis weir. The problem is transformed onto another solved problem using the successive conformal mapping method.

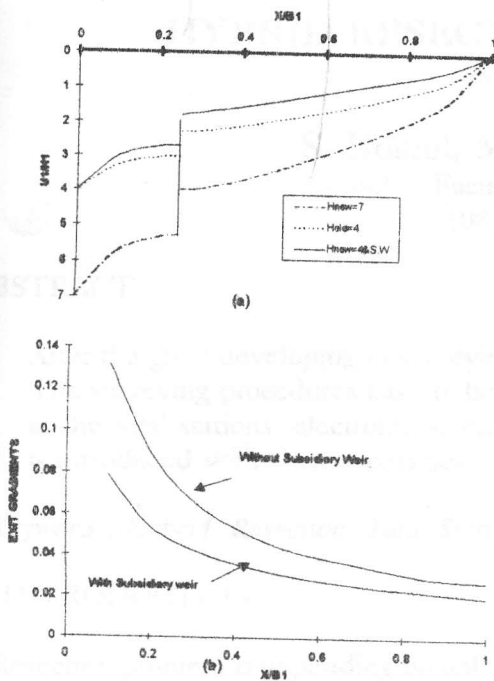


Figure 9. Effect of a proposed subsidiary Glacis weir Downstream Nag Hammadi barrages on uplift pressures and D.S exit gradients. (B1=58.5m; D1=6.5m; b1=13m; Lf=30m; L1=L3=6m; D2=6m; D=3m; L2=21m)

The uplift pressures under the two floors and exit gradients D.S each of them have been calculated, for the considered variables, and the results are plotted on the form of curves. An illustrated example has been prepared to study the effect of a D.S proposed subsidiary glacis weir on Nag Hammadi barrage. From the results, it can be concluded that:

- 1- The closer the two structures the decrease is the uplift pressures underneath the two floors while, the increase is the exit gradients D.S each of them.
- 2- For a constant effective head (designed head) on the existing barrage, the increase of effective head on the subsidiary weir causes a considerable decrease on both of uplift pressures under the two floors and intermediate exit gradients, whereas the D.S exit gradients increase.
- 3- The increase of U.S cutoff depth gives a significant decrease on both of uplift pressures under the existing barrages and intermediate exit gradients.
- 4- The D.S cutoff wall is recommended at the end of D.S floor to reduce both of D.S exit gradients and uplift pressures under the barrage.

- 5- The subsidiary weir should be designed for maximum expected lowering in bed and water levels due to degradation
- 6- The proposed subsidiary glacis weir D.S Nag Hammadi barrage gives an appreciable reduction on both uplift pressures and exit gradients.

REFERENCES

- [1] M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions*, Dover, New York, 1972.
- [2] A.S. Chawla, "Stability of Structures With intermediate Filter", *Journal of Hydraulic Division*, ASCE, Vol. 101, February, 1975, pp 223-241.
- [3] A.E. El-ansary "Evaluation of The Future Degradation in The Nile River Channel In Egypt", *Bulletin of Civil and Mathematical Eng. Depts. of the Faculty of Eng., Alex. University*, Vol. XV :1-1976.
- [4] F..A. Elfitiany, "Seepage Under the Floors of Two closely Spaced Dams", *Proc. of second Fluid mechanics and Hydraulic Conference*, El-Mansoura University, El-Mansoura, Egypt, Dec., 1983, pp 492-514.
- [5] M.A. Elganiny, "Flow underneath a pair of Structures With Intermediate Filter on a Drained Stratum", *Appl. Math. Modeling*, Vol. 10, 1986, pp 394-400.
- [6] M.A. Elganiny, "Seepage underneath Barrages With Downstream Subsidiary Weirs" *Appl. Math. Modelling*, Vol. 11, 1987, pp 423-431.
- [7] V. Halek, J. Sevec (*Ground Water Hydraulics*), Elsevier Scientific Publishing Company, 1979.
- [8] Y.H. Hammad, F.A. Elfitiany, R.I. Nasr, "Stability of Hydraulic Structures With Intermediate Filter and Single Cutoff", *Water Resources Development in Egypt*, Proc. Int. Coif., Cairo, Egypt, June 6-8, 1983.
- [9] M.E. Harr, *Ground Water and Seepage*, Mc Graw-Hill, New York, 1962.
- [10] I.Z. Kenawi and F. Nicola, "Nag hammadi Barrage Research on Remedy of After Construction Problems", (in Arabic), *Ministry of Irrigation*, Cairo, Egypt, 1971.
- [11] A. Kumar, B. Singh and A.S. Chawla, "Design of Structures With Intermediate Filter", *Journal of Hydraulic Division*, ASCE, Vol. 112, March, 1986, pp 206-219.
- [12] W. Willcocks and J.I. Craig, *Egyptian Irrigation*, E&F.N Spoon, London, 1913.