CHARACTERISTIC FUNCTIONS FOR DOSE CALCULATIONS OF HIGH ENERGY ELECTRONS IN WATER

M. Abou Mandour,
Dept, of Nuclear Engineering, University of Alexandria, Alexandria 21544, Egypt.

M.S. Abou Elwafa, M.EL Gammal and Farida Selim
Department of Math.& Eng. Physics, University of Alexandria, Alexandria 21544, Egypt.

ABSTRACT

A Monte Carlo simulation for the transport of high energy electrons in water phantom is developed. The multiple scattering is simulated according to the Goudsmit-Saunderson theory. The energy loss due to collisions is calculated according to a modified Landau distribution and the Bremsstrahlung radiation according to Betha-Heitler theory. The characteristic functions: the lateral radial distribution and the total fluence in depth are calculated and fitted to an analytical expression to facilitate their use in dose calculations in radiation treatment according to the pencil beam model.

Keywords: High energy electrons, Monte Carlo simulation, Multiple Scattering.

1- INTRODUCTION

The penetration, scattering and absorption of high energy electron beams in water has a great importance in radiotherapy for planning the irradiation of malignant tumors with minimal damage to adjacent tissues. It has also a great importance in biophysics to understand the biological effects of radiation. A Monte Carlo code is developed to simulate the transport of the high energy electrons through media and to calculate the desired quantities in radiation dosimetry with great precision and to understand the physical phenomenas affecting the absorption of high energy electrons. The calculations are carried out for water to simulate the human body. Many authors have carried out experiments [16] or done calculations [3 ,4 , 14 , 18 , 20] to determine the dose distributions in water. In the present paper, a study of the absorption of high energy electrons is planned using the Monte Carlo method to achieve two important purposes. First, to get the required information needed to carry out the precise mathematical model [1] to calculate the dose distributions required for radiation planning without the need for detailed and lengthy Monte Carlo calculations. Second, the problem of the presence of inhomogenities in a medium and their effects on the dose distribution has to be considered and to extend the model to take them into considerations. In this paper the results of the first calculations will be presented concerning the developed Monte Carlo simulations and the verification of the results. The developed Monte Carlo simulation is based on the multiple scattering theory of Goudsmit and Saunderson [8,9]. The energy loss due to collisions is simulated according to a corrected Landau distribution [1] and the Bremsstrahlung processes are treated according to the Bethe - Heither theory [17]. Comparing the developed Monte Carlo code to the previous published Monte Carlo codes of other authers, our program is constructed to simulate the electron track with small steps and thus large details about the electron tracks could be obtained.

2- MONTE CARLO SIMULATION

Direct simulation of the physical scattering processes would be laborious because of the large number of Coulomb interactions in a short path length. The transport of electrons is described by considering the multiple scatting and the energy straggling in a thin foil of the matter using the multiple scattering theories through the random sampling techniques. This condesed history is followed through the whole medium until a cut off energy. The cut off energy is selected to be 0.5 Mev. The calculations have been made for a point monodircetional and monenergetic electron beam indicent prependicularly on a semi - infinite water phantom.

(a) Treatment of Multiple Scattering:

In the present Monte Carlo simulation, the treatment of the multiple scattering of electrons in the thin foil is based on Goudsmit Saunderson theory [8, 9] since it considers all angular deflections without restriction to small angles and it can be evaluated for any form of the single scattering cross section.

The GS angular distribution is given in Legendre series as:-

$$A_{GS}(\omega)\sin\omega d\omega = \sum_{\ell=0}^{\infty}(\ell+1)\exp\{-\int_0^s G\ell(s)\,ds\}\,P\ell(\cos\omega)\sin\omega d\omega \quad (1)$$

where

$$G\ell(s) = 2\omega\pi N\int_0^{\pi}\sigma(\theta,s)\{1-P\ell(\cos\theta))\sin\theta\, d\theta \quad (2)$$

N is the number of atoms per unit volume, s is the path length., $\sigma(\theta, s)$ is the single scattering cross section and $P_\ell(\cos\theta)$ is Legendre polynomial of order ℓ.

To include the relativistic and spin effects the Mott corss section is used through considering Rutherford cross section ρ_R and multiplying it by the correction factor σ_M/σ_R as described by Berger [5]:

$$\sigma_R = \frac{Ze^4}{P^2v^2(1-\cos\theta)^2} \quad (3)$$

where P, v are the momentum and velocity of the particle, e is the electron charge, Z is the atomic number. The screening of the nuclear charge by orbital electrons is taken into account by introducing the screening constant h and replacing $(1-\cos\theta)^2$ by $(1-\cos\theta+2\eta)^2$ [7].

The screening constant is obtained from Moliere's multiple scattering theory [15] by this formula:-

$$\eta = \frac{1}{4}\left[\frac{Z^{1/3}}{0.885(137)}\right]^2 T^{-1}(T+2)^{-1} + [1.13+3.76(\frac{Z}{137})^2(T+1)^2T^{-1}(T+2)^{-1}] \quad (4)$$

The factor (Z^2) in Rutherford cross section is replaced by $[Z(Z+1)]$ to take inelastic scattering into account. Combining the Rutherford cross section, the screening parameter and the analytical formula of σ_M/σ_R for small angles, the single scattering cross section can be expressed as follows:

$$\sigma(\theta) = \frac{Z(Z+1)e^4}{P^2v^2(1-\cos+2\eta)} * \left[1+\frac{\pi}{\sqrt{2}}\frac{ZB}{137}\cos\gamma(1-\cos\theta+2\eta)^{1/2}+h(\theta)\right] \quad (5)$$

where

$$h(\theta) = \frac{\sigma_M}{\sigma_R} - 1 - \frac{\pi}{\sqrt{2}}\frac{Z}{137}\cos\gamma(1-\cos\theta+2\eta)^{1/2} \quad (6)$$

$$\cos\gamma = \mathrm{Re}\left[\frac{\gamma(\frac{1}{2}-i\frac{Z}{137B})\gamma(1+i\frac{Z}{137B})}{\mathrm{gamaa}(\frac{1}{2}+\frac{iZ}{137B})\gamma(1-\frac{iZ}{137B})}\right] \quad (7)$$

The function $h(\theta)$ must be approximated by a polynomial and evaluated numerically:

$$h(\theta) = \sum_{j=1}^{J} h_j(1-\cos\theta+2\eta)^{j/2} \quad (8)$$

A computer program is developed to calculate the ratio between Mott cross section and Rutherford cross section. Substituting eq (5), (8) in eq (2), G_ℓ can be expressed as a linear combination of the following form:-

$$P(m,\ell) = \int_{-1}^{1}(1-\cos\theta+2\eta)^m\,[1-P_\ell(\cos\theta)]\,d\cos\theta \quad (9)$$

Spencer [19] has developed recursion relations for the term $P(m,\ell)$ which enables the evaluation of the function G_ℓ By integrating equation (1) for all angles and using the recursion relations derived by Breger

[5] for $\int_{\cos\omega}^{1} P_\ell (\cos\omega)\, d\cos\omega$, the desired cumulative form of the Goudsmit Saunderson distribution is obtained.

(b) Treatment of collision energy loss:

The Landau distribution [12] of energy Loss behind a thin layer of matter of thickness s is given by:-

$$f(s, \Delta)\, d\Delta = \Phi(\lambda)\, d(\lambda) \qquad (10)$$

where $\Phi(\lambda)$ is universal function given by Landau as $\lambda = \frac{\Delta - \Delta_o}{\xi}$ (Δ_o) is the most probable energy loss and

$$\xi = 1.54 * 10^5\ \mu Z / A\ (ev) \qquad (11)$$

where μ is the mass of layer per cm^2, Z, A are the atomic number and weight of the medium.

Instead of using the most probable energy loss given by Landau, we apply the correction suggested in our previous work, Abou Mandour [1] to derive an expression for (Δ_0) which takes into account the density effect and the mean radiative energy loss with energy less than $0.01E_0$. The values of stopping power used for this modification are taken from Berger and Seltzer tables [6] and the mean radiative energy loss was calculated by applying Bethe and Heitler theory [17]

(C) Treatment of radiative energy loss:

In the first calculations the initial electron energies considered are up to 20 Mev. Thus, the use of Bethe-Heilter theory is more convenient since it provides fair results with resonable amount of calculations [11]

According to Bethe - Heitler theory [17] the differerntial radiation probability of electrons ϕ_{rad} (E,E') is given in the following form:

$$\phi_{rad}(E, E')\, dE' = 4\,\alpha\, \frac{N}{A} Z^2 r_e^2 \frac{dE}{E'} F(U,v) \qquad (12)$$

$\phi(E, E')\, dE'\, dx$ is the probability for an electron of kinetic energy E traversing a thickness of dx gm/cm^2 to emit a photon with energy in dE' at E'. Z, A are the charge and mass number of the medium, N Avogadro's, r_e classical raduis of the electron and α=137.

The screening effect of the outer electrons of the atoms is often important and is determined by the quantity

$$\tau = 100\, \frac{m_e c^2}{U} \frac{v}{1-v} Z^{-1/3} \qquad (13)$$

where U is the total electron energy and v is fractional photon energy $E' / m_0 c^2$. The value of g determines the form of the function F (U , v) in equation (12).

By integrating equation (1) along a thin foil dx the total probability has been obtained.

3 - Flunce distributions :

In this paper the most important quantitaive data due to elecron slowing down and electron scattering are calculaed due to the transport of high energy electrons in water. An infinite medium of water is considered where a normal point beam of monenergctic electrons is incident on its surface. An imporant parameter in calculating the dose distributions and also in understanding the behaviour of the electrons inside the medium is the electron fluence. The fluence of particales is defined according to ICRU as the quotient of the number of particales crossing a small sphere of given sectional area by this area. In the code the fluence is calculated at different depths in the medium along the axis of the beam. Both calculations for pencil beam and for a broad beam could be obtained by the program. For a broad beam of infinite extension the reciprcity theorem is used.

In the penetration of electron fluence, or other related quantites, as a function of depth in the medium it is preferable to reduce the depence on the energy of the electron beam by using the relative depth z/r_0, where z is the depth in the direction of beam axis and r_0 is the continous slowing down approximation rang [6].

The simulation has been done for 10,000 histories for monoenergetic electron beam of initial energies

5 , 10 , 20 Mev. In this paper, the secndary electrons are not taken into consideration.

The code was verified by comaring the electron fluence, transmission cofficient and energy deposition with previous calculations the comparisons are given in Figures (1, 2 , 3). The comparison shows good agreement.

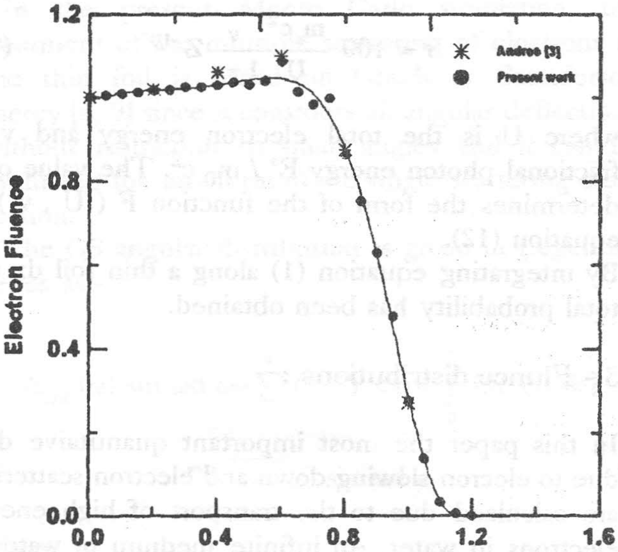

Figure 1. Depth variation of the fluence of 20 MeV electron beam in water.

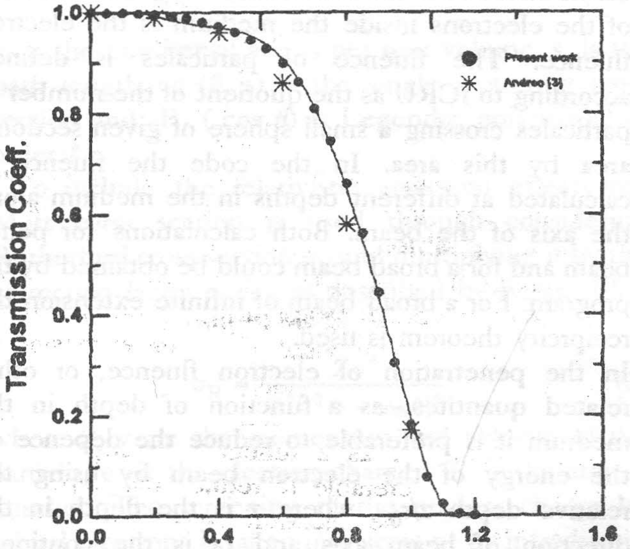

Figure 2. Transmission coefficient of 20 MeV as a function of depth in water.

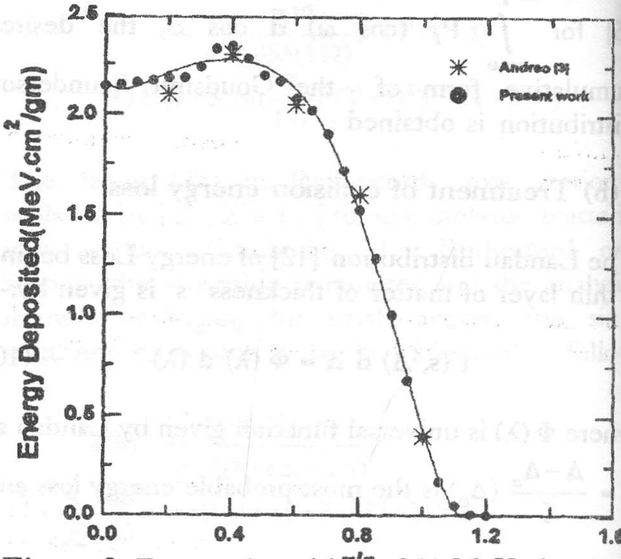

Figure 3. Energy deposition of 20 MeV electron as a function of depth in water.

Figure (4) represents the fluence for electron beam of monoenergetic energies of 5 , 10 and 20 Mev. The fluence exhibits an increase until it reachs a maximum at a depth about 50% to 70% of the range according to electron beam energy. The increase of the electron fluence is due to broadening of the beam due to the electron scattering. The enhancement in the electron flux increases with decreasing the electron's initial energy and its occurrence is at a smaller relative depth. This is due to fact that scattering angles of electrons are higher at lower energies.

The number transmission cofficient i.e planar fluence is calculated and given in Figure (5).

Another imortant parameter in describing the penetration of high energy electrons in the medium is the mean square scattering angle. In the program this is calculated for the electrons penetrating layers of different thicknesses. The results are shown in Figures (6).

It is useful in radiation dosimetry calculations to determine the energy spectrum inside the phantom, the electron spectrum is often described with refrence to a plane surface inside the medium by the distribution in energy. The number of electrons crossing the surafce in one direction N_E is related to the flux $f_{E,W}$ by :-

$$N_E = \int_0^{2\pi} \phi_{E,\Omega} \cos\theta \, d\Omega$$

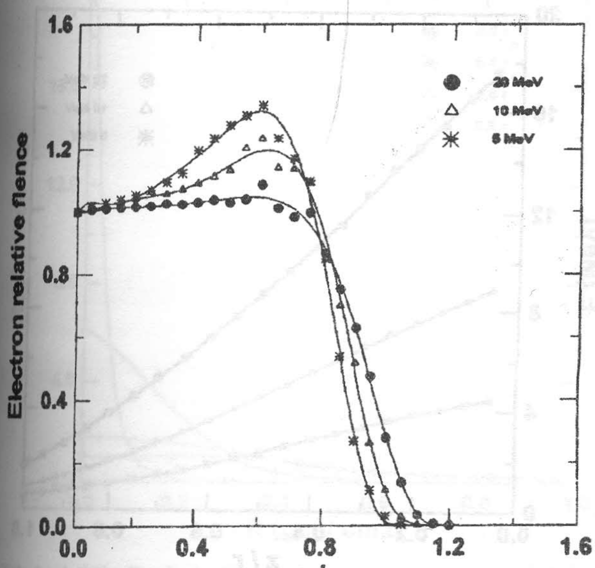

Figure 4. Depth variation of the relative fluence of electron beam in water phantom.

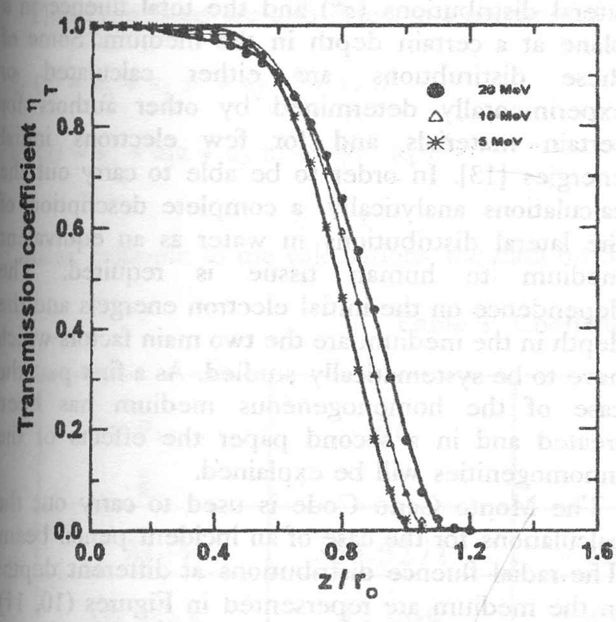

Figure 5. Transmission coefficient of electrons with initial energies 5,10,20 MeV as a function of depth in water.

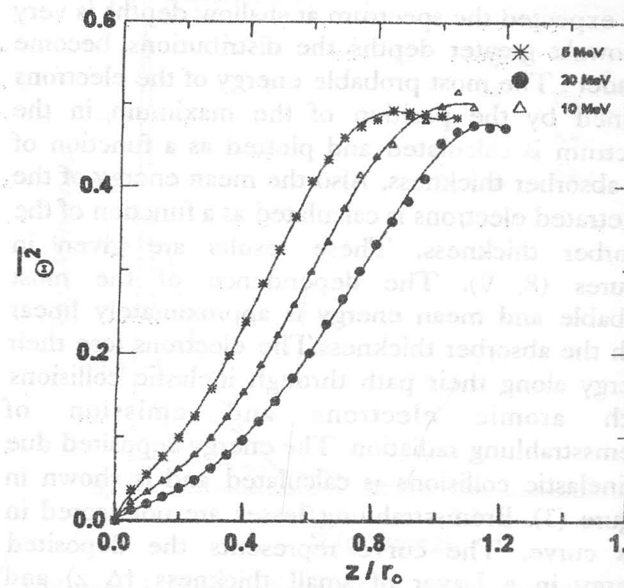

Figure 6. Mean square scattering angle of the electrons behind a slab of different thicknesses.

The energy spectra of 20 Mev electron beam at certain depths in water are calculated and given in Figure (7).

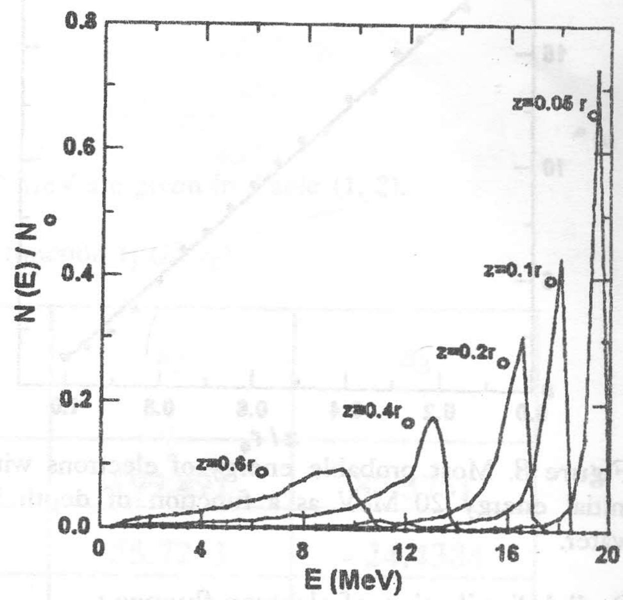

Figure 7. Energy spectra of electrons with initial energy of 20 MeV behind water slabs of different thickness.

As expected the spectrum at shallow depths is very narrow.At greater depths the distributions become broader . The most probable energy of the electrons defined by the position of the maximum in the spectrum is calculated and plotted as a function of the absorber thickness. Also the mean energy of the penetrated electrons is calculated as a function of the absorber thickness. These results are given in Figures (8, 9). The dependence of the most probable and mean energy is approximately linear with the absorber thickness.The electrons lose their energy along their path through inelastic collisions with atomic electrons and emission of Bremsstrahlung radiation. The energy deposited due to inelastic collisions is calculated and is shown in Figure (3). Bremsstrahlung losses are not scored in this curve. The curve represents the deposited energy in a Layer of small thickness (Δ z) and located at a depth z divided by its thickness and normalized to one incident electron.

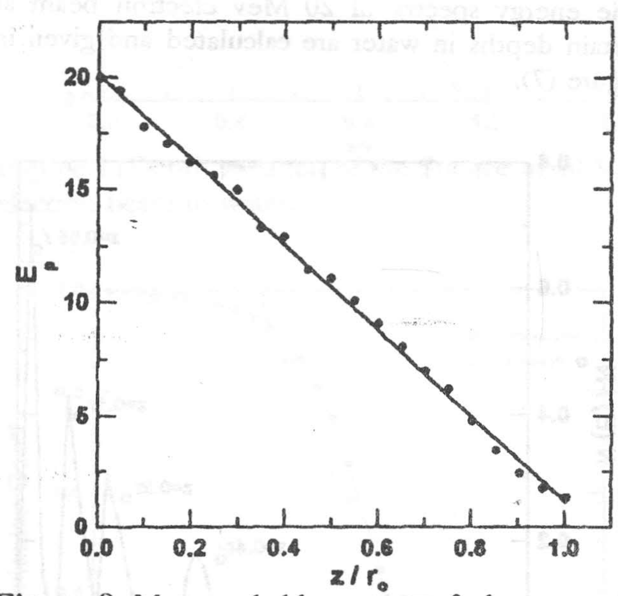

Figure 8. Most probable energy of electrons with initial energy 20 MeV as a function of depth in water.

Radial distribution of electron fluence :

In order to analytically treat the problem of dose calculations in a medium irradiated with high energy electrons the lateral distributions of a pencil beam are required.

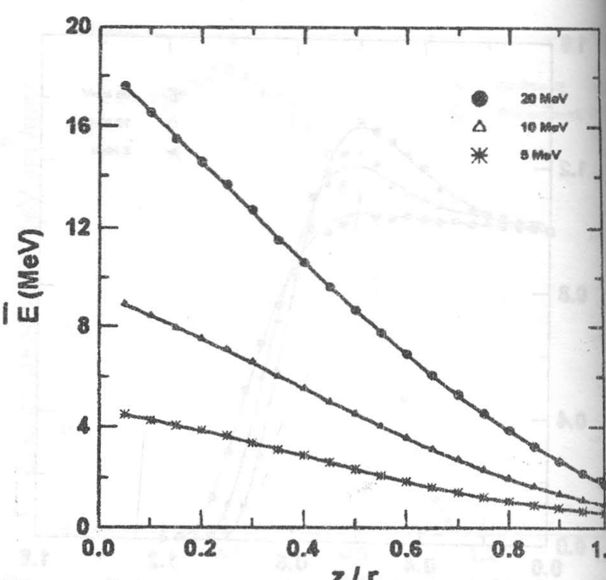

Figure 9. Mean energy of electrons with initial energy 5,10,20 MeV as a function of depth in water.

According to the pencil beam method developed by Abou Madour & Harder [2] two important characteristic functions are needed. These are the lateral distributions (σ^2) and the total fluence in a plane at a certain depth in the medium. Some of these distirubtions are either calculated or experimentally determined by other authors for certain materials and for few electrons initial energies [13]. In order to be able to carry out the calculations analytically, a complete description of the lateral distributions in water as an equivalent medium to human tissue is required. The dependence on the initial electron energeis and the depth in the medium are the two main factors which have to be systematically studied. As a first part the case of the homonogeneous medium has been treated and in a second paper the effects of the inhomogenities will be explained.

The Monte Carlo Code is used to carry out the calculations for the case of an incident pencil beam. The radial fluence distributions at different depths in the medium are repersented in Figures (10, 11). Curve parameter is the relative depth in the medium. The distributions are calculated for all intial energies up to 20 MeV in steps of 1 Mev. A fitting program is used to analytically describe these distributions.These distributions are fitted to a Gaussain distribution of the form

$$\phi\ (Z/r_0,\ R) = f_1\ (z/r_0)\, e^{-R^2/\sigma^2(z/r_0)}$$

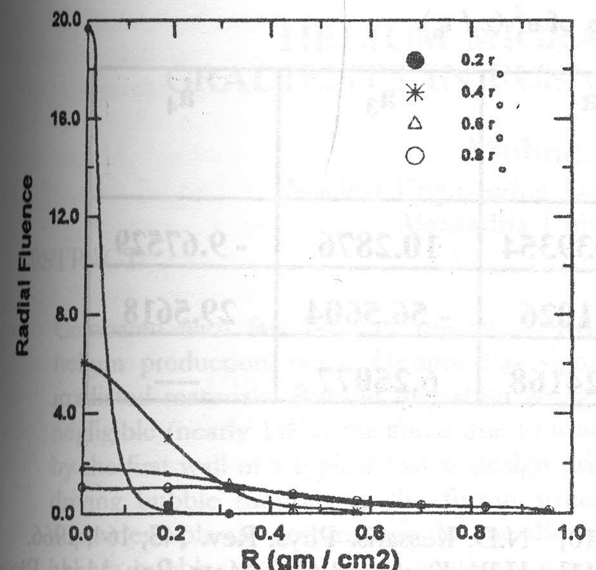

Figure 10. Radial distribution of fluence of 5 MeV electron beam at relative depth of 0.2 r_o, 0.4r_o, 0.6 r_o, 0.8 r_o in water phantom.

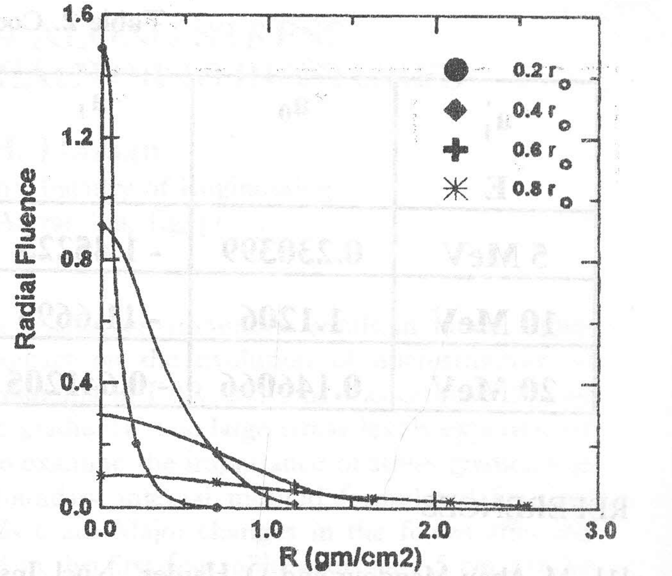

Figure 11. Radial distribution of fluence of 5 MeV electron beam at relative depth of 0.2 r_o, 0.4r_o, 0.6 r_o, 0.8 r_o in water phantom.

The two characteristic functions f_1 (z/r_o) and σ^2 (z/r_o) are determined. These functions are also fitted to a polynomial of the form :

$$f_1 (z) = a_o + a_1 z + a_2 z^2 + a_3 z^3$$

$$\sigma^2 (z) = a_o + a_1 z + a_2 z^2 + a_3 z^3 a_4 z^4$$

As an example to the calculations the data for 5, 10, 20 MeV are given in Table (1, 2).

Table 1. Coefficients of function f_1 (z / r_0).

a_i / E	a_0	a_1	a_2	a_3
5 MeV	38.5271	- 150.544	199.891	87.5751
10 MeV	11.908	- 43.3613	55.7261	- 24.4384
20 MeV	1.84262	- 0.322352	- 8.47469	7.79488

Table 2. Coefficients of σ^2 (z / r_o).

a_i' / E	a_0	a_1	a_2	a_3	a_4
5 MeV	0.230399	- 1.45223	0.0939354	10.2876	- 9.67529
10 MeV	1.1206	-11.6695	41.1026	- 56.5604	29.5618
20 MeV	0.146066	- 0.641205	0.224168	6.25077	----

REFERENCES

[1] M. Abou Mandour and D. Harder, Nucl. Insrt. and Meth . A 300 , 403, 1991.

[2] M. Abou Mandour and D. Harder, Strahlen therapie 754 , 546, 1978.

[3] P. Andro and A. Brahme, Acta Radial . Suppl. 364 proc conf. (stockholm), 1982.

[4] M.J. BergerJ and S.M. Seltzer, Am Acad. Sci (NY); 161, 8, 1969.

[5] M.J. Berger and S.M. Seltzer, Compt. Phys. , 1, 135, 1963.

[6] M.J. Berger and S.M. Seltzer, NASA -SP - 3012, 1964.

[7] H.A. Bethe, Phys. Rev. 89, 1256, 1953.

[8] S.A. Goudsmit and J.L. Saunderson, Phys. Rev. 57, 24, 1940.

[9] S.A. Goudsmit and J.L. Saunderson, Phys. Rev. 58, 36, 1940.

[10] N.D. Kessaris. Phys. Rev. 145, 164, 1966.

[11] H.W. Koch and J.W. Motz, Rev. Mod . Phys. 31, 920, 1959.

[12] L. Landau. Phys. U.S.S. R 8, 201, 1944.

[13] I. Lax and A . Brahame aP. Andreo, Acta , Radial. Suppl. 364 proc conf. (stockholm) 1982.

[14] A.S. Meigooni and I. J Dos,.. phys. Med Biol, vol 32, 761, 1987.

[15] G. Moliere Natur forsh Z A, 133, 1947.

[16] H. Ross and D. Harder, 4 th . Symp. Microdosimetry EUR 5122, 1973.

[17] B. Rossi, High energy particles (Prentice Hall, New york , 1966.

[18] G.A. Sandison and L.S. Papiez, Phys. Med . Biol., vol 35 , 979, 1990.

[19] L.V. Spencer. Phys. Rev . 90 , 1597, 1955.

[20] T. Tabata and to , R . Nucl . Sci . Eng . 53 , 226, 1974.