VELOCITY-DISTRIBUTION COEFFICIENTS IN CURVED OPEN CHANNEL

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ABSTRACT

The use of mean velocity, U, in open channel flow is very common and of general use. It is well known that using U to determine both the momentum and energy leads to an error which can be neglected as far as the channel is straight and prismatic. However, for general use, a momentum coefficient, β, as well as an energy coefficient, α, must be used to account for the difference between both the mean and actual velocity distribution. It is also well known that the normal velocity distribution in open channel of straight longitudinal axis is logarithmic curve. However, in curved open channel, the velocity has two components, longitudinal, u, and radial, v_r. In the present study, the longitudinal as well as the radial velocity distributions are presented. The graphical representation of radial velocity distribution derived by Rozovskii (1957) is converted to a numerical representation for general use. A relationship for the mean radial velocity is derived. The trend between the resultant velocity,V, versus the flow depth, z, is investigated. Moreover, the angle of deviation of the resultant velocity from the radial axis is plotted at different values of water level. Finally, relationships for β and α in curved open channel are derived for smooth bottom by performing a double integration.

Keywords: Longitudinal momentum coefficient, β_l, Longitudinal energy coefficient, α_l, Radial momentum coefficient, β_r, and radial energy coefficient, α_r.

Notation

u	longitudinal velocity at certain depth z
U	depth-average longitudinal velocity or in other words the mean longitudinal velocity
g	acceleration of gravity = 9.81 m/s^2
K	Von Karman constant, K = 0.4, for clear fluid
C	Chezy Coefficient, C = 30, for metric units
η	= z/D
D	total depth of water
v_r	radial velocity
$\bar{u}$	mean velocity
$\bar{v}_r$	mean radial velocity
r_i	inner radius of curvature
r_o	outer radius of curvature
r	rdius of curvature at certain location
β	momentum coefficient
α	energy coefficient
β_l	longitudinal momentum coefficient
α_l	longitudinal energy coefficient
β_r	radial momentum coefficient
α_r	radial energy coefficient
dA	area of a small strip of height dy and width B dA = B dy
A	total water area (A = BD).

INTRODUCTION

LONGITUDINAL AND TRANSVERSAL (RADIAL) VELOCITY DISTRIBUTIONS

Rozovskii (1957) assumed the following logarithmic longitudinal velocity distribution:

$$\frac{u}{U}=1+\frac{\sqrt{g}}{Kc}(1+\ln\eta) \qquad (1)$$

where u is the longitudinal velocity at certain depth z. U is the depth-average longitudinal velocity or in other words the mean longitudinal velocity. g is the acceleration of gravity =9.81 m/s^2. K is the Von Karman constant, K=0.4 for clear fluid, C is the Chezy Coefficient, C=30, for metric units. η=z/D where D is the total depth of water. Rozovskii also derived the following relationship for the transversal, radial, velocity profile in a smooth bottom

$$\frac{v_r}{U}=\frac{1}{K^2}\left(\frac{D}{r}\right)\left[f_1(\eta)-\frac{\sqrt{g}}{KC}f_2(\eta)\right] \quad (2)$$

where $f_1(\eta)$ and $f_2(\eta)$ are defined as following, respectively:

$$f_1(\eta)=\int\left[\frac{2\ln(\eta)}{\eta-1}\right]d\eta \quad (3)$$

$$f_2(\eta)=\int\left[\frac{\ln^2(\eta)}{\eta-1}\right]d\eta \quad (4)$$

The relationships of f_1 and f_2 are represented graphically, as shown in Figure (1). For rough bottom, Rozovskii obtained the following relationship:

$$\frac{v_r}{U}=\frac{1}{K^2}\left(\frac{D}{r}\right)\left[f_1(\eta)-\frac{\sqrt{g}}{KC}\left[f_2(\eta)+0.8(1+\ln\eta)\right]\right] \quad (5)$$

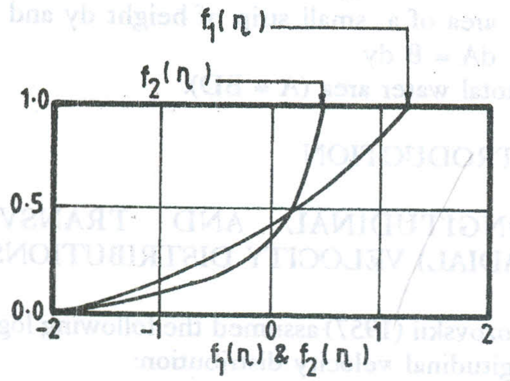

Figure 1. Graphs of functions $f_1(\eta)$ and $f_2(\eta)$ by Rozovskii.

NUMERICAL REPRESENTATION OF RADIAL VELOCITY DISTRIBUTION

For practical use and for purpose of numerical calculation, the functions f_1 and f_2 might be formulated as following, respectively:

$$f_1(\eta) = -1.325\eta^2 + 4.108\eta - 1.575 \quad (6)$$

$$f_2(\eta) = 3.1566\eta^3 - 7.2348\eta^2 + 5.904\eta - 1.3167 \quad (7)$$

Now the relationship of v_r for smooth bottom, Eq. 2, can be rewritten as following:

$$\frac{v_r}{U}=\frac{1}{K^2}\left(\frac{D}{r}\right)[f(\eta)] \quad (8)$$

where

$$f(\eta)=f_1(\eta)-\frac{\sqrt{g}}{KC}[f_2(\eta)] \quad (9)$$

substituting for values of g=9.81, K=0.4, C=30 by metric units and for the relationships of $f_1(\eta)$ and $f_2(\eta)$ as given by Eq.6 and Eq. 7, Eq. 9 can be written as following:

$$f(\eta) = 0.824\eta^3 - 3.214\eta^2 + 5.64\eta - 1.919 \quad (10)$$

Therefore, Eq. 8 can be rewritten as following:

$$\frac{v_r}{U}=\frac{1}{K^2}\left(\frac{D}{r}\right)[0.824\eta^3-3.214\eta^2+5.64\eta-1.919] \quad (11)$$

MEAN RADIAL VELOCITY

It is always known that $\bar{u}$ can be represented by

$$\bar{u}=\frac{1}{D}\int_0^D u\,dz \quad (12)$$

or in other words

$$\bar{u}=\frac{1}{D}\int_0^D U\left[1+\frac{\sqrt{g}}{KC}(1+\ln\eta)\right]dz \quad (13)$$

from which it can be proved that $\bar{u}$ = U.

Similarly, the mean radial velocity can be given by the following equation:

$$\bar{v}_r = \frac{1}{D(r_0-r_i)} \int_0^D \left(\int_{r_i}^{r_o} v_r \, dr \right) dz \tag{14}$$

it is obvious that v_r is function of both r and z. Therefore, the mean radial velocity is an outcome of a double integration. The first is a radial integration from r_i to r_o, while the second is a vertical integration from 0 to D. The radial integration can be done as following:

$$\left(\int_{r_i}^{r_o} v_r \, dr \right) = \int_{r_i}^{r_o} \frac{U}{K^2} \frac{D}{r} f(\eta) dr \tag{15}$$

$$\left(\int_{r_i}^{r_o} v_r dr \right) = \frac{UD}{K^2} f(\eta)[\ln(r_o) - \ln(r_i)] = \frac{UD}{K^2} f(\eta)[\ln(\frac{r_o}{r_i})] \tag{16}$$

substituting in Eq. 14, then

$$\bar{v}_r = \frac{1}{D(r_0-r_i)} \int_0^D \left(\int_{r_i}^{r_o} v_r dr \right) dz = \frac{UD}{K^2} \frac{1}{D(r_0-r_i)} [\ln(\frac{r_0}{r_i})] \int_0^D f(\eta) dz \tag{17}$$

However, the vertical integration can be done as following:

$$^*\int_0^D f(\eta) dz = \int_0^D [0.824(\frac{z}{D})^3 - 3.214(\frac{z}{D})^2 + 5.64(\frac{z}{D}) - 1.919] dz \tag{18}$$

$$^*\int_0^D f(\eta) dz = 0.03567D \tag{19}$$

Therefore,

$$\bar{v}_r = \frac{UD}{K^2} \frac{1}{D(r_0-r_i)} [\ln(\frac{r_0}{r_i})][0.03567D] \tag{20}$$

$$\frac{\bar{v}_r}{U} = \frac{0.03567}{K^2} \frac{D}{(r_0-r_i)} [\ln(\frac{r_0}{r_i})] \tag{21}$$

Substituting for K=0.4 and dividing the numerator and the denominator by r_i, the mean radial velocity may take the following form:

$$\frac{\bar{v}_r}{U} = 0.223 \frac{D/r_i}{(r_o/r_i - 1)} [\ln(\frac{r_0}{r_i})] \tag{22}$$

The relationship between $\bar{v}_r/U$ versus r_o/r_i is graphically shown in Figure (2). It is clear that $-v_r/U$ increases as r_o/r_i decreases, provided that D/r_i is constant. However, at certain value of r_o/r_i, $-v_r/U$ increases as D/r_i increases.

LOCATION OF MEAN RADIAL VELOCITY

To find out the location at which the radial velocity acts, Eq. 11 must equal Eq. 21, from which, it can be concluded that

$$r = (r_i) \frac{(r_o/r_i - 1)}{\ln(r_o/r_i)} \tag{23}$$

which gives the radius at which the mean radial velocity acts. Also to find the height at which the mean radial velocity acts, Eq.11 = Eq. 21, from which it can be concluded that

$$f(\eta) = 0.824\eta^3 - 3.214\eta^2 + 5.64\eta - 1.919 = 0.03567 \tag{24}$$

by trial and error,it can be proved that the height at which $\bar{v}_r$ acts is

$$z = 0.45\ D \tag{25}$$

Eq. 25 means that the mean radial velocity acts almost at the half of the total depth.

RESULTANT VELOCITY IN CURVED OPEN CHANNEL

In open channel of curved longitudinal axis, the resultant velocity, V, can be given as following:

$$V = \sqrt{u^2 + v_r^2} \tag{26}$$

Also, the angle of deviation at which V acts, as shown in Figure (3), can be given as following:

$$\tan\theta = (\frac{u}{U})/(\frac{v_r}{U}) \tag{27}$$

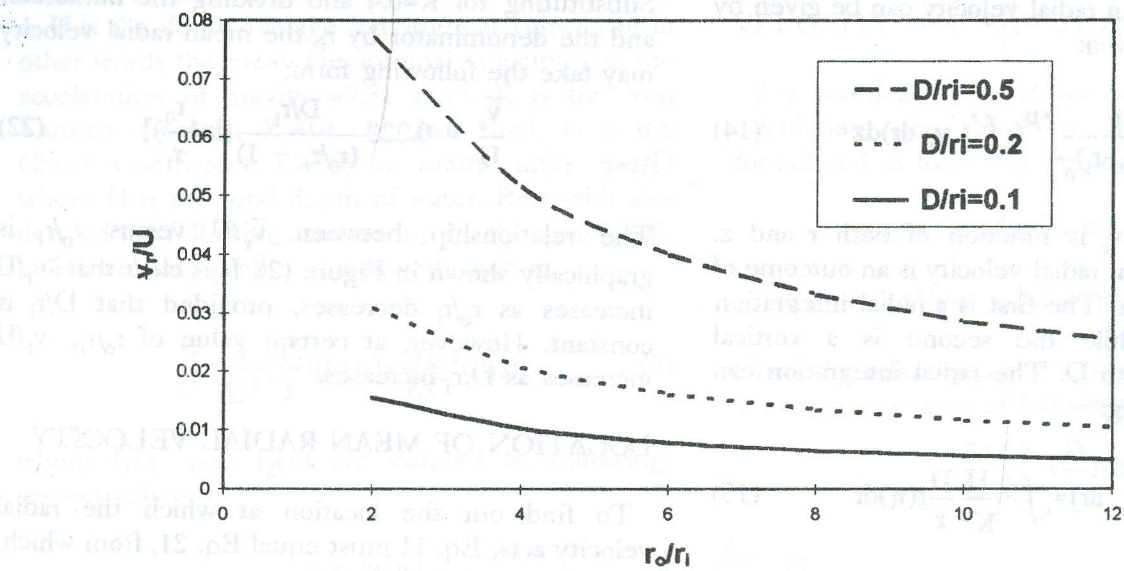

Figure 2 $\bar{v}$/U versus r_o/r_i.

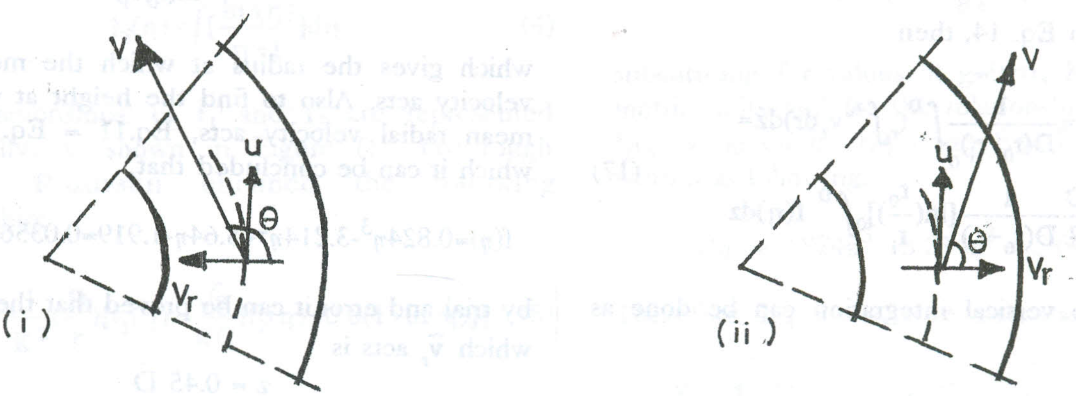

Figure 3. The resultant velocity, V, and its angle of deviation, θ, i-position close to bed, ii- position close to surface.

The relationship between V/U and z/D, at three values of r/D, is shown in Figure (4a). It is clear that as z/D increases, V/U increases. The ratio, V/U, is almost one at z/D = 0.5 which means that $v_r/U = 0.0$ at half the total depth. Also, the relation between z/D and Θ is shown in Figure (4b). It is obvious that as z/D increases, Θ decreases, provided that D/r_i is constant. However, at a certain value of z/D, Θ increases as D/r_i increases provided that z/D > 0.5 and vice-versa at z/D < 0.5.

MOMENTUM AND ENERGY COEFFICIENTS IN OPEN CHANNEL

In open channel of straight longitudinal axis, determination of both the momentum coefficient, β, and the energy coefficient, α, has been established. It is well known that

$$\beta = \int \frac{u^2\, dA}{U^2\, A} \tag{28}$$

and

$$\alpha = \int \frac{u^3\, dA}{U^3\, A} \tag{29}$$

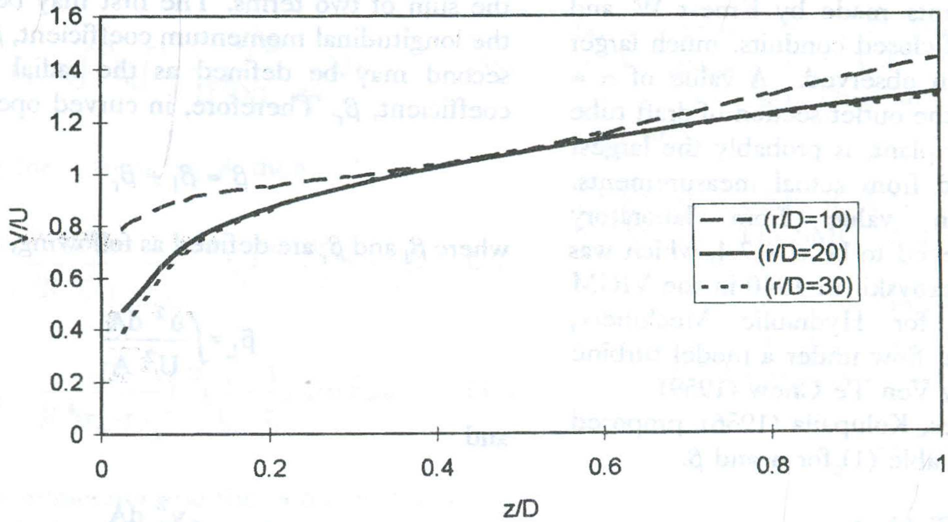

Figure 4a. The relationship between V/U and z/D.

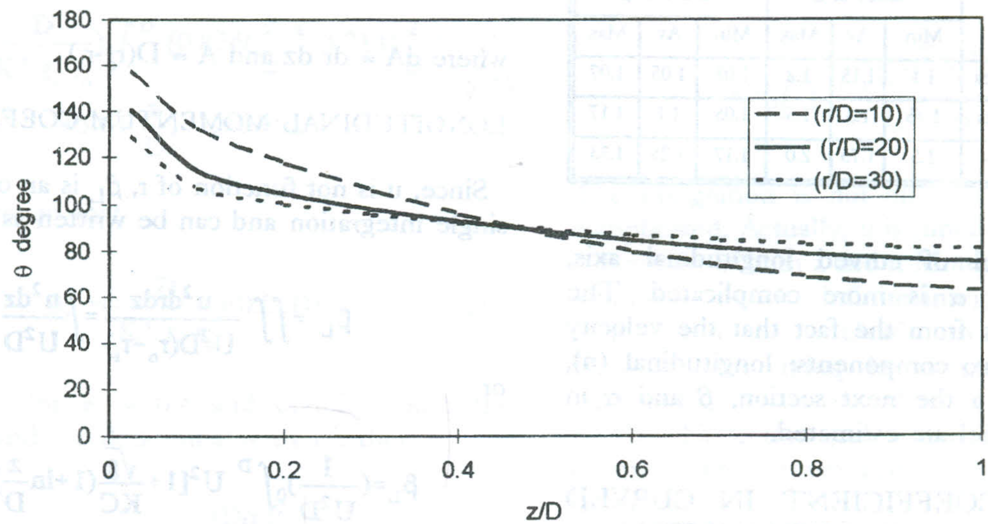

Figure 4b. The relationship between θ versus z/D.

where u and U are local and depth average (mean) velocities, respectively. dA is the area of a small strip of height dy and width B (dA = B dy) and A is the total water area (A = BD).

The two velocity-distribution coefficients are always slightly larger than the limiting value of unity, at which the velocity distribution is strictly uniform across the channel section. For channels of regular cross section and fairly straight alignment, the effect of nonuniform velocity distribution on the computed velocity head and momentum is small, especially in comparison with other uncertainties involved in the computation.

For fairly straight prismatic channels, the momentum coefficient, β, varies approximately from 1.01 to 1.12. In channels of complex cross section , β = 1.2. However, for fairly straight prismatic channels, the energy coefficient, α, varies approximately from 1.03 to 1.36. In channels of complex cross section , α = 1.6. Both α and β can vary quite rapidly from section to section in case of irregular alignment. Upstream from weirs, in the vicinity of obstructions, or near pronounced irregularities in alignment, values of α greater than 2.0 have been observed. For example, a value of α = 2.08 was computed by Linquist (1929) using data from weir measurements made by Ernest W. and Kenneth B. In case of closed conduits, much larger values of α have been observed. A value of α = 3.87, was observed at the outlet section of draft tube in the Rublevo power plant, is probably the largest known value obtained from actual measurements. The largest known value from laboratory measurements is believed to be α = 7.4, which was derived by V. S. Kviatkovskii in 1940 in the VIGM (All-Union Institute for Hydraulic Mechinery, U.S.S.R.) for the spiral flow under a model turbine wheel as presented by Ven Te Chow (1959).

For practical purposes, Kolupaila (1956), proposed the values shown in Table (1) for α and β.

Table 1.

Channels	Values of α			Values of β		
	Min	Av	Max	Min	Av	Max
Regular channels, flumes	1.1	1.15	1.2	1.03	1.05	1.07
Natural streams, torrents	1.15	1.3	1.5	1.05	1.1	1.17
River Valleys, overflood	1.5	1.75	2.0	1.17	1.25	1.33

In open channel of curved longitudinal axis, estimating β and α is more complicated. The complication arises from the fact that the velocity distribution has two components; longitudinal (u), and radial (v_r). In the next section, β and α in curved open channel are estimated.

MOMENTUM COEFFICIENT IN CURVED OPEN CHANNEL OF SMOOTH BOTTOM

The momentum coefficient in curved open channel can be obtained by replacing u in Eq. 28 by V as following:

$$\beta = \int \frac{V^2 \, dA}{U^2 \, A} \tag{30}$$

and substituting for $V^2 = (u^2+v_r^2)$, β can be

$$\beta = \int \frac{(u^2+v_r^2)dA}{U^2 \, A} = \int \frac{u^2 \, dA}{U^2 \, A} + \int \frac{v_r^2 \, dA}{U^2 \, A} \tag{31}$$

Actually, according to Eq. 31, β can represented by the sum of two terms. The first may be defined as the longitudinal momentum coefficient, β_l, while the second may be defined as the radial momentum coefficient, β_r. Therefore, in curved open channel

$$\beta = \beta_l + \beta_r \tag{32}$$

where β_l and β_r are defined as following, respectively

$$\beta_L = \int \frac{u^2 \, dA}{U^2 \, A} \tag{33}$$

and

$$\beta_r = \int \frac{v_r^2 \, dA}{U^2 \, A} \tag{34}$$

where dA = dr dz and A = $D(r_o - r_i)$

LONGITUDINAL MOMENTUM COEFFICIENT

Since, u is not function of r, β_L is an outcome of a single integration and can be written as following:

$$\beta_L = \iint \frac{u^2 dr dz}{U^2 D(r_o - r_i)} = \int \frac{u^2 dz}{U^2 D} \tag{35}$$

or

$$\beta_L = \left(\frac{1}{U^2 D}\right)_0\int^D U^2\left[1+\frac{\sqrt{g}}{KC}\left(1+\ln\frac{z}{D}\right)\right]^2 dz \tag{36}$$

let $e = \frac{\sqrt{g}}{KC}$, then it can be proved that

$$\beta_L = 1 + e^2 \tag{37}$$

for e $=(g)^{0.5}/KC = (9.81)^{0.5}/(0.4)(30) = 0.261$

β_L = 1.068. This value is previously well known.

RADIAL MOMENTUM COEFFICIENT

Since v_r is function of both r and z, β_r , Eq. 34, is an outcome of a double integration and can be written as following:

$$\beta_r = {}_0\int^D [{}_{r_i}\int^{r_o} \frac{v_r^2 dr}{U^2 D(r_o - r_i)}]dz \qquad (38)$$

substituting for v_r from Eq. 8, then

$$\beta_r = \frac{U^2 D^2}{U^2 K^4 D(r_o - r_i)}{}_0\int^D [{}_{r_i}\int^{r_o} \frac{dr}{r^2}]f(\eta)^2 dz \qquad (39)$$

$$\beta_r = \frac{-D}{K^4 (r_o - r_i)}{}_0\int^D [\frac{1}{r_o} - \frac{1}{r_i}]\ f(\eta)^2 dz \qquad (40)$$

dividing the numerator and the denominator by $(r_o - r_i)$ and substituting for $f(\eta)$, then

$$\beta_r = (\frac{D}{K^4\ r_i\ r_o}){}_0\int^D [0.824(\frac{z}{D})^3 - 3.214(\frac{z}{D})^2 + 5.64(\frac{z}{D}) - 1.919]^2 dz \qquad (41)$$

it can be proved that

$$\beta_r = (\frac{D}{K^4\ r_i\ r_o})(0.86D) \qquad (42)$$

Substituting for K = 0.4 and dividing both the numerator and the denominator by r_i^2, then

$$\beta_r = (33.6)\frac{(D/r_i)^2}{(r_o/r_i)} \qquad (43)$$

For smooth bottom, the relationship between β_r versus r_o/r_i is shown in Figure (5) at three different values of D/r_i. It is shown that β_r decreases as r_o/r_i increases provided that D/r_i is constant. At certain value of r_o/r_i, β_r increases as D/r_i increases.

It can now be seen that the momentum coefficient for curved channel, β is represented by the following relationship:

$$\beta = 1.068 + (33.6)\frac{(D/r_i)^2}{(r_o/r_i)} \qquad (44)$$

Unfortunately, values of β reported from previous works were absolute. No relationship was reported to be compared with Eq. 44. However, values reported by Kolupaila (1956), Table (1), have a wide range (β= 1.17 to 1.33). This range might be due to the curvatures.

ENERGY COEFFICIENT IN CURVED OPEN CHANNEL OF SMOOTH BOTTOM

The energy coefficient in curved open channel can be obtained by replacing u in Eq. 29 by the resultant velocity,V, as following:

$$\alpha = \int \frac{V^3\ dA}{U^3\ A} \qquad (45)$$

and substituting for $V = (u^2 + v_r^2)^{0.5}$, α can be

$$\alpha = {}_0\int^z [{}_{r_i}\int^{r_o} \frac{(\sqrt{u^2 + v_r^2})^3\ dr}{U^3\ D(r_o - r_i)}]dz \qquad (46)$$

The integration is not only double but also very complicated. Actually, u is function of z, however, v_r is function of both z and r. Moreover, $(u^2+v_r^2)^{1.5}$ has infinite terms with infinite integrations. Not similar to β, α can not be clearly divided to α_L and α_r. However, simplifying the integration may lead to a satisfactorial result very close to the actual value of α. To achieve this goal, it may be advantageous to examine the convergence between $(u^2+v_r^2)^{1.5}$ and $(u^3+v_r^3)$.

The relationship between $(u^2+v_r^2)^{1.5}$ and $(u^3+v_r^3)$ is shown in Figures (6). At different r/D, best fitting leads to the following results at different values of r/D:

$$r/D = 60,\ (u^3+v_r^3) = 0.994(u^2+v_r^2)^{1.5} \qquad (47)$$

$$r/D = 30,\ (u_3+v_r^3) = 0.977(u^2+v_r^2)^{1.5} \qquad (48)$$

$$r/D = 20,\ (u^3+v_r^3) = 0.952(u^2+v_r^2)^{1.5} \qquad (49)$$

$$r/D = 15,\ (u^3+v_r^3) = 0.921(u^2+v_r^2)^{1.5} \qquad (50)$$

$$r/D = 10,\ (u^3+v_r^3) = 0.853(u^2+v_r^2)^{1.5} \qquad (51)$$

$$r/D = 5,\ (u^3+v_r^3) = 0.754(u^2+v_r^2)^{1.5} \qquad (52)$$

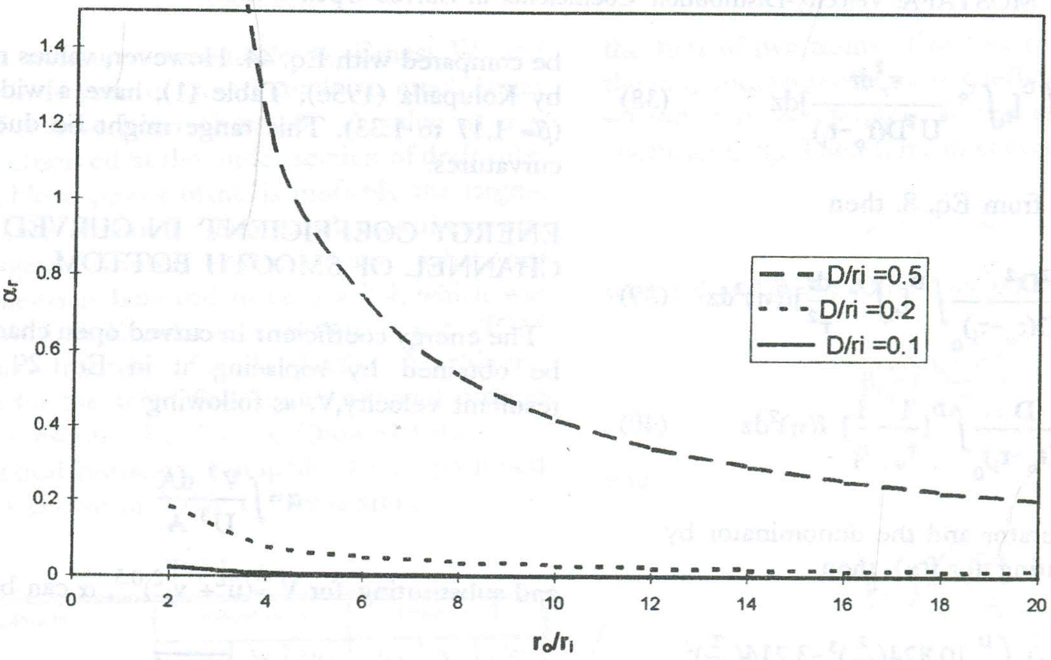

Figure 5. The relationship between β_r versus r_o/r_i at three different values of D/r_i.

It is obvious that the divergence between $(u^2+v_r^2)^{1.5}$ and $(u^3+v_r^3)$ increases as r/D decreases. For r/D=60,30 and 20, estimating α by using $V=(u^3+v_r^3)$ leads to errors 0.6%, 2.3% and 4.8%, respectively. However, for r/D = 15, 10 and 5, the errors are 8%, 15% and 25%, respectively.

To account for this error, it is suggested to use a factor called "m" as following:

$$\alpha = {}_0\int^z \left[\int_{r_i}^{r_o} \frac{(\sqrt{u^2+v_r^2})^3 \, dr}{U^3 \, D(r_o - r_i)} \right] dz = m\left[\int \frac{u^3}{U^3}\frac{dA}{A} + \int \frac{v_r^3}{U^3}\frac{dA}{A}\right] \quad (53)$$

Values of m can be estimated for different values of r/D as following:

$$r/D = 60, \quad m = 1.00 \quad (54)$$

$$r/D = 30, \quad m = 1.02 \quad (55)$$

$$r/D = 20, \quad m = 1.05 \quad (56)$$

$$r/D = 15, \quad m = 1.08 \quad (57)$$

$$r/D = 10, \quad m = 1.17 \quad (58)$$

$$r/D = 5, \quad m = 1.30 \quad (59)$$

Alternatively, the relationship between $(u^2+v_r^2)^{1.5}$ and $(u^3+v_r^3)$ is plotted as shown in Figures (7). for all different values of r/D. In this case, best fitting leads to an average value of m = 1.12 for the range: $60 \geq r/D \geq 5$.

Actually, according to Eq. 53, α can be represented by the sum of α_l and α_r which can be defined as longitudinal energy coefficient and radial energy coefficients, respectively. Therefore, in curved open channel

$$\alpha = m(\alpha_l + \alpha_r) \quad (60)$$

where α_l and α_r are given as following, respectively

$$\alpha_L = \int \frac{u^3}{U^3}\frac{dA}{A} \quad (61)$$

and

$$\alpha_r = \int \frac{v_r^3}{U^3}\frac{dA}{A} \quad (62)$$

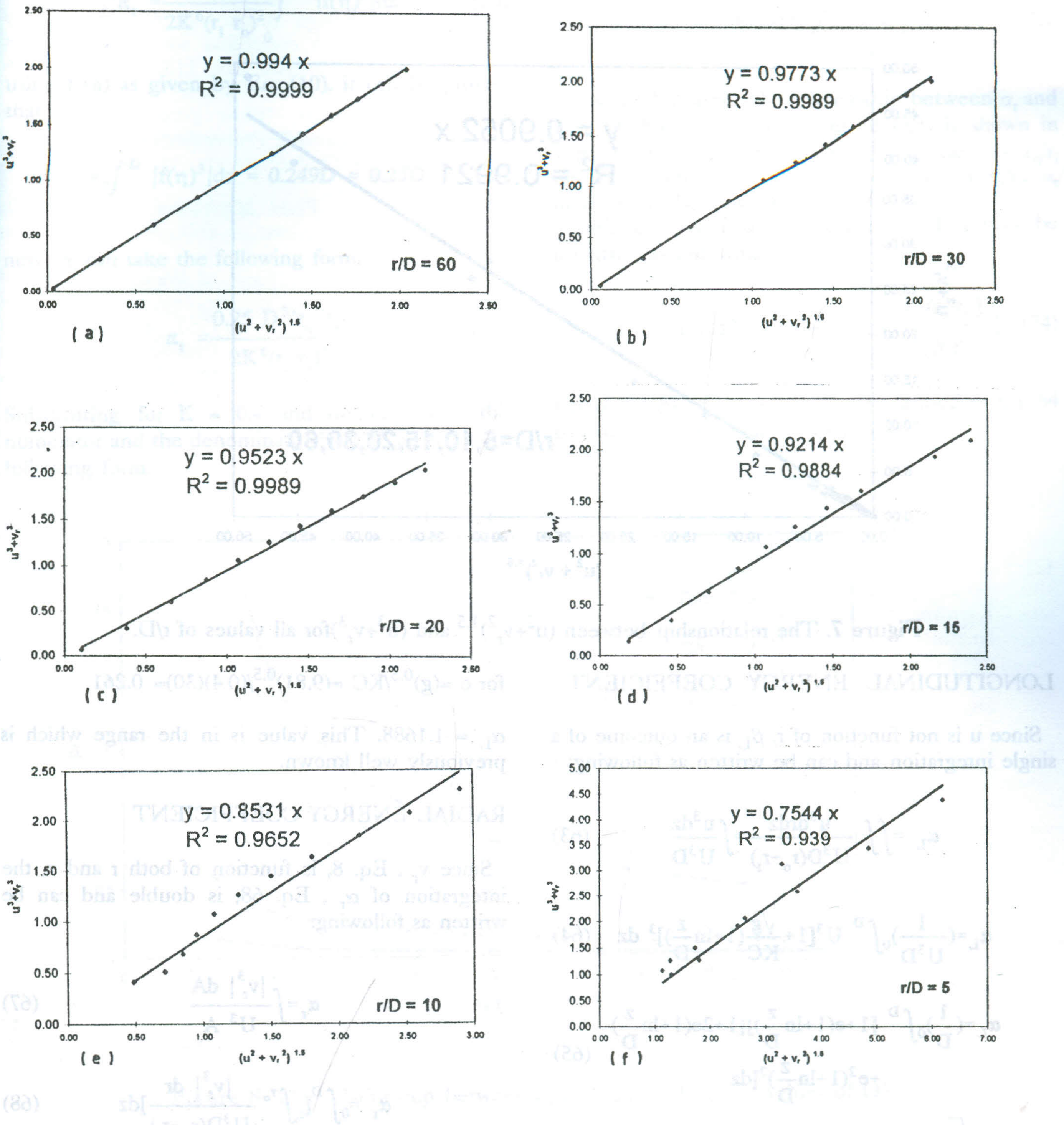

Figure 6. The relationships between $(u^2+v_r^2)^{1.5}$ and $(u^3+v_r^3)$ at different values of r/D.

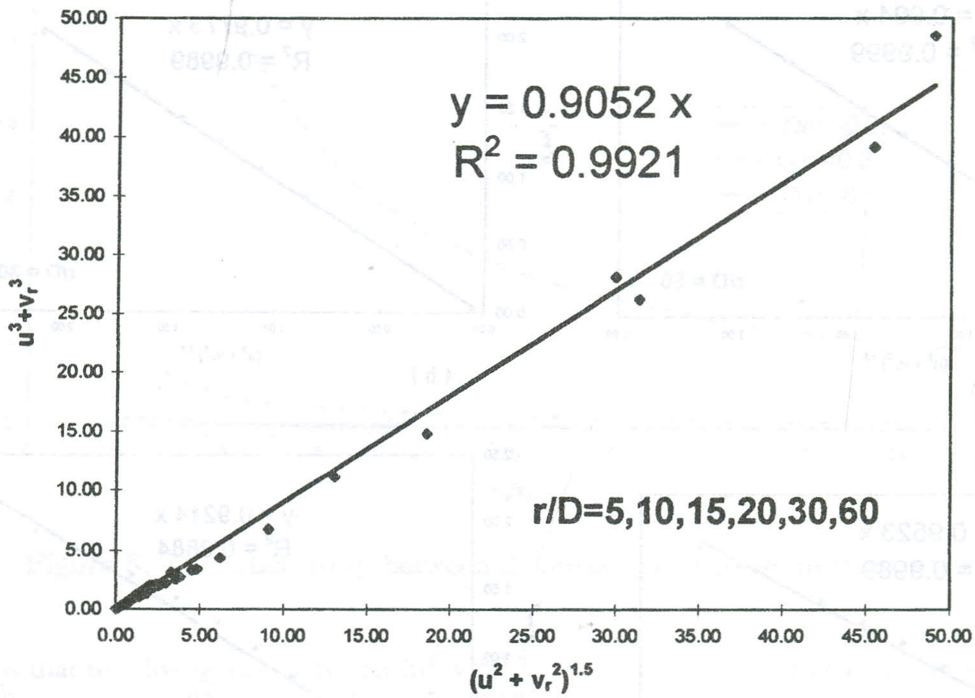

Figure 7. The relationship between $(u^2+v_r^2)^{1.5}$ and $(u^3+v_r^3)$ for all values of r/D.

LONGITUDINAL ENERGY COEFFICIENT

Since u is not function of r, β_L is an outcome of a single integration and can be written as following:

$$\alpha_L = \iint \frac{u^3 dr dz}{U^3 D(r_o - r_i)} = \int \frac{u^3 dz}{U^3 D} \tag{63}$$

$$\alpha_L = \left(\frac{1}{U^3 D}\right) \int_0^D U^3\left[1 + \frac{\sqrt{g}}{KC}\left(1 + \ln\frac{z}{D}\right)\right]^3 dz \tag{64}$$

$$\alpha_L = \left(\frac{1}{D}\right) \int_0^D \left[1 + e\left(1 + \ln\frac{z}{D}\right)\right]\left[1 + 2e\left(1 + \ln\frac{z}{D}\right) + e^2\left(1 + \ln\frac{z}{D}\right)^2\right] dz \tag{65}$$

let $e = \frac{\sqrt{g}}{KC}$, then it can be proved that

$$\alpha_L = (1 + 3e^2 - 2e^3) \tag{66}$$

for $e = (g)^{0.5}/KC = (9.81)^{0.5}/(0.4)(30) = 0.261$

α_L = 1.1688. This value is in the range which is previously well known.

RADIAL ENERGY COEFFICIENT

Since v_r, Eq. 8, is function of both r and z, the integration of α_r, Eq. 68, is double and can be written as following:

$$\alpha_r = \int \frac{|v_r^3|\, dA}{U^3 A} \tag{67}$$

$$\alpha_r = \int_0^D \left[\int_{r_i}^{r_o} \frac{|v_r^3|\, dr}{U^3 D(r_o - r_i)}\right] dz \tag{68}$$

$$\alpha_r = \frac{U^3 D^3}{U^3 K^6 D(r_o - r_i)} \int_0^D \left[\int_{r_i}^{r_o} \frac{dr}{r^3}\right] |f(\eta)^3|\, dz \tag{69}$$

$$\alpha_r = \frac{D^2(r_o+r_i)}{2K^6(r_i\ r_o)^2}\int_0^D |f(\eta)^3| dz \qquad (70)$$

using f (η) as given by Eq. (10), it can be proved that

$$*_0\int^D |f(\eta)^3| dz = 0.249D \approx 0.25D \qquad (71)$$

now α_r can take the following form:

$$\alpha_r = \frac{0.25\ D^3(r_o+r_i)}{2K^6(r_i\ r_o)^2} \qquad (72)$$

Substituting for K = 0.4 and dividing both the numerator and the denominator by $(r_i)^4$, α_r may take following form:

$$\alpha_r = 30.4(D/r_i)^3 \frac{[1+(r_o/r_i)]}{(r_o/r_i)^2} \qquad (73)$$

For smooth bottom, the relationship between α_r and r_o/r_i, at three different values of D/r_i, is shown in Figure (8) It is clear that α_r increases as r_o/r_i decreases. However, for certain value of r_o/r_i, α_r increases as D/r_i increases.

Combining α_l and α_r, the form of α may be rewritten as following:

$$\alpha = m[1.1688 + 30.4(D/r_i)^3(\frac{[1+(r_o/r_i)]}{(r_o/r_i)^2})] \qquad (74)$$

where values of m are previously defined in Eq. 54 to Eq. 59 or an average value, m = 1.12.

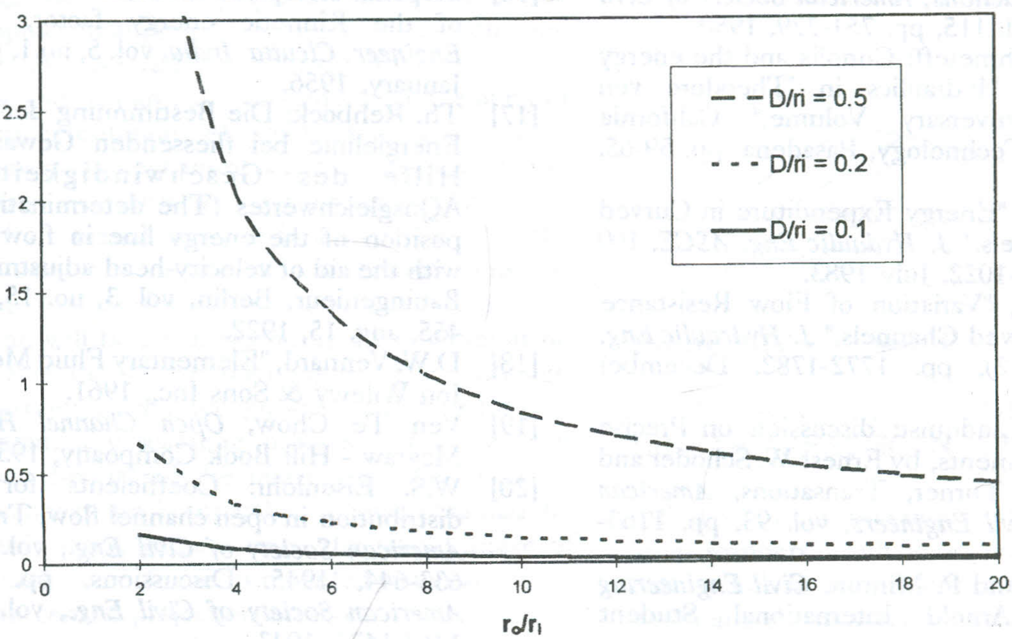

Figure 8. The relationship between α_r and r_o/r_i at different values of D/r_i.

CONCLUSIONS

1- The velocity of the flow in curved open channel has two components, longitudinal and radial distributions. The mean radial velocity seems to have a very small value and acts almost at half of the total flow depth.

2- The resultant velocity of flow in curved open channel deviates from the radial axis of the channel. The angle of deviation is less than 90° close to the water surface and increases as going deep under the water surface to become almost 90° at half the water depth. The maximum angle exists close to the bed surface.

3- The momentum coefficient in curved open channel can be divided to longitudinal, β_l, and radial, β_r, coefficients. β_l = 1.068 while β_r proved to have a relationship . It increases as r_o/r_i decreases, provided that D/r_i is constant. However, at certain value of r_o/r_i, β_r increases as D/r_i increases.

4- The energy coefficient in curved open channel can be divided to longitudinal, α_l, and radial, α_r, coefficients. α_l = 1.1688 while α_r proved to have a relationship. It increases as r_o/r_i decreases, provided that D/r_i is constant. However, at certain value of r_o/r_i, β_r increases as D/r_i increases.

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