STOCHASTIC MODEL FOR VERTICAL DIFFUSION OF SEDIMENTS

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ABSTRACT

Turbulent diffusion process and mixing occurred in open channel flows are one of basic and important subjects in modern hydraulics. Recent needs of quality control and management in water resources urge and accelerate the importance and significance of the research subject. A new stochastic model for vertical distribution of suspended sediment in turbulent open channel flow is developed based on the concept of transient probability and Marcovian process. An expression for the vertical turbulent diffusion coefficient (mass transfer coefficient) is established based on that theoretical approach. A verification analysis, using 248 data, shows a good agreement between predicted and measured values. The effect of grain size of sediments and their fall velocities on the diffusion coefficient is declared. The relation between mass transfer coefficient and momentum transfer coefficient is presented and compared with previous studies in the light of extensive data.

Key words: Turbulent flow, Suspended sediment, Fall velocity, Diffusion process, Momentum transfer.

INTRODUCTION

In the field of sediment dynamics, most of sediments are transported in suspension. Therefore, this part of total load is of special interest. The beneficial aspects of suspended sediment are in their deposition as increasing the amount of land by growing deltas, such like Nile delta in Egypt, and improvement of agriculture soil by deposition of very fine sediments. On the other hand, serious problems come from the diffusion of polluted sediments which requires purification for drinking and industrial water. More severe damage comes from sediments as depositions in reservoirs, harbors, estuaries and coastal areas, such as the recent problems of sedimentation in the area of Damietta port in Egypt.

The movement of sediments in streams has very complex mechanism and its calculation may never be completely possible with the traditional mathematics because there is a great uncertainty of the knowledge of the parameters influencing the transport process. The structure of turbulence and mechanism of diffusion and suspension has a random characteristics. The most fruitful approachs for this study are the stochastic models. They describe the diffusion of solid particles by a statement of probability of the instant movement of a suspended particle which is based on measured turbulence and flow parameters. Such models among others were presented by Liu (1956), Chiu et al. (1969), Willis (1969), Hino (1963), Yalin (1972), Tsujimoto, et al. (1986), and Shao, et al. (1991).

In the previous studies the mechanism governing the vertical turbulent diffusion of suspended sediment in two dimensional open channel flows is often considered as identical to that of momentum transfer of flow. In fact, the size of particles and their concentration in the fluid should have their effect on the diffusion coefficient for sediments.

In this study, a stochastic model is developed in conjunction with the principals of hydrodynamics for a particle motion in two phase flow. In the stochastic analysis, two approaches can be used. The first is stochastic differential equation with Gaussian white noise process. The second approach which is used in this study, is Kolmogrov's equation for transient probability density function. Both of them gave similar results. The physical process is simulated and the diffusion coefficient of particles is determined as a function of flow and sediment properties. The relation between both momentum and mass transfer coefficients is obtain and examined with the experimental and field data collected from other reliable sources. The obtained graphs show a good agreement between estimated and observed data.

Turbulent Diffusion and Mass-Balance Equation

For two dimensional quasi uniform flow under consideration, the sediment concentration varies, and diffusion occurs only along the vertical direction. Under this assumption the influence of both turbulent flow fluctuation and sediment concentration have little bearing on the settling velocity of sediment particles. Therefore, sediment settling does not differ from that in quiescent clear water at the same temperature. The concentration profile *c(y)* is usually calculated by using well known diffusion concept for suspended sediment. The balance between downward sediment settling due to gravity and upward diffusion associated with turbulent fluctuation leads to the equation governing the distribution of suspended sediment which is named mass-balance Eulerian equation, and its final form can be written as

$$w_{oe}\bar{c} + \epsilon_s \frac{\partial \bar{c}}{\partial y} = 0 \qquad (1)$$

in which w_{oe} is the effective terminal fall velocity of the particles, $\bar{c}$ is the mean value of the point volumetric concentration (c), y is the vertical distance measured from the stream bed, ϵ_s is the vertical sediment diffusion coefficient. This coefficient needs to be expressed in appropriate variables instead of using one constant value.

Stochastic Model for Particle Movement in Vertical Direction

A desecrate stochastic model for the movement of a suspended solid can be represented by deterministic and stochastic velocity components. A single particle step can be modeled as

$$y_{i+1} = y_i + \Delta t\,(\bar{v}_s + \eta_t(k)\,|\bar{v}'_s|) \qquad (2)$$

The deterministic velocity is the mean velocity $\bar{v}_s$, v'_s, is the fluctuating vertical velocity component which has the characteristics that its mean value $<v'_s> = 0$. The parameter Δt is the time step, k denotes the location or the flow layer number. The weighting function $\eta_t(k)$ is considered to depend on the difference of relative concentration around site k, and it randomly takes the values of +1 or -1. Thereby, according to the concentration difference, up or down, directed fluctuating velocities v'_s are chosen. The value $|\bar{v}'_s|$ represents the mean of the absolute value of the particle fluctuation v'_s. Dryden and Kuethe, defined the value of $|\bar{v}'|$ in another form as the violence or intensity of the fluid turbulence fluctuations, and it can be expressed as the root-mean-square value of the fluctuation v', see Henze (1975). Similarly, the intensity of sediment particle fluctuation can be presented as; $\tilde{v}_s = |\bar{v}'_s| = \sqrt{\bar{v}'^2_s}$. Equation (2) can be rewritten in the equivalent differential form :

$$dy = \bar{v}_s dt + \tilde{v}_s \eta_t dt \qquad (3)$$

This type of differential equation can be treated as a stochastic process by one of two different techniques; Ito stochastic calculus or Kolmogrov's equation which is used in this study.

Kolmogrov's second equation : For continuous stochastic process *y(t)* during small time intervals without aftereffect, the probability density function satisfies the equation

$$\frac{\partial f(t,y;\Delta t,\Delta y)}{\partial \Delta t} = -\frac{\partial}{\partial \Delta y}[m(\Delta t,\Delta y) f(t,y;\Delta t,\Delta y)] + \frac{1}{2}\frac{\partial^2}{\partial \Delta y^2}[n(\Delta t,\Delta y) f(t,y;\Delta t,\Delta y)] \qquad (4)$$

This equation was derived by Kolmogorov by using Taylor expansion, Gnedenko (1976), where, *m(t,y)* is the average rate of change of *y (t)* .

$$m(\Delta t, \Delta y) = \lim_{\Delta t \to 0} \frac{[y(t) - y(t-\Delta t)]}{\Delta t} = \bar{v}_s + \tilde{v}_s \qquad (5)$$

and, *n (t,y)* is proportional to the mean kinetic energy of the system under study

$$n(\Delta t,\Delta y) = \int_0^{\Delta t} [v_s(t) - \bar{v}_s][v_s(t') - \bar{v}_s]dt' = \tilde{v}_s^2 \Delta t \quad (6)$$

Applying Kolmogorov's equation on Eq. 3, yields

$$\frac{\partial f}{\partial \Delta t} = -\frac{\partial}{\partial \Delta y}(\bar{v}_s + \frac{1}{2}\tilde{v}_s^2 \Delta t \frac{\partial f}{\partial \Delta y}) \quad (7)$$

For a group of particles have the same path, the probability density function is in proportional to the particles concentration *c*. Thus, Eq. 7 can be modified as

$$\frac{\partial c}{\partial t} = -\frac{\partial}{\partial y}(\bar{v}_s + \frac{1}{2}\tilde{v}_s^2 \Delta t \frac{\partial c}{\partial y}) \quad (8)$$

For steady equilibrium condition $\partial c/\partial t = 0$. Thus from Eqs. (1, 8), it is postulated that

$$\bar{v}_s = |w_{\infty}|\ , \quad \varepsilon_s = \frac{1}{2}\tilde{v}_s^2 \Delta t \quad (9)$$

Motion of a Particle Suspended in a Turbulent Stream

The motion of a small spherical particle suspended in a turbulent fluid was formulated by Tchen, see Soo (1967). The resulting differential equation of motion for a particle moving at large particle Reynolds number under the influence of gravity in a fluid not at rest can be derived as

$$\frac{\pi d^3}{6}\rho_s \dot{v}_s = \frac{\pi d^3}{6}\rho \dot{v} + k\frac{\pi d^3}{8}\rho(\dot{v} - \dot{v}_s) + C_D \frac{\pi d^2}{8}\rho(v - v_s)^2$$

$$+ \frac{3\pi \mu d^2}{2\sqrt{\pi v_o}}\int_{t_o}^{t} dt' \frac{\dot{v}(t)' - \dot{v}_s(t)}{\sqrt{t - t'}} - \frac{\pi d^3}{6}(\rho_s - \rho)g \quad (10)$$

where d is the suspended particle diameter, $v(t)$ and $v_s(t)$ are the velocity of flow and solid particle, respectively, $\dot{v}$ and $\dot{v}_s$ are their respective accelerations $(\partial v/\partial t)$ and $(\partial v/\partial t)$. The parameters ρ and ρ_s are the density of flow and solid particles, respectively, t is the time, t_o is the starting time, v_o is the kinematic viscosity, μ is the dynamic viscosity, g is the gravitational acceleration, k is the virtual mass coefficient = 0.5 for spherical particle, and C_D is the particle drag coefficient which depends mainly on particle Reynolds number R_n^*.

In the equation, the left-hand side term represents the force required to accelerate the particle. The first term on right-hand side represents the resultant pressure due to fluid acceleration, the second term represents the force to accelerate the apparent mass of the particle relative to the fluid, the third term represents the drag force at steady state, the fourth term represents the Basset term (history of particle), and the last term is due to gravitational force.

The mathematical difficulties of integrating the general equation of motion forced us to exclude not effective terms of the equation. It is seen that the Basset force term becomes important only if the density of the fluid has similar or higher order of magnitude than that of solid particles, hence it can be omitted for our case of study without fear of accuracy.

Let $v_r = v - v_s$ and $v_r' = v' - v_s'$, thus

$$\frac{\pi}{6}d^3(\rho_s + \frac{\rho}{2})v_r' = \frac{\pi}{6}(\rho_s - \rho)gd^3 - C_D\frac{\pi}{8}d^2\rho v_r^2 \quad (11)$$

$$\frac{dv_r}{dt} = \frac{3C_D}{4d(\frac{\rho_s}{\rho} + \frac{1}{2})}\left[\frac{4d}{3C_D}(\frac{\rho_s}{\rho} - 1)g - v_r^2\right] \quad (12)$$

$$\frac{dv_r}{dt} = A(B^2 - v_r^2) \quad (13)$$

where

$$A = 3C_D/4d(\frac{\rho_s}{\rho} + \frac{1}{2}) \text{ and } B = \sqrt{\frac{4d}{3C_D}(\frac{\rho_s}{\rho} - 1)g} \quad (14)$$

When the time t = 0, the velocity of fluid is v and particle velocity v_s = 0. When the time is Δt the fluid velocity remains v, while the particle velocity becomes v_s. Thus, the solution of Eq. (13) may be derived as

$$\int_{v}^{v_r} \frac{dv_r}{B^2 - v_r^2} = \int_{o}^{\Delta t} A\, dt \qquad (15)$$

The particle velocity yields,

$$v_s = \frac{(v^2 - B^2)[1 - Exp(2BA\Delta t)]}{(v - B) - [(v + B)Exp(2BA\Delta t)]} \qquad (16)$$

Since $v_s = \bar{v}_s + v'_s$ for sediment particles, and $v = \bar{v} + v'$ for flow,

$$\bar{v}_s + v'_s = \frac{\bar{v}^2 + v'^2 - B^2}{\bar{v} + v' - B\dfrac{[1 + Exp(2BA\Delta t)]}{1 - Exp(2BA\Delta t)}} \qquad (17)$$

Assuming two dimensional flow, the main vertical velocities $\bar{v}$ can be neglected, and the sediment particles turbulence intensity $\tilde{v}_s = \sqrt{v'^2_s}$ may be derived in the form

$$\tilde{v}_s = \sqrt{\left[\frac{v'^2 - B^2}{v' - B\dfrac{[1 + Exp(2BA\Delta t)]}{[1 - Exp(2BA\Delta t)]}} - \bar{v}_s\right]^2} \qquad (18)$$

Time Scale Δt

A time scale Δt which is used for simulation represents the life time of the eddy to move the suspended particle one step above. It may be obtained by regarding the Lagrangian time scale of turbulence. It shows that the momentum transfer coefficient ε_m is given by

$$\varepsilon_m = v'^2\ \Delta t \qquad (19)$$

Various distributions of fluid mixing coefficient (momentum transfer coefficient) can be found in the previous studies. The parabolic distribution is in the form

$$\varepsilon_m = \kappa\, u_*\, h\, \frac{y}{h}(1 - \frac{y}{h}) \qquad (20)$$

where κ is von Karman constant, which is equal to 0.32 ~ 0.4 for water with low concentration of sediments, h is the total depth, y is the depth of water from the bed. Thus,

$$\Delta t = \kappa\, u_*\, h\, \frac{y}{h}(1 - \frac{y}{h}) / v'^2 \qquad (21)$$

Development of the Model

If turbulence flow field is steady, the velocity with respect to time can be averaged. Taking the average over large number of velocities which have the same initial and boundary conditions (ensemble-mean value), yields

$$\bar{v}_s = \lim_{t_1 \to \infty} \frac{1}{t_1} \int_{o}^{t_1} v_s dt \qquad (22)$$

where $t_1 > \Delta t$, thereby $\partial \bar{v}_s / \partial t$ should be equal zero , Henze (1976). Applying such rules on the above equation (12), the particle mean velocity yields,

$$\bar{v}_s = B = \sqrt{\frac{4d}{3C_D}(\frac{\sigma}{\rho} - 1)g} \qquad (23)$$

Applying equations (9, 18, 21 and 23), the particles diffusion coefficient ε_s yields

$$\varepsilon_s = \frac{1}{2} K u_* h \frac{y}{h}\frac{(1-y)/h)}{v'^2}\left[\frac{v'^2 - B^2}{v' - B\dfrac{[1 + Exp(2BA\Delta t)]}{[1 - Exp(2BA\Delta t)]}} - B\right]^2 \qquad (24)$$

The intensity v' is adopted by Shinohara and Tsubaki (1959)as $v' = 0.93\ u_*$.

For drag coefficient C_D in Eq. 14, it is usually assumed that for small particle Reynolds number R_n^*, the stokes linear approximation to the drag coefficient *($C_D = 24/R_n^*$)* will be valid. This is not always the case and at sufficiently high Reynolds number, very large errors will result from the use of this formula. In this paper, the values of C_D will be adopted according to Morsi-Alexander (1972), see Appendix.

Examining the previous equation (24) shows that, Exp (2B A Δt)>> 1.0, and the term .

$$\frac{[1+\mathrm{Exp}(2BA\Delta t)]}{[1-\mathrm{Exp}(2BA\Delta t)]}=-1.$$

From Eqs. (9 and 23), it can be postulated that the mean velocity of the particle moving upward by diffusion is equal to particle fall velocity in value,

$$\overline{v}_s = B = -w_{oe} \tag{25}$$

Thus, equation (24) may be simplified as follows,

$$\varepsilon_s = 0.18\, u_*\, h\, \frac{y}{h}\left(1-\frac{y}{h}\right)\left[1+2.15\left(\frac{w_{oe}}{u_*}\right)\right]^2 \tag{26}$$

Effect of Particles Size on Diffusion Coefficient

Re-examining particles fall velocity equations: In the case of study of vertical diffusion, the fall velocity of particles in moving fluid is considered as in the case of still fluid. The particle fall velocity was extensively studied by several investigators. The most fruitful studies were as follows :

1- Stokes law : In this method, the value of particle fall velocity w_o depends mainly on the mean particle diameter d_{50}, Reynolds number for particles, $R_n^* = w_o\, d_{50}/v$, and water temperature T_c. In which v is the kinematic viscosity of the fluid. For $R_n^* < 1$, *or* $d_{50} < 62$ μm., the fall velocity can be obtained from

$$w_o = \frac{g}{18}\left(\frac{\rho_s}{\rho}-1\right)\frac{d_{50}^2}{v_o} \tag{27}$$

2- *Rubey's formula [1936]* : Rubey presented one general explicit equation to get the suspended particles fall velocity in still clear water.

$$\frac{w_o}{\sqrt{s g d_{50}}} = \sqrt{\frac{2}{3}+\frac{36 v_o^2}{s g d_{50}^3}} + \sqrt{\frac{36 v_o^2}{s g d_{50}^3}} \tag{28}$$

in which s is the submerged specific weight of sediments = 1.65.

3- Simons et al. chart [1977] : The value of w_o can be obtained directly from the common chart which was presented by Simons et al. (1977).

4- Zanke's formula [1977] : For suspended sand particles in the range 100-1000 μm, the following type of equation is proposed by Zanke, see van Rijn (1982),

$$w_o = 10\,\frac{v_o}{d_{50}}\left\{\left[1+\frac{0.01 s g d_{50}^3}{v_o^2}\right]^{0.5}-1\right\} \tag{29}$$

5- Yalin's equation [1977] : For laminar motion ($R_n^* < 1$), the value of fall velocity can be obtained from stocks law. For turbulent flow ($R_n^* > 1000$), the fall velocity equation is

$$w_o = 1.825\sqrt{s g d_{50}} \tag{30}$$

6- van Rijn's formula [1982] : For particles larger than about 1000 μm, he proposed,

$$w_o = 1.1\sqrt{s g d_{50}} \tag{31}$$

The previous methods for calculating fall velocity is compared with each other, and examined in the light of extensive data at temperature 20°c and different values of specific gravity. In Figures. (1, 2), it can be noticed that for relatively big size particles, Rubey's curve gives less values than the others. Yalin' s formula is applicable for $d_{50} > 2000$ μm, while the application of van Rijn's formula is limited in the range 1000 μm < d_{50} < *2000* μm.

Effect of concentration : Experimental research by Richardson and Zaki, {see van Rijn (1982)}, has shown that for high sediments concentration, the fall velocity is not only affected by the displaced fluid but also by additional factors such as; particle collisions, particle-induced turbulence and modified drag coefficient. The overall effect can be represented by,

$$w_{oe} = (1-\overline{c})^{\alpha} w_o \tag{32}$$

where w_{oe} is the effective particle fall velocity , and α is a coefficient (4 to 5 for particles in the range of 50 to 500 μm). In this paper, Equations 27, 29, 30, 31 and 32 will be used to represent sediments fall velocity for different size of particles.

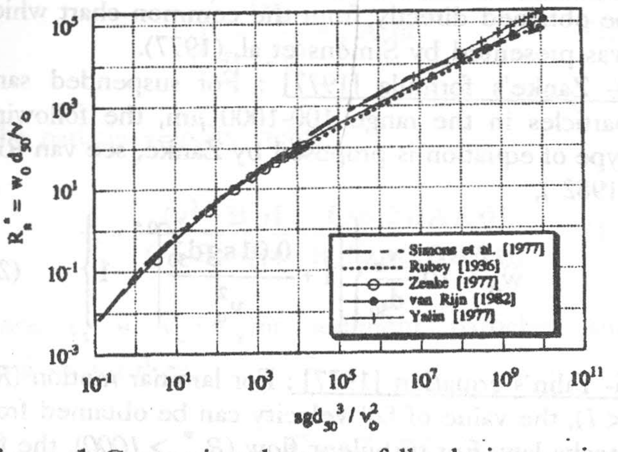

Figure 1. Comparison between fall velocity previous equations.

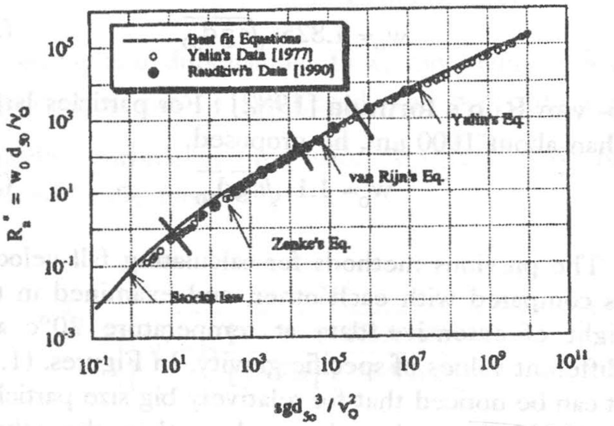

Figure 2. Examining selected fall velocity equations with data.

Figure (3) shows a relation between the dimensionless water y/h and dimensionless diffusion coefficient $\varepsilon_s/h\ u_*$, with the suspension parameter w_{oe}/u_*. The curves and plotted data show that the increase of fall velocity, which indicates the increase of suspended particle size, is in proportional to the increase of diffusivity coefficient.

Verification of the Model

The presented equation for diffusion coefficient is examined with a group of different data collected by Anderson (1942), Vanoni (1946), and Coleman (1970). Figure (4), shows the relation between 248 observed data and calculated ones for dimensionless parameter ε_s/hu_*. The figure shows a good agreement, and the presented equation is promising.

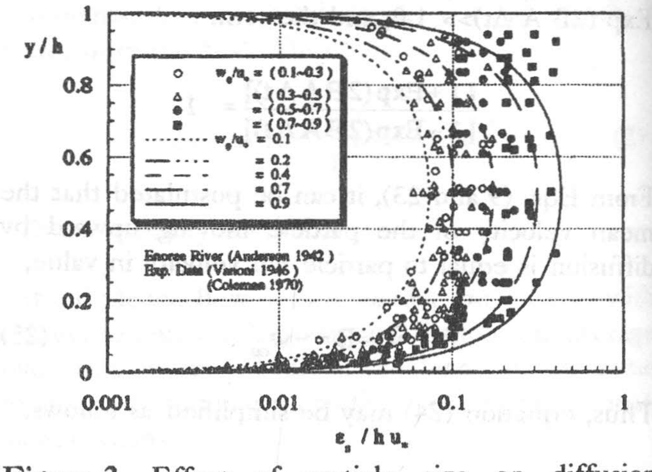

Figure 3. Effect of particle size on diffusion coefficient.

The Relation between Momentum and Mass Transfer Coefficients

The fluid momentum coefficient is previously described in the parabolic form

$$\varepsilon_m = \kappa\ u_*\ h\ \frac{y}{h}(1-\frac{y}{h}) \tag{33}$$

The relation between mass and momentum diffusion coefficients was given as

$$\varepsilon_s = \beta\ \varepsilon_m \tag{34}$$

Thus, from equations (26 and 33)

$$\beta = 0.5\ [1 + 2.15\ (\frac{w_{oe}}{u_*})]^2 \tag{35}$$

Comparison with previous studies:

Most of previous other studies considered β as constant equal to 1.2. van Rijn (1984) defined the ratio factor β as, $\beta = \varepsilon_{s\ max}/\ \varepsilon_{m\ max}$, and the computed value was described by,

$$\beta = 1 + 2\ (\frac{w_o}{u_*})^2 \quad \text{for} \quad 0.1 < \frac{w_o}{u_*} < 1 \tag{36}$$

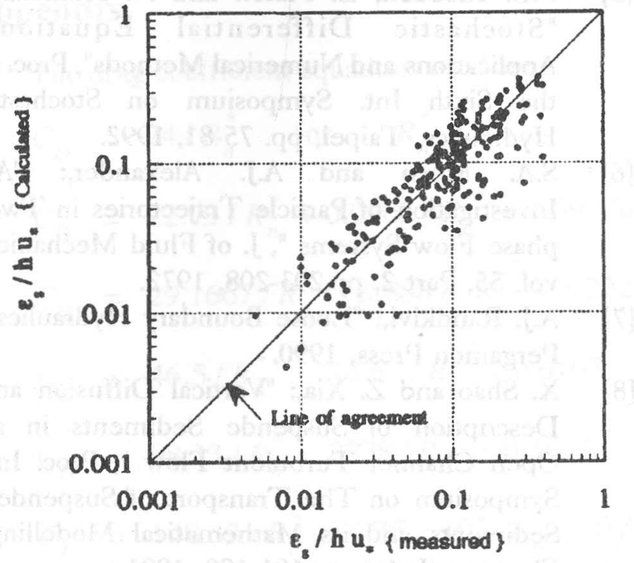

Figure 4. Verification of the presented equation.

Tsujimoto et al. (1986) presented β factor in the following form,

$$\beta = 1 + 1.56\left(\frac{W_o}{u_*}\right)^2 \qquad (37)$$

Shao et al. (1991) presented expression for mass transfer coefficient. From his results, the β factor can be expressed as

$$\beta = 1500\, s^{0.2}\, \frac{d_{50}}{h} \frac{1}{\sqrt{w_o/u_*}} \qquad (38)$$

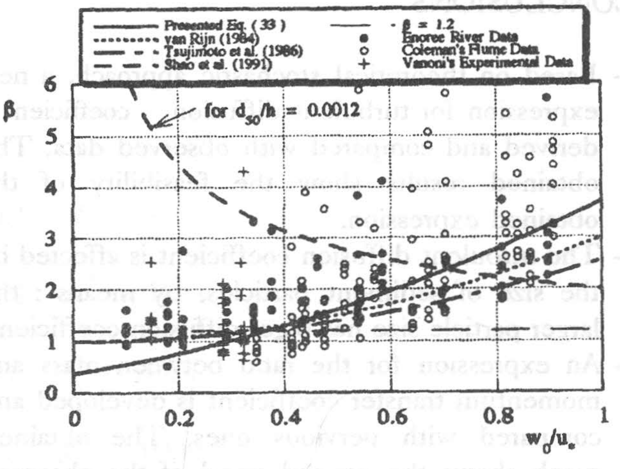

Figure 5. Comparison between presented equation and previous studies.

Comparing all of these expressions with the presented Eq. (26), in Figure (5), shows the general trend of the observed data to the presented equation more than the others, especially for $w_{oe}/u_* > 0.2$. Also, it can be noticed some scattering for the observed data may be because of the effect of sediments concentration in the fluid mixture, and it should be considered preciously in further research.

Table 1 Error analysis for β ratio in the presented equation and the previous studies

Authors	Mean error	Standard deviation	Scores of errors in intervals, as a percentage									
			- 5,5	- 10,10	- 15,15	-20,20	-25,25	-30,30	-35,35	-40,40	-45,45	-50,50
1) The prsented equation	0.03	0.28	13.94	20.72	31.08	42.23	56.18	63.35	68.53	75.30	79.68	84.08
2) Tsujimoto et al. [1986]	- 0.158	0.32	10.76	19.52	29.08	39.44	49.40	58.17	66.93	73.31	78.88	82.87
3) van Rijn [1984]	- 0.0968	0.34	11.55	21.51	30.68	38.65	47.01	54.58	58.96	63.35	74.90	80.08
4) $\beta = 1.2$	- 0.25	0.37	8.76	13.94	21.91	28.29	36.25	42.23	50.20	55.78	62.55	65.34

CONCLUSIONS

1- Based on theoretical stochastic approach, a new expression for turbulent diffusion coefficient is derived and compared with observed data. The obtained results show the feasibility of the obtained expression.
2- The turbulent diffusion coefficient is affected by the size of sediment particles, by means : the larger particle size has biger diffusion coefficient.
3- An expression for the ratio between mass and momentum transfer coefficient is developed and compared with pervious ones. The obtained graph shows the general trend of the observed data to the presented equation.
4- Further research is needed to study the effects of particles concentration and particles shape factor on mass transfer coefficient and turbulent characteristics.

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Appendix

The drag coefficient equations used are

$$C_D = 24 / R_n^* \quad \text{for} \quad R_n^* < 0.1,$$

$$C_D = 22.73 / R_n^* + 0.0903 / R_n^{*2} + 3.69 \quad \text{for} \quad 0.1 < R_n^* < 1,$$

$$C_D = 29.1667 / R_n^* - 3.8889 / R_n^{*2} + 1.222 \quad \text{for } 1 < R_n^* < 10,$$

$$C_D = 46.5 / R_n^* - 116.67 / R_n^{*2} + 0.6167 \quad \text{for } 10 < R_n^* < 100,$$

$$C_D = 98.33 / R_n^* - 2778 / R_n^{*2} + 0.3644 \quad \text{for } 100 < R_n^* < 1000,$$

$$C_D = 148.62 / R_n^* - 4.75 \times 10^4 / R_n^{*2} + 0.357 \quad \text{for } 1000 < R_n^* < 5000,$$

$$C_D = -490.546 / R_n^* + 57.87 \times 10^4 / R_n^{*2} + 0.46 \quad \text{for } 5000 < R_n^* < 10000,$$

$$C_D = -1662.5 / R_n^* + 5.4167 \times 10^6 / R_n^{*2} + 0.5191 \quad \text{for } 10000 < R_n^* < 50000,$$