RELIABILITY OF INITIALLY IMPERFECT PLATE PANELS UNDER BIAXIAL LOADING

Y.A. Abdel-Nasser and H.W. Leheta
Marine Engineering and Naval Architecture Department,
Alexandria University, Alexandria, Egypt.

ABSTRACT

The reliability of perfect plate panels modelled with the Idealized Structural Unit Method (ISUM) and considered uniaxially loaded has been previously studied. The First Order Reliability Method (FORM) was selected to calculate the safety index of perfect plates during buckling , post-buckling and yielding stages. In this paper, the influence of initial deflection and residual stresses on the reliability assessment of plate panels subjected to biaxial loading is studied. Before the reliability analysis is carried out, suitable probabilistic models are selected for random variables involved in the performance functions. These performance functions are calculated at each stage of collapse. Examples of analyses are presented showing the influence of the initial deflection, residual stresses and biaxial loading on the safety index of the plate panels.

Keywords: ISUM, Yielding, Buckling, Reliability, Initial deflection, Residual stresses, Biaxial loading, FOI

Nomenclature

B	Strain-displacement matrix
D	Stress-strain matrix
E	Young's modulus of elasticity
H	Strain hardening stiffness matrix
K^B	Post-buckling stiffness matrix
K^P	Elastic-plastic stiffness matrix
M	Safety margin
p_f	Probability of failure
w	Added deflection due to load
W_o	Initial deflection
W_{om}	Amplitude of initial deflection
β	Safety index
Γ	Gamma function
Γ_B	Buckling function
Γ_Y	Yielding function
ε	Strain vector
Φ_i	$\{\Gamma/\partial R\}_i$
υ	Poisson's ratio
σ	Stress vector
σ_o	Material yield stress
σ_{rx}	Residual stress along the x- axis
σ_{ry}	Residual stress along the y- axis
σ_x	Normal stress directed along the x axis
σ_{xav}	Maximum allowable average stress in x direction
σ_{yav}	Maximum allowable average stress i direction
σ_{xcr}	Critical buckling stress in x direction
σ_{ycr}	Critical buckling stress in y direction
τ_{xy}	Average shear stress in x and y directions

1. INTRODUCTION

In a previous study [1], the reliability of p panels modelled with the Idealized Structural I Method (ISUM) and considered uniaxially loa was studied. ISUM was used to assess the strer while a First Order Reliability Method (FORM) selected to calculate the safety index of plate pa for both failure modes; namely, yielding buckling. The random variables involved v defined and suitable probabilistic models v selected.

Practically, plate panels used in ship and fixed floating platforms, unavoidably have initial deflec and residual stresses. These imperfections produced during fabrication processes, in partic during welding. In this paper, the response of p panels subjected to biaxial loading is evalu taking into account the effect of initial imperfect then the reliability of these plate panels is estimated.

Firstly, a strength analysis of plate panels with initial imperfections and subjected to biaxial loading is carried out using the Idealized Structural Unit Method (ISUM). Here, the presence of the initial imperfections in the plate panels is idealized and its effect on the plate stiffness matrix is formulated. That means, the response of plate panels may be calculated during each stage of collapse e.g. buckling, post-buckling and yielding stages.

Secondly, a reliability analysis of plate panels is performed using a First Order Reliability Method (FORM). The statistical distributions of random variables such as dimensions, material properties, loading and initial imperfections are modelled. Also, possible performance functions are calculated during the history of collapse.

Several examples of square plate panels with initial imperfections and subjected to biaxial loading are used and calculations carried out, and the results are presented.

2. STRENGTH ANALYSIS

2.1 *Effect of Initial Deflection and Residual Stresses*

As mentioned before, usually plate panels in marine structures have initial deflections and residual stresses. These initial imperfections are produced at the fabrication stage, in particular during welding. In this section, the effect of initial deflection and residual stresses on the strength of the plate panels is studied.

First, initial deflection is considered. Initial deflection may be expressed in the form of fourier series as follows [2],

$$w_o = \Sigma W_{om} \sin(m\pi x/a) \sin(n\pi y/b) \quad (1)$$

where

W_{om} = the amplitude of a component of initial deflection similar to the buckling mode, and

m = the number of half buckling waves of the buckling mode.

A plate with initial deflection and subjected to biaxial compression exhibits an increase of deflection from the beginning of the loading process. At the beginning, the magnitude of all fourier components of the initial deflection increases. Close to the critical buckling load, unless some other component has an extremely large magnitude, the magnitude of the component similar to the buckling mode of the corresponding perfect plate continues to increase at a higher rate, while the magnitudes of other components start to decrease. Strictly speaking, bifurcation at the critical load is not observed. The behaviour is accompanied by the effect of large deflection from the beginning of loading. Yielding starts at a load lower than that for a perfectly flat plate (without initial deflection) and ultimate strength is also reduced. Only one component of initial deflection similar to the buckling mode has an appreciable effect on plate behavior and needs to be taken into account. Initial deflection may then be expressed as,

$$w_o = W_{om} \sin(m\pi x/a) \sin(\pi y/b) \quad (2)$$

The value of W_{om} to be used in design is given in [3], when average measured values are not available. The value of m depends on the plate aspect ratio and the ratio of $\sigma_{xav} / \sigma_{yav}$.

Additional deflection due to the applied load may be assumed in the same form as follows,

$$w = W_m \sin(m\pi x/a) \sin(\pi y/b) \quad (3)$$

where

W_m = the amplitude of the additional deflection.

Next, welding residual stresses are considered. These usually take the distribution as in Figure (1a) and may be idealized as in Figure (1b). This distribution is characterized by two tension bands near the edges where the stress reaches the yield stress, and a compressive region in the middle portion of the breadth of the plate. This stress distribution is in self-equilibrium. The effect of residual stress is directly related to the magnitude σ_{rx} of the compressive region.

The effect of these residual stresses is to reduce the buckling load, the load at which first yielding occurs, as well as the ultimate strength and post-ultimate strength carrying capacity.

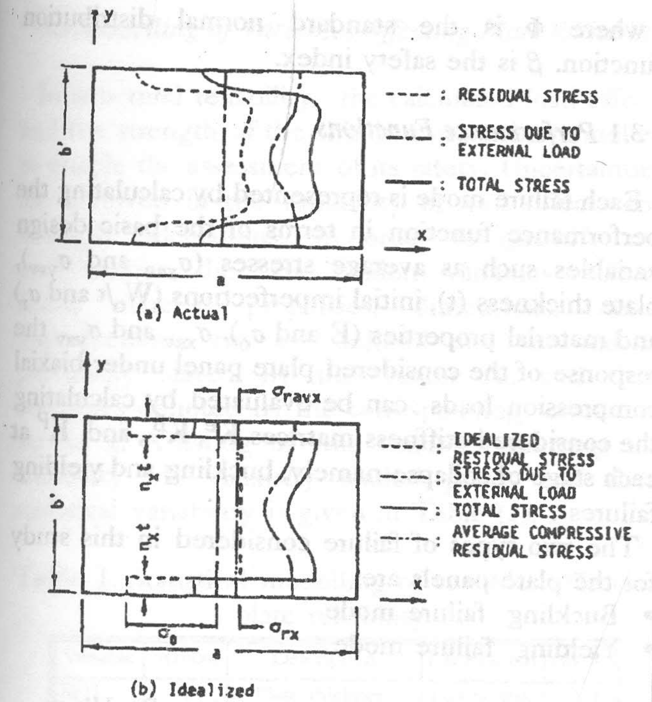

Figure 1. Longitudinal stress distribution in a welded plate subjected to uniaxial compression.

In the present formulation, when evaluating the large deflection behaviour of the plate, effective compressive residual stresses distributed uniformly in x and y directions are assumed as follows [4],

$$\sigma_{rex} = \sigma_{rx}\,(1\text{-}0.5\;\sigma_{rx}/(\sigma_{rx} + \sigma_o))$$

$$\sigma_{rey} = \sigma_{ry}\,(1\text{-}0.5\;\sigma_{ry}/(\sigma_{ry} + \sigma_o)) \qquad (4)$$

where, σ_{rx} and σ_{ry} are magnitudes of the compressive residual stresses in x and y directions, respectively.

As mentioned above, the deflection of a plate with such imperfections, when subjected to external loads, starts to increase from the beginning of the loading process and the bifurcation at buckling is unclear. The behaviour of such a plate may be treated in the same way as the post-buckling behaviour of perfectly flat plates. In the following, the elastic stiffness matrix, ultimate strength condition and the post-ultimate strength elastic-plastic stiffness matrix are evaluated.

2.2 *Elastic Stiffness Matrix*

The stress distribution around the edges of a plate with initial imperfection is nonlinear from the beginning of the loading process. That means, a non-linear stiffness matrix has to be considered from the beginning of loading. To evaluate the stifness matrix, an imaginary flat plate is employed, using linear displacement functions [5]. Stress distributions are linear in this imaginary plate and the material properties are determined so that the plate exhibits similar stiffness to that of the deformed plate. Under loading, shortening in x and y directions and shear strain of the deflected plate may be evaluated, and the relation between average strain, ε_{av} and average stress, σ_{av} may be written as follows,

$$\varepsilon_{xav} = (\sigma_{xmax} - \upsilon\sigma_{yav})\,/E$$

$$\varepsilon_{yav} = (\sigma_{ymax} - \upsilon\sigma_{xav})\,/E$$

$$\gamma_{xyav} = \tau_{xy}\;/\,G_e \qquad (5)$$

where σ_{xav} and σ_{yav} are the average stresses in x and y directions, respectively and σ_{xmax} and σ_{ymax} are the maximum membrane stresses at the plate edges. In order to determine the maximum membrane stresses σ_{xmax} and σ_{ymax}, Galerkin's method is applied to solve the equilibrium and compatibility equations of the plate [6]. Thus, the average stress -strain relationship is,

$$\sigma_{av} = D^B\,\varepsilon_{av} \qquad (6)$$

where, D^B is the stress- strain relationship.

The post-buckling stiffness matrix K^B may be written as follows,

$$K^B = \int B^T\,D^B\,B dV \qquad (7)$$

where, B is the strain- displacement relationship.

2.3 *Ultimate Strength Condition and Elastic-Plastic Stiffness Matrix*

In presence of residual stresses, tension bands as shown in Figure (1) exist along the edges. Therefore, initial yielding may start just on the inside of these tension bands rather than the outer edges.

Yielding is assumed to start at any location when Mises yield condition is satisfied, that is,

$$\Gamma_y = \sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y - 3\tau_{xy}^2 \; \sigma_o^2 = 0 \qquad (8)$$

Plastic nodes are inserted where the yield condition is satisfied. Having D^B of Eq. (6), the elastic plastic stiffness matrix K^P may be rewritten as follows,

$$K^p = K^B - K^B \Phi_i \Phi_i^T K^B / S_{pi} \qquad (9)$$

When yielding occurs at m nodes, K^P may similarly be derived as follows,

$$K^p = K^B - K^B \Phi S_p^{-1} \Phi^T K^B \qquad (10)$$

where, $\Phi = [\Phi_1, \Phi_2, ..., \Phi_m]$, $S_p = \Phi^T (K^B + K^o) \Phi$

$K^o = \int B^T H B \, dV$ and H is an equivalent strain hardening matrix ,see Ref. [5].

3. RELIABILITY ANALYSIS

When the reliability of a structure is dealt with, the structure is assumed to be either in a safe state or a failed state. This state may be expressed through a performance function g(x). Usually, g(x) is defined in terms of the set of basic variables x, describing loads, material properties, geometry, scantlings, initial imperfections, etc. The limit state function satisfies:

$$g(x)\begin{cases} <0 & x \text{ in failure set} \\ =0 & x \text{ on limit state surface} \\ >0 & x \text{ in safe set} \end{cases} \qquad (11)$$

The First Order Reliability Method (FORM) is adopted whereby the limit state function is transformed and linearized [7,8 and 9].

$$G_Y(y) = g_X(x) = 0 \qquad (12)$$

The approximation to the failure probability, p_f is [10]:

$$p_f \approx \Phi(-\beta) \leftrightarrow \beta \approx -\Phi^{-1}(p_f) \qquad (13)$$

where Φ is the standard normal distribution function. β is the safety index.

3.1 *Performance Functions.*

Each failure mode is represented by calculating the performance function in terms of the basic design variables such as average stresses (σ_{xav} and σ_{yav}), plate thickness (t), initial imperfections (W_o/t and σ_r) and material properties (E and σ_o). σ_{xav} and σ_{yav} the response of the considered plate panel under biaxial compression loads can be evaluated by calculating the considered stiffness matrices K^e, K^B , and K^P at each stage of collapse namely, buckling and yielding failures

The two types of failure considered in this study for the plate panels are:

- Buckling failure mode
- Yielding failure mode

3.1.1 *Performance Functions against Buckling*

The buckling condition Γ_B representing the performance function of the rectangular plate panel under consideration may be written in terms of the average normal stress σ_{xav} in the x direction, and the average stress σ_{yav} in the y-direction. The safety margin M is,

$$M = 1 - \frac{\sigma_{xav}}{\sigma_{xcr}} - \frac{\sigma_{yav}}{\sigma_{ycr}} \qquad (14)$$

where, σ_{xcr} and σ_{ycr} are the critical buckling stresses in x and y directions respectively.

3.1.2 *Performance Functions against Yielding*

When the maximum equivalent stress reaches the material yield stress, a substantial loss of stiffness occurs and the plate becomes unable to carry further load. The yielding condition Γ_Y of the rectangular plate element is the Von Mises criterion, which may be written as in Eq. (8). The safety margin, M is,

$$M = \sigma_o - \sigma_{xmax}^2 - \sigma_{ymax}^2 + \sigma_{xmax}\sigma_{ymax} + 3\tau_{xy}^2 \qquad (15)$$

where σ_{xmax} and σ_{ymax} are the maximum stresses induced at the plate edges.

3.2 *Modelling of Variables Affecting Plate Reliability*

In structural reliability, the calculated load effects and the strength of the structure must be modelled to enable the assessment of its safety. Uncertainties are involved in the variability of load effects and strength calculations. In order to quantify the uncertainties, the basic random variables such as loads, material properties, dimensions, initial imperfections, etc. are defined and denoted as previously stated by the vector X, and their outcomes denoted by the corresponding lower case vector, x. A listing of the variables affecting the reliability of biaxially loaded plates and their statistical variability is given in Table (1).

Table 1. Statistical modelling of variables affecting plate reliability.

Variable	Symbol	Description	Distribution	cov %
X(1)	t	Plate thickness	Log-normal	0.5-4
X(2)	σ_x	Normal stress in X-direction	Normal	15-40
X(3)	σ_y	Normal stress in y-direction	Normal	15-40
X(4)	E	Young's modulus	Normal	1-2
X(5)	σ_o	Yield stress	Normal	4-10
X(6)	w_o	Initial deflection	Normal	4-10
X(7)	σ	Residual stress	Normal	10-20

4. PARAMETRIC STUDY

A parametric study is performed on initially imperfect square plate panels (1000x1000 mm). A plate panel of 16mm thickness and a yield strength of 24 kg/mm^2 is considered. The reliability of plate panels with different plate thicknesses has been clarified in Ref. [1].

ISUM calculates the response of plate panels at each loading stage, then FORM calculates the safety index or the probability of failure at this stage of loading. Probabilistic models of the variables used in the performance functions are as shown in Table (1). In these analyses, the Young's modulus and the yield stress are modelled by a normal distribution with 2% and 10% coefficient of variation respectively. The plate thickness is modelled by a log-normal distribution with a 2% coefficient of variation. σ_{xav} and σ_{yav} are assumed to have a normal distribution with a 15% cov.

The results of the parametric study are presented in the following subsections.

4.1 *Uniaxial Loading*

4.1.1 *Effect of Initial Deflection*

The analysis is performed on plate panels with different values of the initial deflection W_o/t under uniaxial compression along the x direction. Practical values of W_o/t equal to 0.1, 0.2, 0.3, 0.4 and 0.5 are considered. Figures (2) and (3) show the relationship between the safety index β and the normalized load σ_{xav}/σ_o when the plate fails in the buckling and in the yielding modes, respectively.

The higher values of W_o/t predict a decrease of the safety index of the plate panels.

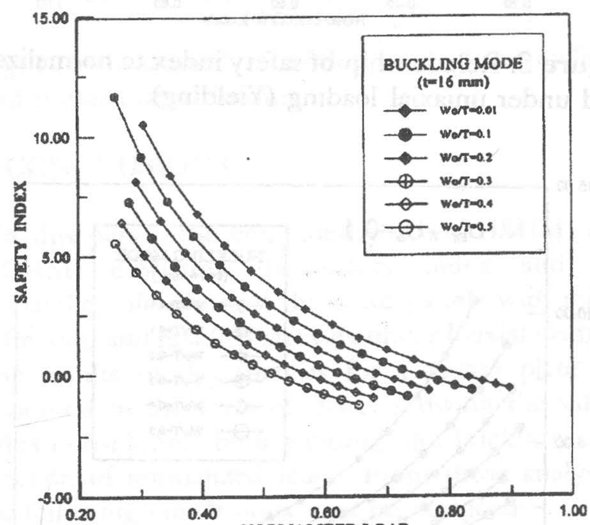

Figure 2. Relationship of safety index to normalized load under uniaxial loading (Buckling).

4.1.2 *Effect of Residual Stresses*

The analysis is performed on plate panels with initial deflection W_o/t equal 0.1, 0.2, 0.3, 0.4 and 0.5 and a residual stress σ_{rx} under uniaxial compression. Two cases of σ_{rx}/σ_o are considered. Figures (4) and (5) show the relationship between the safety index β and the normalized load σ_{xav}/σ_o of the plate panels when σ_{rx}/σ_o equals 0.1. Figures (6) and (7) show the same relationship of the plate panels when σ_{rx}/σ_o equals 0.2. The influence of the higher value of the residual stress on the safety index of the plate panels is obvious from these analyses.

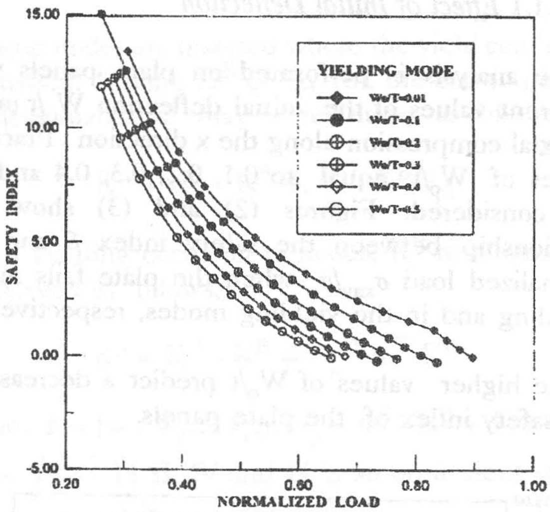

Figure 3. Relationship of safety index to normalized load under uniaxial loading (Yielding).

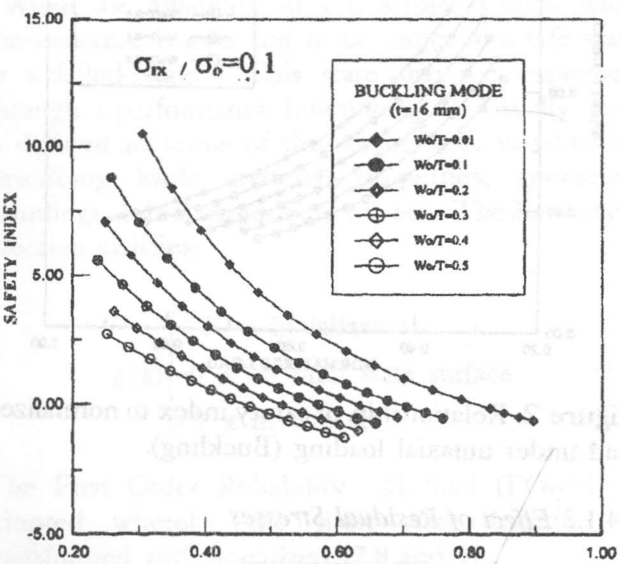

Figure 4. Relationship of safety index to normalized load under uniaxial loading (Buckling).

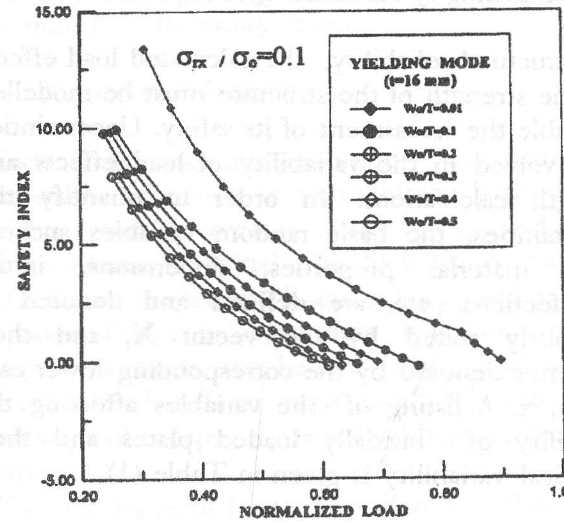

Figure 5. Relationship of safety index to normalized load under uniaxial loading (Yielding).

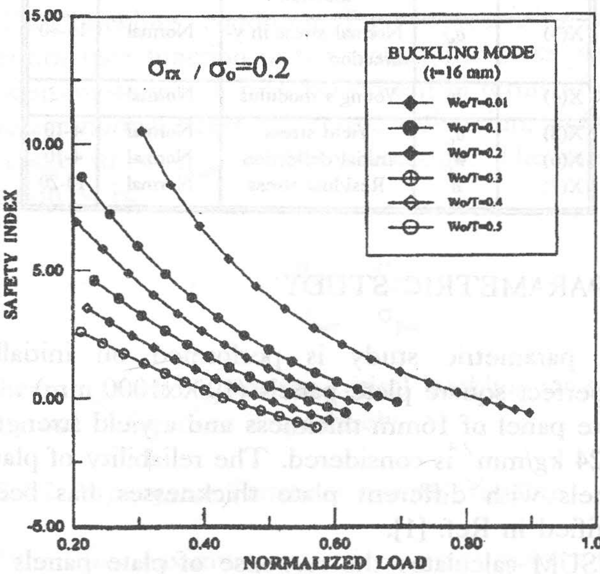

Figure 6. Relationship of safety index to normalized load under uniaxial loading (Buckling).

panels under biaxial compression loading has smaller values than that of initially imperfect plate panels under uniaxial compression load. Increasing the load in the y-direction will decrease the safety index.

4- It is essential when assessing the safety of plate panels of marine structures to include the effects of biaxial loading and initial imperfections such as initial deflection and residual stresses induced in the plate during fabrication.

It is to be noted that further studies should be carried out to include the effects of other types of loading such as shear loading and lateral load, and combined load effects.

REFERENCES

[1] H.W. Leheta and Y.A. Abdel-Nasser, "Reliability of ISUM Modelled Uniaxially Loaded Plates" *Alexandria Engineering Journal*, vol. 35, No. 1, Jan. 1996.

[2] Y. Ueda and T. Yao, "The Influence of Initial Deflection upon Compressive Ultimate Strength of Long Rectangular Plates" Transactions of JWRI, vol. 13, No. 1, 1984.

[3] Y. Ueda, S.M.H. Rashed and J.K. Paik, "Plate and Stiffened Plate Units of the Idealized Structural Unit Method (1st report)", *Jl. of Soc. of Naval Architects of Japan*, vol. 156, 1984 (in Japanese).

[4] Y. Ueda and T. Yao, "Ultimate Strength of Rectangular Plate under Thrust wi Consideration of the Effects of Initi Imperfections due to Welding" Transactions JWRI, vol. 8, No. 2, 1979.

[5] Y. Ueda, S.M.H. Rashed and Y. Abdel-Nasse "An Improved ISUM Rectangular Pla Element: Taking Account of Post-Ultimat Strength Behavior", *Jl. of Society of Nav Architects of Japan*, vol.171, 1992.

[6] Y. Ueda, S.M.H. Rashed and J.K. Paik, "A Incremental Galerkin Method For Plates An Stiffened Plates", *Computers & Structures*, vo 27, No.1, pp. 147-156, 1987.

[7] P. Thoft-Christensen and M. Baker, *Structur Reliability Theory and its Application* Springer-Verlag, Berlin, 1982.

[8] H.O. Madsen, S. Krenk and N.C. Lin *Methods of Structural Safety*, Prentice-Hall, N 1986.

[9] E. Nikolaidis and P. Kaplan, "Uncertainties i Stress Analysis on Marine Structures", Repo SSC-363, 1991.

[10] A.D. Kiureghian and P.-L. Liu, "Structura Reliability under Incomplete Probability Information", *ASCE, J. EMD*, vol. 112, No. January 1986.