DISCRETIZED MODEL OF DIESEL ENGINE INCLUDING SAMPLING EFFECT DUE TO FUEL INJECTION

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ABSTRACT

A linear time invariant model of marine Diesel engine is presented. A sampling effect due to the discontinuity of the fuel injection into the cylinders and short injection period is introduced. The proposed discrete model consists of a sampler and zero-order-hold mechanism, representing the fuel injection. The design of the discrete controller was based on the pole assignment of the characteristic polynomial of the discrete closed loop transfer function with the goal of achieving zero steady state error, and design specifications. The discrete model representation of the model process and the controller is recast in state space form. Performance was illustrated by simulation and design results for a two stroke Sulzer-engine of type 5RTA58.

Keywords: Digital Simulation, Diesel Engine Dynamics, Discrete Control, State Variable Technique.

NOMENCLATURE

- $A(z)$ Characteristic polynomial of the discrete open loop transfer function
- $\underline{A}_c$ System matrix of the digital controller
- $\underline{A}_s$ System matrix of the discrete control system
- a_i Coefficients of $A(z)$
- $B(z)$ Nominator polynomial of the discrete open loop transfer function
- $\underline{b}_c$ Control vector of the digital controller
- b_i Coefficients of $B(z)$
- $\underline{b}_s$ Control vector of the discrete control system
- $\underline{c}$ Output vector of the sampled process
- $\underline{c}_c$ Output vector of the digital controller
- $\underline{c}_s$ Output vector of the discrete control system
- c.v. Calorific value of fuel (kJ/kg)
- $D(z)$ Characteristic polynomial of the discrete closed loop transfer function
- d Scalar denoting transmission value of the sampled process
- d_c Scalar denoting transmission value of the digital controller
- d_s Scalar denoting transmission value of the discrete control system
- $E(z)$ z-Transform of error signal
- $e(k)$ Discretization of the error signal
- F_r Fuel pump rack position
- $G(s)$ Transfer function of the process with z.o.h
- $G_h(s)$ Transfer function of the zero-order-hold
- $G_p(s)$ Transfer function of the model process
- $G(z)$ Discrete transfer function of the sampled process
- $G_c(z)$ Discrete transfer function of the controller
- $G_w(z)$ Discrete transfer function of the closed loop control system
- $\underline{h}$ Control vector of the sampled process
- $\underline{I}$ Identity matrix
- J Equivalent moment of inertia of crank shaft and rotating parts ($kg.m^2$)
- K_c Controller gain
- K_l Propeller law constant
- K_s Steady state gain of the model process
- N Number of cylinders
- m_f Fuel mass flow rate (kg/s)
- $P(z)$ Denominator polynomial of the digital controller transfer function
- P_e Engine power (kW)
- P_l Propeller load (kW)
- p_i Coefficients of $P(z)$
- $\underline{q}(k)$ State vector of the sampled process
- $\underline{q}_c(k)$ State vector of the digital controller
- $\underline{q}_s(k)$ State vector of the discrete control system
- $R(z)$ Nominator polynomial of the digital controller transfer function

r_i	Coefficients of R(z)	
s	Laplace operator	
T	Sampling period	(s)
T_1, T_2	Time constants	(s)
U(s)	Laplace transform of input variable of the open loop system	
U(z)	z-Transform of the discrete input signal of the open loop system	
u(k)	Discretized input signal of the open loop system	
W(z)	z-Transform of the discrete reference value	
w(k)	Discretized desired signal	
X(s)	Laplace transform of output variable	
X(z)	z-Transform of the discrete output signal	
x(k)	Discretized output signal	
z{.}	z-Transformation of {.}	
z	Complex variable of z-transformation	
α_i	Coefficients of the closed loop characteristic polynomial D(z)	
Δ..	Small excursions of the variable from the equilibrium point	
Φ	System matrix of the sampled process	
η_{bth}	Engine thermal efficiency	
η_m	Mechanical efficiency of the engine	
η_{tr}	Transmission efficiency of the propeller shaft	
ω	Frequency	(rad/s)
ω_e	Shaft angular velocity	(rad/s)
ω_w	Transformed frequency	

INTRODUCTION

Automatic control systems, including controller design with fast response, have been widely in marine Diesel engine. The automatic control of engine and processes for the purpose of maintaining speed at a desired value has achieved a position of great importance in modern control technology. The degree of regulation of ship machinery is a natural result of economics optimization of the machinery configuration and operation.

One key factor is that machinery systems are becoming increasingly complex in the attempt to regulate the engine speed. The need for automatic control is obvious, both to optimize power, and enhance safety. In the area of marine power plant, many researches based on various control theories such as optimal control, adaptive control, state feedback control and modern control theories have been reported [1-4]. The utilization of control systems in the marine generating systems is rapidly increasing with the present trend of using computer for nearly every unit. Recent extensive theoretical studies incorporating the basic sampling action of the engine and subsequently produced practical techniques for identifying the dynamics of the engine should lead to much more systematic design methods for speed control. The major dynamic problems of running Diesel engines have, in many cases, been related to the operation of the governor. Therefore, Diesel engine dynamics have mainly been investigated by governor manufactures. In order to predict how a specific controller will behave, it is necessary to apply an engine model; including the physical relations on which the controller is based. The designing of such control systems is intimately connected to the concept of system modeling. Such model may be derived in an experimental or an analytical manner. Experimental approaches have been used based on identification procedures[5-10]. There is a number of alternative analytical methods to simulate Diesel engine behavior, ranging from complicated models that calculate details of internal processes such as that of the combustion processes or thermodynamic considerations [11-13], to less complicated dynamic models that can represent the engine by means of simple transfer function [4,14-16]. In the light of the many published papers on the Diesel engine, models based on continuous time techniques have been treated. These models are primarily intended for the analysis of the performance of governor controls. The main drawback of the above models is that the sample effect due to discontinuity of the fuel injection is not taken into account. Only a limited number of more technical papers and reports concerning the sampled data techniques, where these models are primarily intended for analysis of the performance of internal combustion engines exist [17-18]. This flows from the fact that the fuel pump virtually samples the position of the fuel rack in the short injection period. The fuel index does not directly influence the cylinder processes in the crank angle intervals when no injections are taking place [11, 16]. For this reason some of the effects known in sampled data systems may be found in Diesel engines. This means that the sample effect has to be taken into account. Furthermore, in recent years significant progress has been made in discrete-data and digital control systems. These systems have gained popularity and importance in all industries due in part to the advances made in digital computers, and more recently in microcomputers, as well as the advantages found in working with digital signals. A digital controller has the versatility that its control function can be easily modified by changing a few program instructions or even the entire program and a changing in instruction can be accomplished either manually or automatically under control of supervisory function. Also, as the electronic devices become cheaper and more reliable, it becomes worth considering their use for controller which, traditionally, have been hydraulic or mechanical. Current hydromechanical control units are of highly complex design and, in addition to being expensive, require a high degree of skill in their manufacture, assembly and maintenance to achieve the required performance and reliability.

In this paper, the goal is to examine the sample effect due to the fuel injection discontinuity on the controller design in discrete time.

SIMULATION MODEL

A mathematical model to simulate the transient behavior of marine Diesel engines has been developed for the purpose of evaluating various designs of control algorithms. For this purpose the simulation should be able to predict the engine rpm output, from the changes in fuel rack position on the input side.

The turbocharger responds normally slowly to changes in the operating point of the engine. This slow response of the turbocharger does not influence the engine [19]. This means that no significant difference was found between the complete description model including turbocharger and the simplified model used here when small changes were considered.

The simulation equations are grouped according to functional blocks which listed below. These blocks, which consist of the fuel pump, the engine cylinders, propulsion shaft and propeller are described by the following equations:

<u>Fuel pump:</u>

The fuel mass flow rate (m_f) injected into the engine in each cycle is controlled by the fuel rack position F_r. In addition, the fuel flow rate is dependent on the shaft speed ω_e. Hence

$$m_f = K_p . F_r . \omega_e \qquad (1)$$

The gradual pressure built up in the cylinder is reflected in a gradual change of the shaft torque. To accommodate the time required for the pressure built up inside the cylinder, a time constant may be introduced as

$$\tau_c = \frac{\alpha_v}{\omega_e} \qquad (2)$$

where α_v is the crank angle in radians corresponding to the pressure build up time. This angle is assumed to be 75^o [11].

<u>Engine cylinders:</u>

The engine may be modeled with the fuel flow rate as input variable and the effective power P_e as output variable. Considering constant calorific value of fuel (c.v.), then

$$P_e = \eta_{bth} . \eta_m . (c.v.) . m_f \qquad (3)$$

where the engine's running conditions and gas properties are included in η_{bth} and η_m.

<u>Propulsion shaft:</u>

The mathematical modeling of the propulsion shaft is based on the traditional differential equation of dynamics related to the balance of the powers. Assuming that the ship is moving straight-ahead, the crank shaft dynamics equation will be

$$\frac{d}{dt}\left(\frac{1}{2} J . \omega_e^2\right) = \eta_{tr} . P_e - P_l \qquad (4)$$

Propeller:

The load power absorbed by the propeller may be approximated by the propeller law [5,16]

$$P_l = K_l . \omega_e^3 \qquad (5)$$

Assuming constant brake thermal efficiency η_{bth}, using nominal values for F_r, m_f, P_e and ω_e and applying Laplace-transformation to the non-linear previous equations, the solution of the previous equations leads to a linear model, which has the process model transfer function [11]

$$G_p(s) = \frac{X(s)}{U(s)} = \frac{K_s}{(1+sT_1)(1+sT_2)} \qquad (6)$$

where the input variable U(s) is the change in the rack position, $\Delta F_r(s)$, and the output variable X(s) is the change in the shaft rpm, $\Delta n(s)$.
In general, the simplified model of Diesel engine is a second order system having two simple negative real poles.

DISCRETIZATION OF LINEAR TIME INVARIANT MODEL

In the dynamic condition, the engine will not respond to a change in the fuel index until an injection has occurred. This will point out to the presence of a sampling effect.

The sampling effect of the proposed model can be easily introduced by the use of a sampler and zero-order-hold mechanism. The sampler converts the fuel index signal, when an injection takes place, into a digital signal, and the hold device maintains this digital value as the gas in the cylinder delivers power to the crank shaft, i.e. for a prescribed time duration T. Because the fuel pump virtually samples the fuel rack position in the short injection period and the fuel index does not directly influence the cylinder processes in the crank angle intervals when no injections occur, the sampling period T may be taken equal to the firing interval

T = Firing interval
$= 2\pi / N.\omega_e$ for 2-stroke engine
$= 4\pi / N.\omega_e$ for 4-stroke engine.

The simplest form, which will be adopted in this study is shown in Figure (1). In general, the overall transfer function of the discrete system will be

$$G(s) = G_h(s).G_p(s) \qquad (7)$$

Since, the transfer function of the zero-order-hold is given by

$$G_h(s) = \frac{1-e^{-sT}}{s} \qquad (8)$$

The z-transformation of equation (7) will be

$$Z\{G(s)\} = Z\{\frac{1-e^{-sT}}{s}.G_p(s)\}$$

$$G(z) = (1-z^{-1}).Z\{G_p(s)/s\} \qquad (9)$$

Expanding $G_p(s)/s$ into the following form by partial-fraction expansion

$$\frac{G_p(s)}{s} = \frac{A_0}{s} + \sum_{i=1}^{l} \frac{A_i}{s+s_i}$$

where A_0 and A_i are residues of poles at $s = -s_i$, and $-s_i$ (i=1,2,...,l) are simple negative real poles. The z-transform of equation (9) is

$$G(z) = A_0 + (1-z^{-1}) \sum_{i=1}^{l} \frac{A_i}{z-e^{-s_i T}}.z \qquad (10)$$

which has simple positive real poles have to be inside the unit circle | z | = 1. This transfer function of equation (10) can be expressed as a quotient of two polynomials in z^{-1},

$$G(z)=\frac{X(z)}{U(z)}=\frac{b_0+b_1z^{-1}+...+b_mz^{-m}}{a_0+a_1z^{-1}+...+a_nz^{-n}},m\leq n \qquad (11)$$

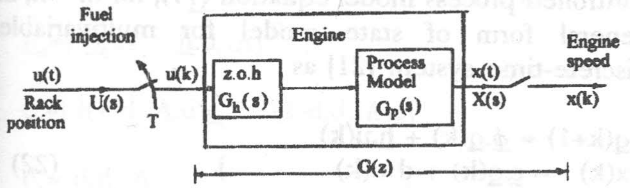

Figure 1. A digital with sampler-and hold.

DIRECT DISCRETE CONTROLLER DESIGN

The direct design methods are based on the basic requirements of finding a controller which achieves the desired closed-loop characteristics. The estimation of the controller parameters satisfying the desired dynamic response for the controlled system can be obtained by the pole assignment of the characteristic polynomial of the closed-loop transfer function. It has been a practical practice to attempt to cancel the undesirable poles and zeros of the controlled process transfer function by the poles and zeros of the controller, to satisfy the design specifications. Consider the discrete time feedback control system comprised of a controller and a plant as shown in Figure (2), with a digital controller located in the forward path.

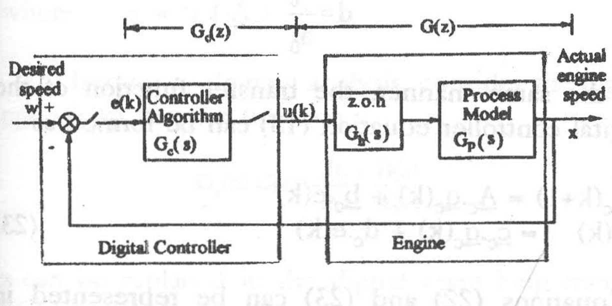

Figure 2. A digital control system with a digital controller.

Now we let in equation (11), $b_0 = 0$, $a_0 = 1$ and m=n, the sampled-data process with zero-order hold will have the form

$$G(z)=\frac{X(z)}{U(z)}=\frac{b_1z^{n-1}+...+b_n}{z^n+a_1z^{n-1}+...+a_n}=\frac{B(z)}{A(z)} \qquad (12)$$

In general, consider the controller's transfer function

$$G_c(z)=\frac{U(z)}{E(z)}=\frac{r_0+r_1z^{-1}+...+r_\nu z^{-\nu}}{1+p_1z^{-1}+...+p_\mu z^{-\mu}},\nu\leq\mu \qquad (13)$$

For $\nu=\mu=n-1$,

$$G_c(z)=\frac{U(z)}{E(z)}=\frac{r_0z^{n-1}+r_1z^{n-2}+...+r_{n-1}}{z^{n-1}+p_1z^{n-2}+...+p_{n-1}}=\frac{R(z)}{P(z)} \qquad (14)$$

According to Figure (2), the overall transfer function of the discrete control system has the characteristic polynomial

$$\begin{aligned}D(z) &= P(z).A(z) + R(z).B(z)\\ &=(z^{n-1}+p_1z^{n-2}+..+p_{n-1}).(z^n+a_1z^{n-1}+..+a_n)\\ &+(r_0z^{n-1}+r_1z^{n-2}+..+r_{n-1}).(b_1z^{n-1}+..+b_n)\end{aligned} \qquad (15)$$

In general form, it has the form

$$D(z)=\prod_{i=1}^{2n-1}(z-z_i)=\alpha_0+\alpha_1z+..+\alpha_{2n-2}z^{2n-2}+z^{2n-1} \qquad (16)$$

Comparing the coefficients of both equations (15) and (16) represents directly the linear system of equations given by

$$\begin{bmatrix} a_n & 0 & \cdots & 0 & b_n & 0 & \cdot\cdot & \cdot\cdot & 0\\ a_{n-1} & a_n & \cdots & 0 & b_{n-1} & b_n & \cdot\cdot & \cdot\cdot & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ a_3 & \vdots & \vdots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots\\ a_2 & \vdots & \vdots & a_n & b_2 & \vdots & \vdots & \vdots & 0\\ a_1 & a_2 & \vdots & a_{n-1} & b_1 & \vdots & \vdots & \vdots & b_n\\ 1 & \vdots & \vdots & \vdots & 0 & \vdots & \vdots & \vdots & b_{n-1}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & a_1 & \vdots & \vdots & \vdots & \vdots & b_2\\ 0 & \vdots & \vdots & 1 & 0 & \vdots & \vdots & \vdots & b_1 \end{bmatrix} \begin{bmatrix} p_{n-1}\\ p_{n-2}\\ \vdots\\ \vdots\\ p_1\\ r_{n-1}\\ \vdots\\ \vdots\\ r_1\\ r_0 \end{bmatrix} = \begin{bmatrix} \alpha_0\\ \alpha_1\\ \vdots\\ \vdots\\ \alpha_{n-2}\\ \alpha_{n-1}-a_n\\ \vdots\\ \vdots\\ \alpha_{2n-3}-a_2\\ \alpha_{2n-2}-a_1 \end{bmatrix} \qquad (17)$$

By pole assignment for the closed-loop transfer function, the coefficients $\alpha_i(i = 0,1,..,2n-2)$ can be obtained. Solving equation (17) will produce the controller parameters, $r_i(i = 0,1,..,n-1)$ and $p_i(i = 0,1,..,n-1)$.

The process of choosing the poles for the overall characteristic polynomial will be shown in details in the example below. It should be emphasized that the location of the poles will be determined so as to satisfy a predetermined damping ratio, as will be shown. In addition, one of the basic requirements of many control systems, zero steady state error in closed-loop, has to be taken into account. This means that the steady state error of the digital system can be expressed using the final value theorem

$$\lim_{k\to\infty} [e(k)] = 0$$

or in z-plane

$$\lim_{z\to 1} [(z-1).E(z)] = 0 \qquad (18)$$

The overall transfer function of the system will be

$$G_w(z)=\frac{X(z)}{W(z)}=\frac{G_c(z).G(z)}{1+G_c(z).G(z)} \qquad (19)$$

The error E(z) can be determined simply as

$$E(z) = W(z) - X(z)$$

$$E(z)=\frac{1}{1+G_c(z).G(z)}.W(z) \qquad (20)$$

For a unit step input, W(z)= z /(z-1). Substituting equation (20) into equation (18), we get

$$\frac{P(1)}{P(1)+R(1).G(1)}=0$$

Since G(1) is the steady state gain factor A_0 (as indicated in equation (10)), then P(1) has to be zero. In other words, the requirement of zero steady state error ensures that the controller has a pole at z =1 (an integrator property) [20], i.e.

$$G_c(z)=\frac{R(z)}{P(z)}=\frac{R(z)}{(z-1).\overline{P}(z)} \qquad (21)$$

The direct design method thus forces the controller to incorporate an integrator if required. The implementation of the closed-loop transfer function by a digital computer can be done by reforming the controlled process model equation (11), for m = n, in general form of state model for multivariable discrete-time system [21] as

$$\left.\begin{aligned} \underline{q}(k+1) &= \underline{\phi}.\underline{q}(k) + \underline{h}.u(k) \\ x(k) &= \underline{c}.\underline{q}(k) + d\,u(k) \end{aligned}\right\} \qquad (22)$$

where

$$\underline{\phi}=\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ \frac{a_n}{a_0} & \frac{a_{n-1}}{a_0} & \frac{a_{n-2}}{a_0} & \dots & \frac{a_1}{a_0} \end{bmatrix},$$

$$\underline{h}=[0\ 0\ 0\ \dots\ \frac{1}{a_0}]^T,$$

$$\underline{c}=[(b_n-\frac{b_0}{a_0}a_n)\ (b_{n-1}-\frac{b_0}{a_0}a_{n-1})\ \dots\ (b_1-\frac{b_0}{a_0}a_1)],$$

and

$$d=\frac{b_0}{a_0}$$

In the same manner, the transfer function of the digital controller equation (13) can be formed as

$$\left.\begin{aligned} \underline{q}_c(k+1) &= \underline{A}_c.\underline{q}_c(k) + \underline{b}_c.e(k) \\ u(k) &= \underline{c}_c.\underline{q}_c(k) + d_c.e(k) \end{aligned}\right\} \qquad (23)$$

Equations (22) and (23) can be represented in block diagram as shown in Figure (3).
Both the model process and controller equations can be easily introduced in one form of state space [22], whereas the output signal x(k) can be derived directly with respect to the reference signal w(k). The overall state space equations will be

$$\underline{q}_s(k+1) = \underline{A}_s.\underline{q}_s(k) + \underline{b}_s.w(k)$$

$$x(k) = \underline{c}_s.\underline{q}_s(k) + d_s.w(k) \quad \} \qquad (24)$$

where,

$$\underline{A}_s = \begin{bmatrix} (\underline{A}_c - \underline{b}_c.\Lambda.d.\underline{c}_c) & (-\underline{b}_c.\Lambda.\underline{c}) \\ (\underline{h}.\underline{c}_c - \underline{h}.d_c.\Lambda.d.\underline{c}_c) & (\underline{\phi} - \underline{h}.\Lambda.d_c.\underline{c}) \end{bmatrix},$$

$$\underline{b}_s = [\ \underline{b}_c.\Lambda \quad \underline{h}.d_c.\Lambda]^T \ ,$$

$$\underline{c}_s = [d(1-d_c.\Lambda.d)\underline{c}_c \quad (1-d.d_c.\Lambda)\underline{c}] \ ,$$

$$d_s = d.d_c.\Lambda \ ,$$

$$\Lambda = (1 + d.d_c)^{-1} \ ,$$

and $\underline{q}_s(k) = [\ \underline{q}(k) \quad \underline{q}_c(k)\ ]^T$.

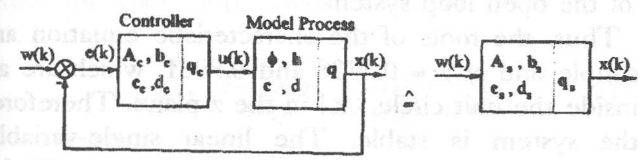

Figure 3. Representation of equations (22) and (23) in block-diagram form.

The solution of the linear time-invariant discrete state equations, can be carried out by taking the z-transform on both sides of equations(24), considering X(0) = 0 and solving for X(z), we get

$$X(z) = [\underline{c}_s.\underline{\psi}^{-1}.\underline{b}_s + d_s]\ W(z) \qquad (25)$$

where $\underline{\psi} = (z.\underline{I} - \underline{A}_s)$

For frequency domain analysis, consider the digital open loop transfer function

$$G_c(z).G(z) = \frac{R(z)}{P(z)}.\frac{B(z)}{A(z)} \ , \qquad (26)$$

z can be replaced in the digital open loop transfer function by the relation

$$z = \frac{1 + j.\omega_w}{1 - j.\omega_w} \qquad (27)$$

which is obtained through the bilinear transformation of the interior of the unit circle onto the left half of a complex variable plane [23], where

$$\omega_w = \tan[\frac{\omega.T}{2}]$$

which establishes the relation between the frequency ω and the transformed frequency ω_w. Once the expression of $G_cG(j.\omega_w)$ is obtained from equations (26) and (27), the magnitude and phase of $G_cG(j.\omega_w)$ can be plotted by varying ω_w between zero and infinity introducing the bode diagram.

IMPLEMENTATION AND RESULTS

1. Simulation Model

The present investigation will be applied to model a two stroke Sulzer 5RTA58 marine Diesel engine running at 115 rpm rated for 5.963 MW at 73 % part load with 0.2958 kg/s fuel rate and 68504.8 kg.m^2 equivalent moment of inertia of crank shaft and rotating parts. The transmission efficiency is assumed to be 98 %. With the previous nominal values, the model can be linearized for small changes of fuel rack position and engine speed as mentioned through equations (1) to (5) as follows:

$$\Delta m_f = 0.4052\ \Delta F_r + 0.0246\ \Delta\omega_e$$

$$\tau_c = 0.1087$$

$$\Delta P_e = 20.1589 * 10^6.\Delta m_f$$

$$\Delta\omega_e = 1.1879 * 10^{-6}.\Delta P_e - 1.2121 * 10^{-6}.\Delta P_l$$

$$\Delta P_l = 1.4558 * 10^6.\Delta\omega_e$$

From equation (6), the transfer function will be

$$G_p(s) = \frac{X(s)}{U(s)} = \frac{78.767}{(1+0.1013s)(1+0.9118s)} \qquad (28)$$

The open-loop poles of the system are -9.8717 and -1.0967. Since they lie on the negative part of the real axis in the s-domain, the system is consequently stable and over-damped. The system has a steady state gain of 78.767. The transient response of the speed change with respect to a unit step change in the fuel rack position as input signal is shown in Figure (4). The response time is normally defined in terms of the rise time which here equals 2.2 seconds and with the settling time which is 3.6 seconds.

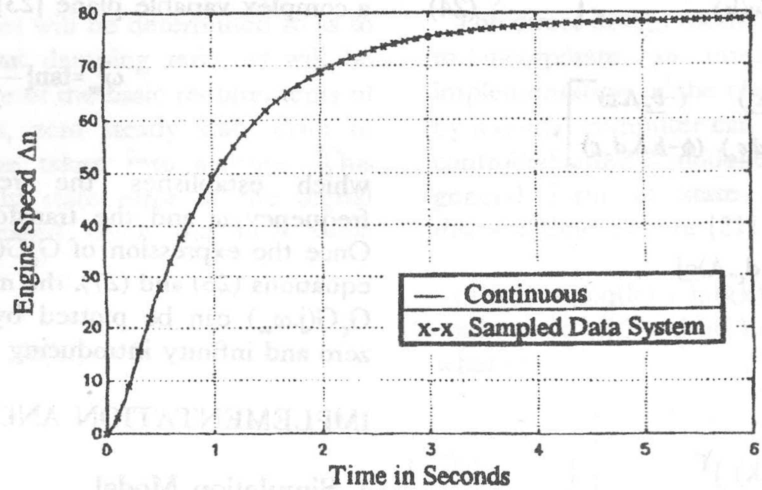

Figure 4. Unit step response of the open loop system.

2. Model Discretization

This model can be described easily in linear discrete-time form by the use of a sampler and zero-order-hold mechanism as stated in equation (10) as

$$G(z)=\frac{X(z)}{U(z)}$$

$$= 78.767+(1-z^{-1})\left[\frac{9.8494z}{z-e^{-9.8678T}}-\frac{88.6163z}{z-e^{-1.0968T}}\right]$$

Let the sampling period T be nearly equal to the firing interval

$$T = 0.1 \quad [s]$$

The transfer function in z-domain can be expressed as a quotient of two polynomials in z^{-1} as

$$G(z)=\frac{X(z)}{U(z)}=\frac{3.0274z^{-1}+2.1046z^{-2}}{1-1.2689z^{-1}+0.3341z^{-2}} \quad (29)$$

or

$$G(z)=\frac{X(z)}{U(z)}=\frac{3.0274(z+0.6952)}{(z-0.3728)(z-0.8961)}$$

Thus, the roots of the characteristic equation are simple and at z = 0.3728 and 0.8961, which are all inside the unit circle | z | in the z-plane. Therefore, the system is stable. The linear single-variable digital system is described using equation (29) by the difference equation

$$x(k)-1.2689\ x(k-1)+0.3341\ x(k-2)$$
$$= 3.0274\ u(k-1)+2.1046\ u(k-2).$$

This is a recursion relation from which the response of the system for k = 0,1,2,.... can be computed. The unit step response of the digital system is shown in Figure (4), which is identical to that for continuous system response in the given points at t = kT. Also, it is shown that the system has a steady state value of 78.767.

It is to be noted that as the sampling period approaches zero, the response of the digital control system approaches that of the continuous data system.

3. Controller Design

Since the sampled data process with zero-order-hold has the transfer function of second order which is stated in equation (29), the desired controller will have a transfer function of first order as

$$G_c(z)=\frac{r_0+r_1z^{-1}}{1+p_1z^{-1}}=K_c\frac{z+\overline{z_1}}{z+p_1} \quad (30)$$

where K_c is the controller gain ,and z_1 is a controller zero.

To obtain zero static error, as mentioned after equation (21), the controller has to have an integrator property. This means that $p_1 = -1$. The open-loop system now has an extra pole at $z = -p_1 = 1$ corresponding to the integral action of the controller, and in addition a zero at $z = -r_1/r_0$. This new zero can be placed anywhere along the real axis by simply choosing the value $-r_1/r_0$ used in the control program. Since the simple design method calls for a controller which cancels the undesired plant dynamics, the controller zero will be placed at

$$\overline{z_1} = -0.8961$$

Now, the open loop transfer function will be

$$G_cG(z) = K_c \frac{(z-0.8961)}{(z-1)} \cdot \frac{3.0274(z+0.6952)}{(z-0.3728)(z-0.8961)} \quad (31)$$

From the root locus view point, Figure (5) shows how the system open loop poles are moved to desirable closed loop locations as the controller gain is increased. Referring to Jury contours lines of constant damping ratio on the z-plane [24], the intersection of the root locus with the contours lines of the desired damping ratio value of $\zeta = 0.7$ on the z-plane presents $K_c = 0.04086$ and a pole pair of

$$z_{1,2} = 0.625 \pm j0.262$$

Since the overall control system will have 3 poles z_1, z_2 and z_3 which represent the characteristic polynomial as

$$D(z) = \prod_{i=1}^{3} (z - z_i) = \alpha_0 + \alpha_1 z + \alpha_2 z^2 + z^3 \quad (32)$$

substituting for the parameters
$b_1 = 3.0274$ $b_2 = 2.1046$
$a_1 = -1.2689$ $a_2 = 0.3341$
in the matrix equation (17), it gives

$$\begin{bmatrix} 0.3341 & 2.1046 & 0 \\ -1.2689 & 3.0274 & 2.1046 \\ 1 & 0 & 3.0274 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ r_1 \\ r_0 \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \alpha_1 - 0.3341 \\ \alpha_2 + 1.2689 \end{bmatrix} \quad (33)$$

The coefficients of the characteristic polynomial of equation (32) will be

$$\alpha_0 = -0.4593\ .z_3, \quad \alpha_1 = -0.4593 + 1.25\ .z_3$$
$$\alpha_2 = -(1.25 + z_3)$$

Solving the matrix equation (33) will give $z_3 = 0.8961$, and the controller parameters are:

$$r_0 = 0.04086 \quad \text{and} \quad r_1 = -0.0366$$

Thus the desired controller has the transfer function

$$G_c(z) = \frac{U(z)}{E(z)} = \frac{0.04086 - 0.0366 z^{-1}}{1 - z^{-1}} \quad (34)$$

In form of difference equation , it will be
$u(k) - u(k-1) = 0.04086\ e(k) - 0.0366\ e(k-1)$

or $u(k+1) - u(k) = 0.04086\ e(k+1) - 0.0366\ e(k)$
Now let, $q_c(k) = u(k) - 0.04086\ e(k)$
or. $q_c(k+1) = u(k+1) - 0.04086\ e(k+1)$
i.e. $q_c(k+1) = u(k) - 0.0366\ e(k)$
Thus the controller transfer function can be given as

$$\left.\begin{aligned} q_c(k+1) &= q_c(k) + 4.26 \times 10^{-3}\ e(k) \\ u(k) &= q_c(k) + 0.04086\ e(k) \end{aligned}\right\} \quad (35)$$

which have the form of equation (23).
Also the state space representation for the transfer function stated in equation (29) can be obtained as stated in equation (22) as

$$\begin{bmatrix} q_1(k+1) \\ q_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.3341 & 1.2689 \end{bmatrix} \begin{bmatrix} q_1(k) \\ q_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$x(k) = [2.1046 \quad 3.0274] \begin{bmatrix} q_1(k) \\ q_2(k) \end{bmatrix} \quad (36)$$

Substituting equations (35) and (36) in equations (24), we get

$$\underline{q}_s(k+1) = \begin{bmatrix} 1 & -8.97 \cdot 10^{-3} & -0.0129 \\ 0 & 0 & 1 \\ 1 & -0.4201 & 1.1452 \end{bmatrix} \underline{q}_s(K) + \begin{bmatrix} 4.26 \cdot 10^{-3} \\ 0 \\ 0.04086 \end{bmatrix} w(k) \quad (37)$$

$$x(k) = [0 \quad 2.1046 \quad 3.0274]\, \underline{q}_s(k)$$

where $\underline{q}_s(k) = [q_1(k) \quad q_2(k) \quad q_c(k)]^T$

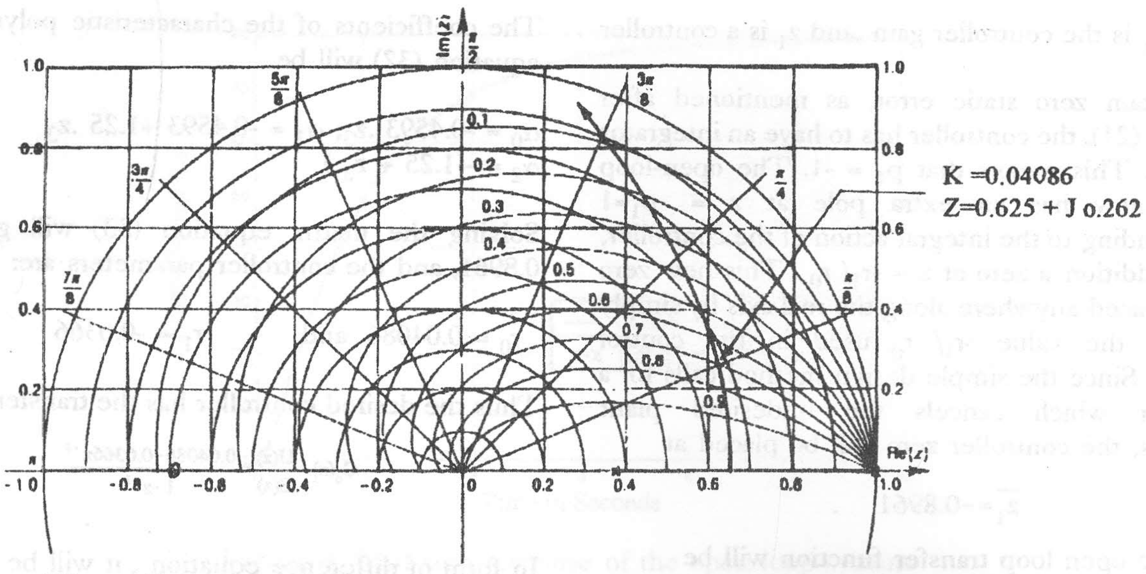

Figure 5. Root-Loci of the digital system with the jury contours lines of constant damping ratio (ζ) on the Z-plan

Equation (25) gives the solution of the previous state space equations for the output speed of the engine X(z) with respect to the reference value W(z), which will be

$$x(z) = \frac{0.12367z^2 - 0.02485z - 0.07704}{z^3 - 2.14523z^2 + 1.57815z - 0.41114} W(z) \tag{38}$$

Figure (6) shows the simulated step response of the closed-loop discrete time system obtained with the designed controller. The system shows a slight overshoot. With the controller as described by equation (34), the unit step response has a maximum overshoot of about 5 %, although achieving the relative damping ratio of $\zeta = 0.7$. It is to be noted that the response is fast although the existing of the overshoot.

Thus, the digital engine control system is capable of giving the desired speed value after a short time of about 1.3 seconds without any steady state error.

Figure (7) illustrates the Bode diagram of $G_c.G(j\omega_w)$ of equation (31) for the value of $K_c = 0.04086$. From Figure (7), the gain margin is about 17.25 db at the transformed frequency of $\omega_w = 0.78$, and the phase margin is nearly 63° at transformed frequency of $\omega_w = 0.16$.

CONCLUSION

The engine dynamics simulating the engine rpm output due to a change in fuel rack position on the input side can be modeled by means of a simple transfer function, taking into account time delay corresponding to the pressure build up in the cylinders. The discretization of this model can be easily introduced by the use of a sampler and zero-order-hold mechanism. The sampler converts the fuel index signal when an injection takes place into a digital signal, and the hold device maintains this digital value as the gas in the cylinder delivers power to the crank shaft. This time interval which is the sampling duration may be taken equal to the firing interval. The digital controller can be designed directly by the pole assignment of the discrete closed loop characteristic polynomial. The pole assignment is dependent upon the requirement of obtaining zero steady state error and achieving system specifications. This has been done by the cancellation the undesirable poles and zeros of the controlled process transfer function by poles and zeros of the controller, and new open loop poles and zeros are added at more advantageous locations to satisfy the design specifications. The new poles are assigned at the intersection between the root loci and the constant damping locus in the z-plane for desired damping value to find a digital controller which realizes the required damping. The closed loop control system has been reformed in state space form. The digital control system of a two stroke Sulzer engine of type 5RTA58 has been illustrated and analyzed with respect to unit step response in time domain and frequency domain. The obtained resulting performance gives reasonable values of system specifications.

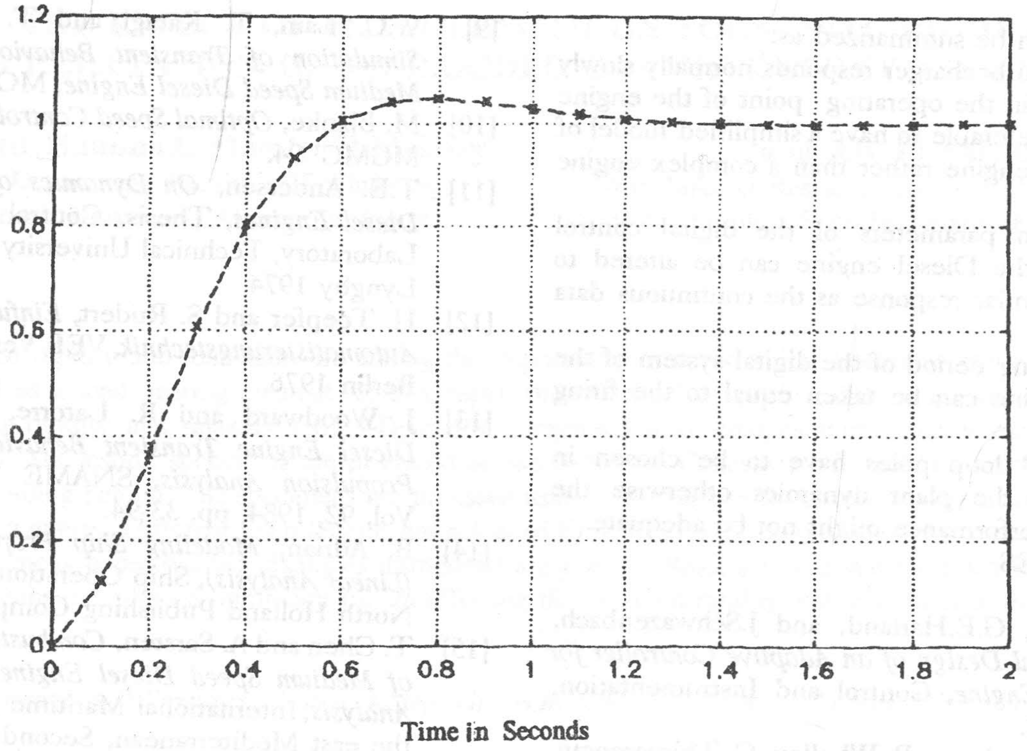

Figure 6. Unit step response of closed loop digital system.

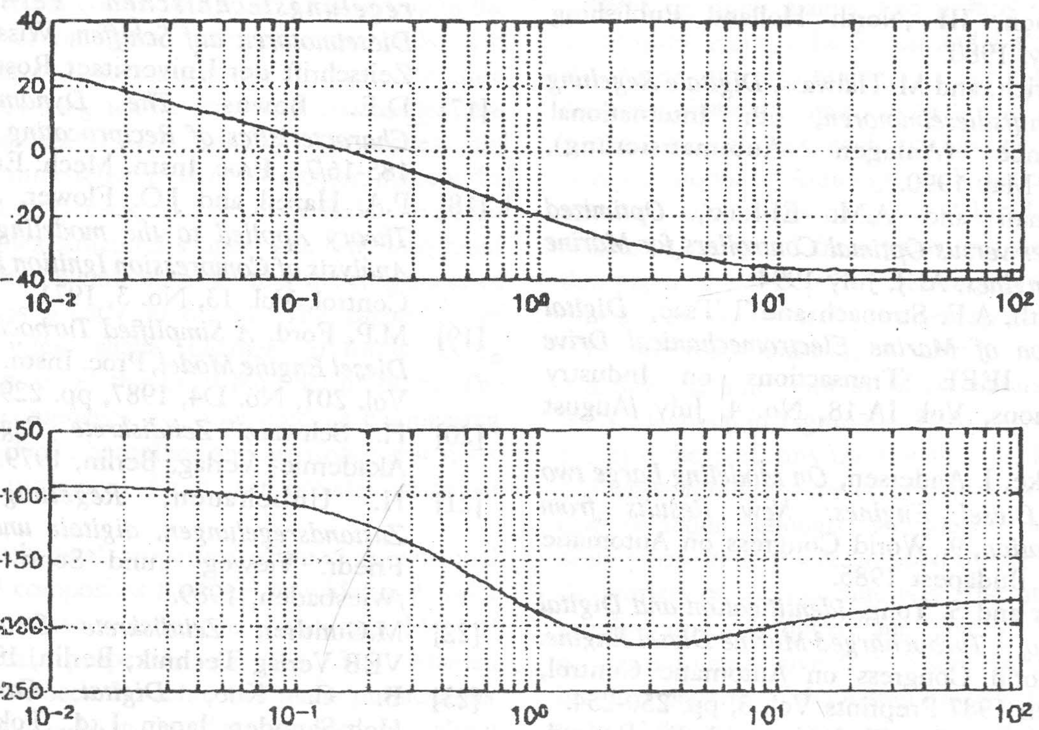

Figure 7. Bode diagram of the digital system.

The results can be summarized as:

- Since, the turbocharger responds normally slowly to changes in the operating point of the engine [19], it is preferable to have a simplified model of the Diesel engine rather than a complex engine model.
- The system parameters of the digital control system of the Diesel engine can be altered to render a similar response as the continuous data system.
- The sampling period of the digital system of the Diesel engine can be taken equal to the firing interval.
- The closed loop poles have to be chosen in relation to the plant dynamics otherwise the resulting performance might not be adequate.

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