COMPUTER ANALYSIS OF INDETERMINATE FRAMES LOADED NORMAL TO THEIR PLANE USING THE METHOD OF VIRTUAL WORK

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INTRODUCTION

The auther has previously presented a computer program to solve indeterminate frames loaded in their planes using the well known method of "VIRTUAL WORK". The method can be used to all types of loads, settlements of supports, temperature changes, and several cases of loadings, acting simultanuouslg. [1]. This paper gives the solution when loads are normal to the plane of the structure. It is assumed that this paper uses all principles, and sign conventions presented before, which will be briefly given later.

ASSUMPTIONS

The assumptions adopted here will be.

1. The members of the frame are considered each as two members when using the previous program "VW".

The first of these members, with two various end moments will resist the developing moments in the frame and the second with two equal twisting moments to resist the internal torsional moments in the frame.

2. The elastic lengthes for these two members will be:
   a. For first members resisting the bending moments $L1(1) = (L/EI)\ (1)$
   b. For second members resisting the torsional moments $TL1(I) = (L/EI)*T1(I)$

where

$T1 = (EI/GJ)$
i.e
$TL1 = L1*T1$

Hence, in the computer program "VWN" presented in this paper, n = 4 x no. Of members in the frame.

MORE ABOUT SIGN CONVENTION

In addition to program 'VW"[1], notice that:

1. The bending moments developing in the member will act in a plane normal to that of the frame.

Notice that all moments acting above the plane of the frame will be positive.

If we rotate these bending moment diagrams to become drawn in the plane of the frame on the left of the vertical members and on top of all other members, horizontal or inclined, they will become negative moments.

These moments are produced by moment vectors normal to the axis of the member. Moment vectors acting on the right end of the member, (looking from inside)produce moments on the other side of the member, i.e, negative moments.

On the other hand, the torsion in the members is produced from moment vector along the axis of the member. If these vectors appear as if giving tension in the member, the resulting torsion will be positive. More details will be shown in the following examples.

RESOLUTION OF MOMENT VECTORS

Figure (1) shows member CD, with resultant action at end D consisting of moment vector V, making an angle ϕ w.r.t the member, together with a concentrated force P normal to the plane downwards. The resolution of V gives:

a- Moment vector, normal to CD; $M = V \sin \phi$
b- Torsion vector, along CD; $T = V \cos \phi$

Now M is to the right of CD, hence the rotated B.M. Diagram appears on the other side of the member, i.e.; negative moment.

Due to the presence of P, normal to the plane, (downwards) the bending moment at the far end to the left, will be:

$$M1 = M + P * L$$

The vector T, giving torsion, appears axial to the member, producing axial tensile force, hence gives positive torsion.

Figure (2) shows a member EF to the right where M = V sin ϕ, and T = - V cos ϕ

The B.M. at E will be; M = V sin ϕ, and at the far end M1 = M + P * L, all negative. The application of the method is clarified by following examples.

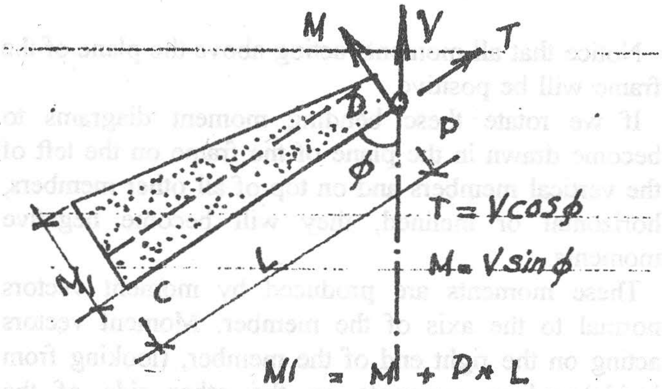


Fig. (1)
Resolution of end vectors. (A) (Member CD)

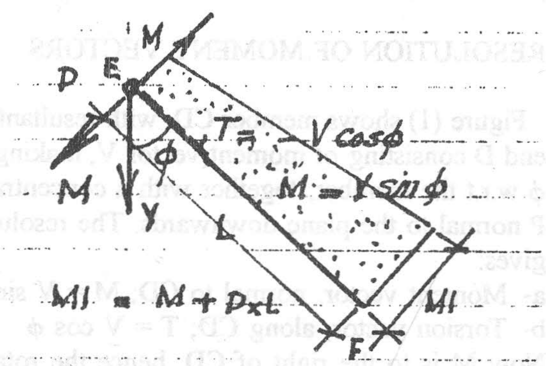


Fig. (2)
Resolution of end vectors (B) (Member EF)

Step of Solution

The main programs are first copied onto the disckets used in the solution. These programs are:

1. Appendix (1) : "VWNM"
2. Appendix (2) : "DATVWNM"

Next

1. Load the data VWNM program . [write F1 "DATVWNM"]
2. Fill in the necessary data in the diagram.
3. Save the diagram as : F4/ DATVWN4M; 4 being any chosen number.
4. Add [MERGE "VWNM"] when (OK), run the program as (F2); to get the final requirement which askes whether you nead the moments and torsion produced on the screen (1), or printer (2), or not (3). chose (1), then (2), then (3).

Direction of Sight

As previously given "VW"(1), we shall look to vertical members from left, and to other members upwards.

EXAMPLES

Example (1)

The data for example (1) is shown in program "DATVWN_4M". In this data program we see the following remarks.

In 3010, notice that the value of N equals 4 x no of members = 4 * 4 = 16

In 3030 the no of elastic lengthes = 2 * no of member = 2 * 4 = 8. For each member we give two elastic lengths, the second of which corresponds to the member resisting torsion for which L1 = L1 * T1.

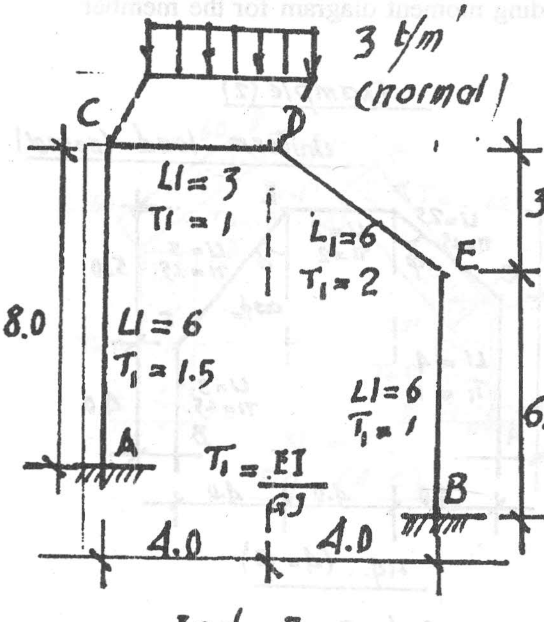
In 3040, we see the values of the moments, then the torsion in each members due to the given loads on the statically determinate frame obtained by producing a cut section in the second member in the frame. [Figure (3b)].

In this diagram the bending moments are drawn as shown. Posite moments are drawn to the right of vertical members, and on bottom of all other member.

In 3080 we show the additional average ordinates of the bending moments. $A = pL^2/12$ for uniform loads These values appear only in bending moments, while the values are zero for torsions. For this example, A = 3 * 16/12 = A in member O.

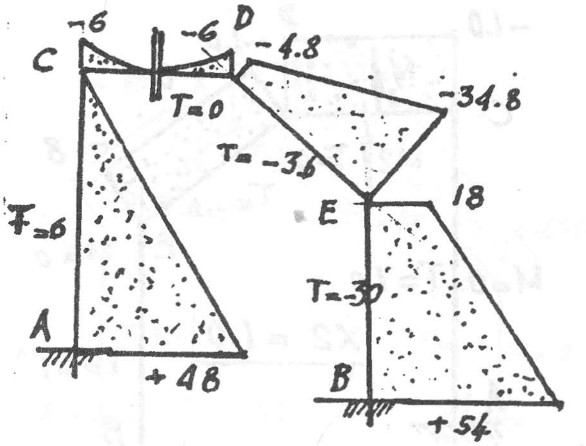
Lines 3250, 3270 refer only to the cases of settlements of supports and temperature changes or ties and hence will be zero in this example, the member equals to the R value for each line.

Example (1)



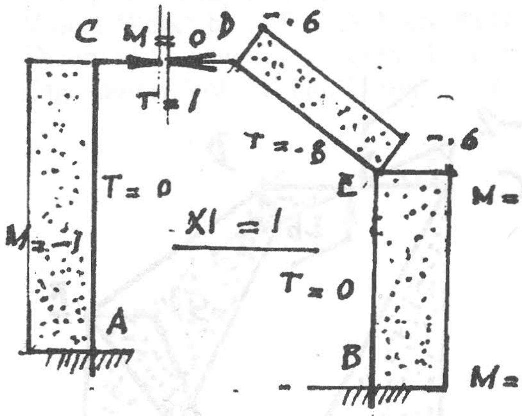
Ind. Frame

Fig. (3.a)



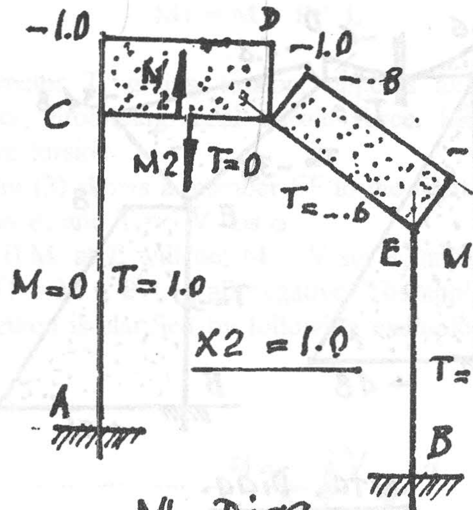
MTO. Diag.

Fig. (3.b)



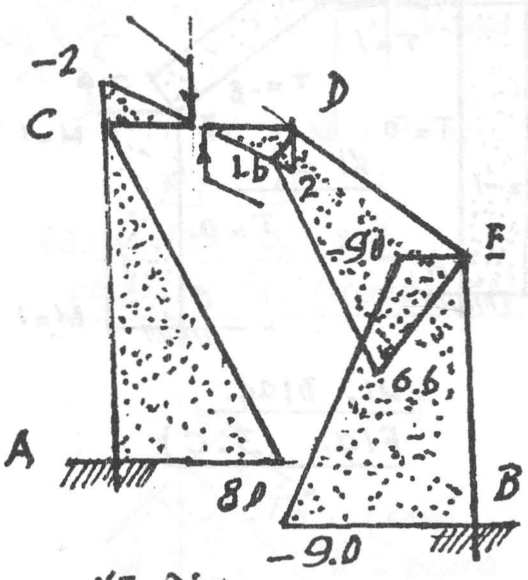
MI. Diag.

Fig. (3-c)



MI. Diag.

Fig.(3.d)



X3. Diag.

MI- Diag.

Fig.(3_e)

Example (2)

This is shown in Figures (4a,b,c,d and e). Figures (1) and (2) will be useful when computing the MT0 diagrams.

We remember the following remarks:

1. The uniform load normal to each member must be considered when evaluating the M_{To} values at the corners. On any member the uniform load will give a concentrated load at the mid-point equal to pL, and hence an end moment equal to $pL^2/2$.
2. On each member there will be an additional average value equal to $pL^2/12$ with opposite sign to the bending moment diagram for the member

Example (2)

Uniform load (normal)

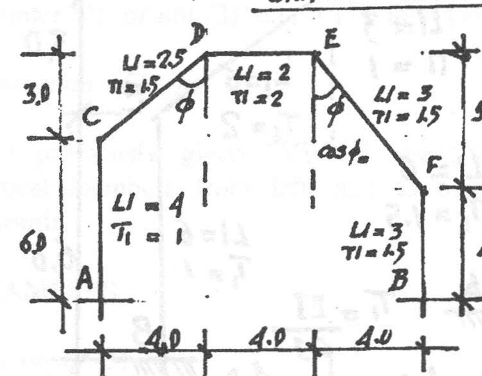
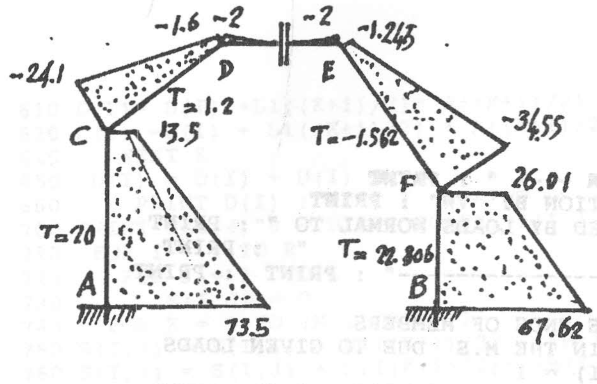


Fig. (4-a)

Ind. Frame

p = 1 t/m'



MTO. DIAG.
Fig. (4-b)

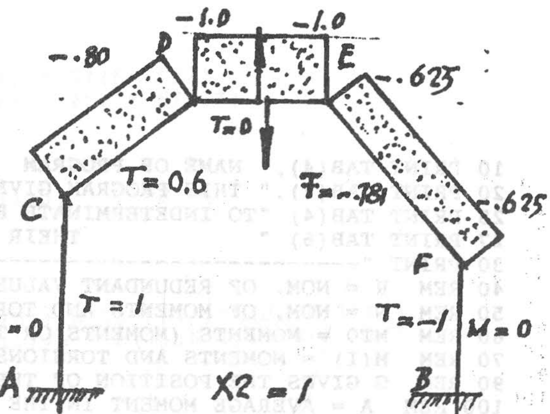


Fig. (4-d)

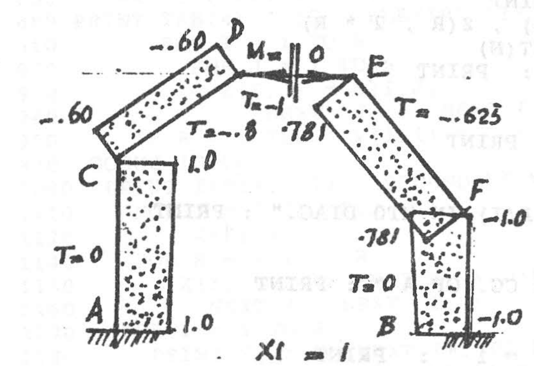


Fig. (4.c)

REFERENCES

[1] "Computer Analysis Applied To the Method of Virtual Work" by A.F.S. DIWAN Ph.D. Prof. of Theory of Structures. Alexandria University Alex. Eng. Journal Vol. 31, No. 2, April, 1992.

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10 PRINT TAB(4)," NAME OF PROGRAM  ----    VWMM ---- " : PRINT
20 PRINT TAB(4)," THIS PROGRAM GIVES THE SOLUTION BY  VW" : PRINT
25 PRINT TAB(4) "TO INDETERMINATE FRAMES LOADED BY LOADS NORMAL TO " : PRINT
27 PRINT TAB(6) "                  THEIR PLANES                   "  : PRINT
30 PRINT "--------------------------------------------------" : PRINT  : PRINT
40 REM  R = NOM. OF REDUNDANT VALUES
50 REM  N = NOM. OF MOMENTS AND TORSION AT THE ENDS OF MEMBERS
60 REM  MTO = MOMENTS (MOMENTS OR TORSIONS ) IN THE M.S  DUE TO GIVEN LOADS
70 REM  M(I) = MOMENTS AND TORSIONS DUE TO X(I) = 1
90 REM  G GIVES THE POSITION OF THE C.G. OF ADDITIONAL MO DIAG.
100 REM  A = AVERAGE MOMENT IN THE ADDITIONAL MTO DIAG.
110 REM  D(I) = DISPLACEMENT COEFS. FROM MTO & M(I) DIAGS.
120 REM  S(I,J) = .................... FROM APPROPRIATE M(I,J) DIAGS.
140 REM T(I) = ADDITIONAL VALUE TO S(I,J) DUE TO TIE IF ANY
150 REM U(I) = ..................... D(I)  DUE TO SETTLEMENT IF ANY
180   READ R , N
190  PRINT " R = " , R  : PRINT " N = " , N  : PRINT
210        DIM L1(N/2) , MTO(N) , A(N/2) , G(N/2) , Z(R , 2 * R)
220        DIM D(R) , M(R,N) , S(R,R), C(R,R) , MT(N)
240 PRINT  TAB(4)" 1 - ELASTIC LENGTHES L1(I) "  :  PRINT
250       FOR I = 1 TO (N /2)
260 READ L1(I) : PRINT L1(I),: NEXT I : PRINT
280  PRINT TAB(4)" 2 - MOMENTS MTO IN  M.S. "  :  PRINT
290    FOR I = 1 TO N
300 READ MTO(I) : PRINT  MTO(I),: NEXT  I : PRINT
320  PRINT TAB(4) " 3 - AVERAGE ADDITIONAL AREAS A(I) IN MTO DIAG." : PRINT
330  FOR I = 1 TO N / 2
340  READ A(I): PRINT A(I),:NEXT I : PRINT
360 PRINT TAB(4)" 4 - POSITION OF CENTROID  G FOR CG. OF A " : PRINT
370  FOR  I = 1 TO N / 2
380  READ G(I): PRINT  G(I),: NEXT  I : PRINT
400 PRINT  TAB(4)" 5 - MOMENTS M(I,J) DUE TO M(I) = 1 " :  PRINT
410   FOR I = 1 TO R
420     FOR J = 1 TO N
430   READ M(I,J) : PRINT M(I,J), : NEXT J
440    PRINT  : NEXT  I : PRINT
460 PRINT TAB(4)  " 6 - ADDITIONAL DISP. COEF. U(I) DUE TO SETTLEMENT " : PRINT
470               FOR I = 1 TO R
480  READ U(I) : PRINT  U(I),: NEXT I : PRINT
500 PRINT TAB(4)  " 7 - ADDITIONAL DISP. COEF. T(I) DUE TO TIES       " : PRINT
510   FOR I = 1 TO R
520     READ T(I) : PRINT  T(I),: NEXT I : PRINT
550 PRINT TAB(6) " 8 - DISP. COIFS. D(I) DUE TO MTO , M DIAG.":   PRINT
560   FOR I = 1 TO R
570    D(I) = 0
580     FOR K = 1 TO (N - 1)  STEP 2
590   D(I) = D(I) + L1((K+1)/2) * ( MTO(K)*M(I,K) /3 +  MTO(K+1) * M(I,(K+1))/
)
600 D(I)= D(I) +L1((K+1)/2) *(MTO(K)* M(I,(K+1))/6  + MTO(K+1)* M(I,K )/6)
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610 D(I)= D(I) +L1((K+1)/2)* A((K+1)/2) * M(I,(K+1)) * G((K+1)/2)
620 D(I) =D(I) + L1((K+1)/2) * A((K+1)/2) * (1-G((K+1)/2)) * M(I,K)
640     NEXT K
650  D(I) = D(I) + U(I)
660     PRINT D(I) : NEXT I : PRINT
700 PRINT TAB(4)"9 - DISP. COEFS. S(I,J)  DUE TO M(I,J) " : PRINT
710  FOR I = 1 TO R
720    FOR J = 1  TO R
730       S(I,J) = 0
740   FOR K = 1 TO (N-1) STEP 2
750 S(I,J) = S(I,J) + L1((K+1)/2)  *(M(I,K)*M(J,K)/3 + M(I,(K+1))*M(J,(K+1))/3
760 S(I,J) = S(I,J) + L1((K+1)/2)* (M(I,K)*M(J,(K+1))/6 + M(I,(K+1))* M(J,K)/6
770     NEXT K
780      IF J<>I THEN GOTO 800
790   S(I,J) =  S(I,J) + T(I)
800    NEXT J   :  NEXT I
820   FOR I = 1 TO R
830    FOR J = 1 TO R
840  PRINT S(I,J), : NEXT  J : PRINT
850   NEXT I : PRINT  : PRINT
880 PRINT TAB(4) " 10 - FLEXIBILITY MATRIX V = INVERS OF S " :  PRINT
910       FOR I = 1 TO R
920         FOR J = 1 TO R
930           Z(I,J) = S(I,J)
940              NEXT  J  :  NEXT I : PRINT
950     IF R = 1 THEN  V(1,1) = 1 / S(1,1)  : PRINT V(1,1) :GOTO 1000
980  GOSUB 10000
1000  PRINT TAB(4)" 11 - REDUNDANT VALUES X (R) " : PRINT
1110    FOR I = 1 TO  R
1120        X(I) = 0
1140    FOR K = 1 TO    R
1150       X(I) = X(I) -V(I,K) * D(K)
1160         NEXT K : NEXT  I
1170   FOR I = 1 TO R
1180     PRINT X(I)   : NEXT I : PRINT
1200     PRINT TAB(4)" 12 - FINAL B.MOMENTS MT(I) " : PRINT
1210  FOR I = 1 TO N
1220         MT(I) = 0
1230    FOR J = 1 TO R
1240     MT(I) = MT(I)  + X(J) * M(J,I)
1250         NEXT  J
1260     MT(I) = MT(I) + MT0(I)
1270      NEXT  I
1272 INPUT "YOU WANT SCREEN (1) OR PRINTER (2) , OR END (3) " , SP
1274    ON SP GOTO 1280 , 1360 , 1500
1280  FOR I = 1 TO (N-1) STEP 2
1290   PRINT    MT(I) , MT(I+1)
1310     NEXT I : PRINT
1320   PRINT TAB(4) " THIS IS ALL NOW " : PRINT
1340      GOTO 1272
1360  FOR I = 1 TO (N-1)  STEP 2
1370   LPRINT   MT(I) , MT(I+1)
1380    NEXT I   :  LPRINT
1400   LPRINT TAB(4) " THAT  IS  ALL  NOW " :   LPRINT
1500       END
```

```
10000      FOR I = 1 TO R
10010        FOR J = (R+1) TO  2 * R
10020         Z(I,J) = 0
10030           NEXT J
10035       Z(I , R+I) = 1
10040           NEXT I
10050      P = 0
10060         P = P + 1
10070      C(P,P) = Z(P,P)
10080        FOR     J = 1 TO (2 * R)
10090      Z(P,J) = Z(P,J) / C(P,P)
10100          NEXT J
10110      FOR I = (P + 1)  TO  R
10120         C(I,P) = Z(I,P)
10130      FOR J = P  TO 2 * R
10140       Z(I,J) = Z(I,J) - C(I,P) * Z(P,J)
10150             NEXT J
10160               NEXT I
10170            IF P = 1  THEN 10060
10180       FOR I = 1 TO  (P-1)
10190         C(I,P) = Z(I,P)
10200      FOR J = P TO (2 * R)
10210       Z(I,J) = Z(I,J) - C(I,P) * Z(P,J)
10220          NEXT  J
10230            NEXT I
10240      IF P = R THEN GOTO 10260
10250       GOTO  10060
10260    FOR I = 1 TO R
10270      FOR J = 1 TO R
10280      V(I,J) = Z(I,R+J)
10290   PRINT V(I,J), : NEXT J
10300       PRINT   : NEXT  I : PRINT
10320          RETURN
```

DATVWN M

```
3000 PRINT " ENTER DATA FOR  R , N "
3020 PRINT " ENTER DATA FOR ELASTIC LENGTHES L(I) "
3040 PRINT " ENTER DATA B.MOMENTS AND TOESION MTO(I) IN M.S. "
3080 PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
3120 PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
3160 PRINT " ENTER DATA FOR M(I,J) DUE TO M(I) = 1 "
3240 PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. U(I) "
3260 PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. T(I) "
```

Example (1) "DATVWN4M"

```
3000 PRINT " ENTER DATA FOR  R , N "
3010 DATA 3 , 16
3020 PRINT " ENTER DATA FOR ELASTIC LENGTHES L(I) "
3030 DATA 6,9, 3,3, 6,12, 6,6
3040 PRINT " ENTER DATA B.MOMENTS AND TORSIONS MTO(I) IN M.S. "
3050 DATA 48,0, 6,6, -6,-6, 0,0, -4.8,-34.8, -3.6,-3.6
3060 DATA 18,54, -30,-30
3080 PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
3090 DATA 0,0, 4,0, 0,0, 0,0
3120 PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
3125 DATA 0.5,0.5, 0.5,0.5, 0.5,0.5, 0.5,0.5
3160 PRINT " ENTER DATA FOR M(I,J) DUE TO M(I) = 1 "
3170 DATA -1,-1, 0,0, 0,0, 1, 1,-.6,-.6, .8,.8, 1,1, 0,0
3180 DATA 0,0, 1,1, -1,-1, 0,0,-.8,-.8, -.6,-.6, 0,0, -1,-1
3190 DATA 8,0, 2,2, -2,2, 0,0, 1.6,6.6, 1.2,1.2, -3,-9, 6,6
3240 PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. U(I) "
3250 DATA 0 , 0 , 0
3260 PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. T(I) "
3270 DATA 0 , 0 , 0
```

```
MERGE  [ VWNM ]

R  U  N

 66.85296    -.7879229
 1.545921     1.545921
-1.545921     8.274521
 .7879229     .7879229
 6.146863    -11.57759
 5.595051     5.595051
 11.42259     32.69193
-5.905039    -5.905039

THAT  IS  ALL  NOW
```

Example (2) "DATVWN6M"

```
3000 PRINT " ENTER DATA FOR  R , N "
3010 DATA 3 , 20
3020 PRINT " ENTER DATA FOR ELASTIC LENGTHES L1(I) "
3030 DATA 4 , 4 , 2.5 , 3.75 , 2 , 4 , 3 , 4.5 , 3 , 4.5
3040 PRINT " ENTER DATA B.MOMENTS MTO(I) IN M.S. "
3050 DATA  73.5 ,  13.5,  20 ,  20 ,  -24.1 , -1.6,  1.2 , 1.2 , -2 , -2
3055 DATA 0 , 0 ,  -1.245 , -34.555 ,  -1.562 , -1.562
3057 DATA 26.008 , 67.62 , -22.806 , -22.806
3080 PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
3090 DATA -3, 0 , 2.083 , 0 ,  1.33 , 0 ,  3.417 , 0 ,  -1.33 , 0
3120 PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
3125 DATA .5 , .5 , .5 , .5 , .5 , .5 , .5 , .5 , .5 , .5
3160 PRINT " ENTER DATA FOR M(I,J) DUE TO M(I) = 1 "
3170 DATA  1, 1, 0,0,  -.6,-.6, -.8,-.8,  0,0, -1,-1,  .781,.781
3172 DATA -.625,-.625 , -1,-1, 0,0
3180 DATA 0,0, 1,1,  -.8,-.8, .6,.6,  -1,-1 , 0,0
3182 DATA -.625, -.625,-.781,-.781, 0,0, -1,-1
3190 DATA  9, 3, 6,6,  -6.6,-1.6, 1.2,1.2,  -2,2, 0,0
3192 DATA 1.249,7.652, 1.56,1.56,  -5,-9, 6,6
3240 PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. U(I) "
3250 DATA 0 , 0 , 0
3260 PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. T(I) "
3270 DATA 0 , 0 , 0
```

```
MERGE   [ VWNM ]

R   U   N

 66.85296     -.7879229
 1.545921      1.545921
-1.545921      8.274521
 .7879229      .7879229
 6.146863     -11.57759
 5.595051      5.595051
 11.42259      32.69193
-5.905039     -5.905039

THAT   IS   ALL   NOW
```