RESPONSE OF 3-D TALL BUILDINGS TO EARTHQUAKE GROUND MOTION INCLUDING FOUNDATION INTERACTION

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ABSTRACT

A computationally efficient procedure for analyzing the response of 3-D tall buildings to earthquake ground motion including foundation interaction is presented. The building is modelled as a lumped mass system considering 3 dynamic D.O.F (two orthogonal horizontal translations and rotation about the vertical axis) at each floor level. The building base is modelled as a rigid circular disc attached to the surface of a linearly elastic half space. Five base D.O.F are considered (horizontal translations in two orthogonal axes,rocking about these axes and twisting about the vertical axis). The structural displacements are transformed to the normal modes of vibration of the fixed base building. Frequency dependent impedance functions of the foundation are considered in the analysis. The seismic excitation is represented by the horizontal components of ground acceleration which are considered as stationary random processes. Spectral analysis is conducted in the frequency domain to evaluate the r.m.s values and mean peak values of the structural responses. Some parametric studies are carried out on an example building to study the effect of soil and structure parameters on the response.

Keywords: Structural analysis, Earthquake response analysis, Tall building.

INTRODUCTION

It is well known that the deformability of the foundation affects the structural response [1] and this effect depends on the properties of structure relative to its foundation. Also,when a heavy structure is supported on a deep soft soil layer,a considerable energy will be transferred from the structure to the soil and the base motions may differ drastically from the free field motions. In general,both the ground motion modification and interaction effects may be important. These effects can be incorporated using finite element method (FEM) of analysis of the entire soil-structure system. Gupta et al [2] employed 3-D finite element model to study the rigid surface foundation,and developed a hybrid method which partitions the soil into a near field part (that is combined with the structure as a super structure) and a far field part. However,FEM becomes costly for complex problems having large number of D.O.F, and the modal analysis does not offer any significant reduction in the computational effort. Recently,the boundary element method (BEM) has been developed [3]. This method requires only the discretization of the surface of domain. The interaction effects are included by modelling the contact surface between the structure and soil as a rigid plate supported on the surface of the soil layer (that is idealized as an elastic half space) and the interaction forces are obtained using impedance coefficients which are evaluated [4,5] by solving a mixed boundary value problem. Gutierrez and Chopra [6] presented a substructure method for analyzing structure with interaction effect,in which the dynamic of a rigid plate on the soil layer is analyzed separately from the structure;the resulting impedance coefficients appear in the structural equations. These equations are then analyzed to find the structural response.

Different techniques are employed for analyzing the earthquake response of tall buildings. Step-by-step numerical integration scheme was used by Parmelee et al [7] for multistory building on flexible foundation assuming frequency independence of foundation stiffness and damping;this approach is general but involves large computational effort. Moreover,the foundation impedance functions relating the forces and displacements are frequency dependent, therefore the analysis is better to be conducted in frequency domain. In the Fourier analysis [6] the ground motion is transformed into its Fourier components and the seismic response is obtained by combining the steady state responses of the frequency components whose contribution is significant. Chopra and Gutierrez [1] developed an efficient method combining the mode superposition and the Fourier transform technique for 2-D earthquake analysis of multistory building including foundation interaction. But,the 2-D idealization is not appropriate neither for unsymmetric buildings having non-coincident centres of mass and stiffness nor for symmetrical buildings subjected to rotational component of ground motion [8]. In such cases,the lateral motions in the two orthogonal directions and torsional motion are coupled and a 3-D model must be used [9,10]. When the foundation interaction is considered,the standard modal analysis is not directly applicable because the building does not possess classical normal modes due to the frequency dependence of foundation properties. Even if these properties are approximated by frequency independent values, the damping of structure and foundation will not usually be so related to permit classical normal modes. However,the modal analysis has been applied by establishing equivalent modal damping values for the interaction system that is based on energy ratio criterion [11].

In seismic design practice of tall buildings,3-D analysis using response-spectrum is more preferable than an elaborate analysis based on a given time-history. On the other hand,in recognition of the fact that the earthquake is a random phenomenon,the structural response should be treated as a random vibration problem and a stochastic analysis has to be conducted to evaluate the statistical descriptions of the response.

In this paper,the effect of soil-structure interaction on the response of 3-D tall building to earthquake random ground motion is investigated. The system considered is a shear building on a circular disc footing attached to the surface of a linearly elastic half space. 3 dynamic D.O.F are assigned at each floor level,namely two horizontal translations in x,y directions and rotation about the vertical axis. 5 base D.O.F are considered,i.e two horizontal translations in two perpendicular directions,two rocking components about these directions and twisting about the vertical axis. Frequency dependent impedance functions of the foundation are considered in the analysis. The structural displacements are transformed to the normal modes of vibration of fixed base building and the complex frequency response functions of the response quantities due to unit harmonic ground acceleration (expressed in complex form) are obtained by solving only 5 linear complex equations. The seismic input is represented by the horizontal component of earthquake ground acceleration incident at an angle w.r.t x direction and is modelled as stationary random process characterized by a Power Spectral Density Function (PSDF). Stochastic analysis is carried out in the frequency domain to evaluate the r.m.s values and mean peak values of the building displacements and base forces.

IDEALIZATION OF ANALYTICAL MODEL

The building-foundation system considered is a linear n-storey 3-D building on flexible foundation (Figure (1)). The mass of the building is assumed to be concentrated at the floor levels. The floor slabs are assumed to be rigid in their own planes,thus leading to 3 dynamic D.O.F per floor,namely,two lateral translations in two orthogonal directions x and y and a rotation about the vertical axis. The structural damping is assumed to be proportional to mass and stiffness. The base of the building is idealized as a rigid circular disc of negligible thickness resting on the surface of a linearly elastic half space. The system is subjected to the horizontal free field ground motions $u_{gx}(t)$,$u_{gy}(t)$ in x and y directions. The response of the system is completely defined by N displacements of n floors (N=3n) plus the five interaction displacements at the base.

EQUATIONS OF MOTION

The N differential equations expressing the dynamic equilibrium of n floor masses can be written as

$$[m](\ddot{u})^t+[C]\{\dot{u}\}+[K]\{u\}=\{0\} \quad (1)$$

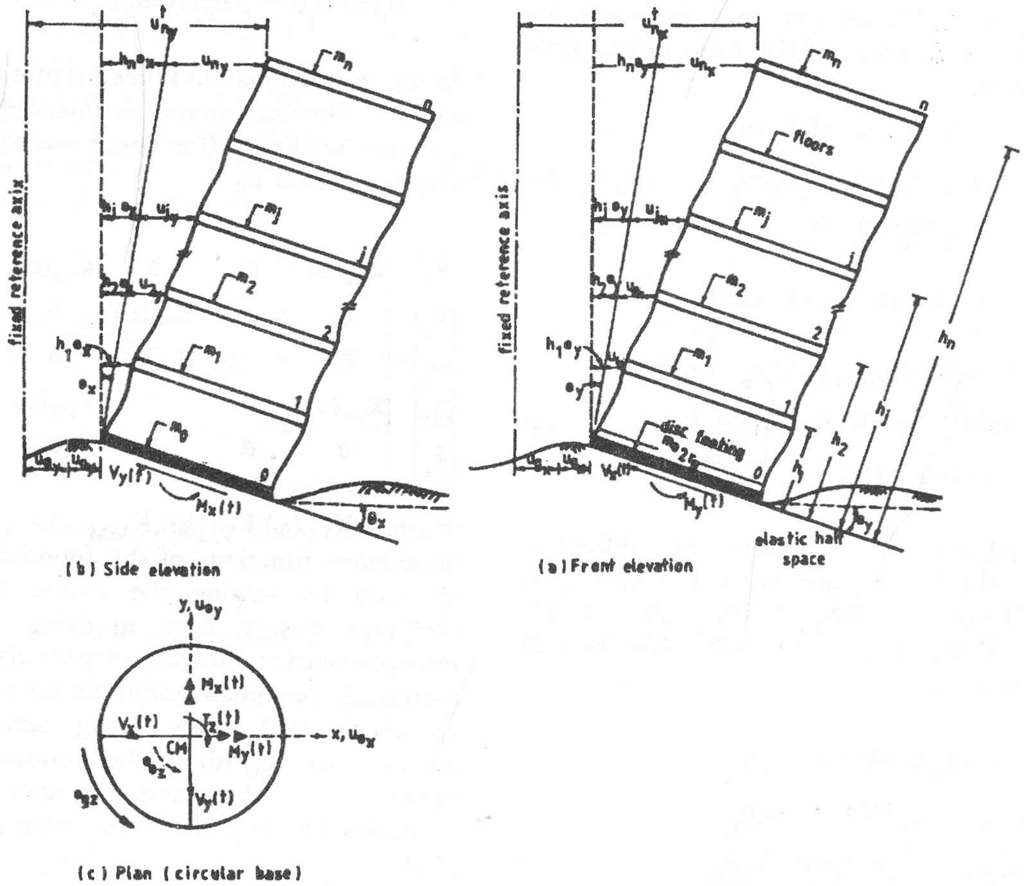

Figure 1. Idealized building-foundation system.

In addition,there are 5 equations expressing the equilibrium of the building-foundation system in translations in x and y directions,rocking about x,and y axes,and rotation (twisting) about the vertical axis z:

$$\begin{aligned}
&\sum_{j=1}^{n} m_j \ddot{u}^t_{j_x} + m_0(\ddot{u}_{g_x} + \ddot{u}_{0_x}) + V_x(t) = 0 \\
&\sum_{j=1}^{n} m_j \ddot{u}^t_{j_y} + m_0(\ddot{u}_{g_y} + \ddot{u}_{0_y}) + V_y(t) = 0 \\
&\sum_{j=1}^{n} m_j h_j \ddot{u}^t_{j_y} + I_{t_x} \ddot{\theta}_x + M_x(t) \qquad = 0 \qquad\qquad (2) \\
&\sum_{j=1}^{n} m_j h_j \ddot{u}^t_{j_x} + I_{t_y} \ddot{\theta}_y + M_y(t) \qquad = 0 \\
&\sum_{j=1}^{n} I_{j_z} \ddot{\theta}^t_{j_z} + I_{0_z}(\ddot{\theta}_{g_z} + \ddot{\theta}_{0_z}) + T_z(t) = 0
\end{aligned}$$

where {u} is a vector containing floor displacement; u_{jx},u_{jy} and rotations Θ_{jz} ,j=1,....n; [m] is the diagonal mass matrix; [K] is the dynamic stiffness matrix of the fixed base building; [C] is the viscous damping matrix; $V_x(t)$, $V_y(t)$ are the base shears in x,y directions; $M_x(t)$,$M_y(t)$ are the base moments about x,y directions; $T_z(t)$ is the base torque about the vertical axis z; m_0,I_{0z} are the mass and mass moment of inertia of the footing; m_j,h_j are the lumped mass and height of jth floor; and I_{tx},I_{ty} are the total mass moment of inertia about x,y axes:

$$I_{t_x} = \sum_{j=0}^{n} I_{j_x} \quad , \quad I_{t_y} = \sum_{j=0}^{n} I_{j_y}$$

in which I_{jx},I_{jy},I_{jz} are the mass moment of inertia of j th floor about its own x,y,z axes,respectively; u_{gx},u_{gy},Θ_{gz} are the free field translations (in x,y directions) and rotation (about z axis); u_{0x},u_{0y},Θ_{0z} are the rigid body translations (in x,y directions) and rotation (about z axis) of the footing; Θ_x, Θ_y are the rotations of the footing due to rocking about x,y axes;and $u^t_{jx}, u^t_{jy}, \Theta^t_{jz}$ are the total displacements and rotations of the centre of the mass of j th floor and are given by

$$\begin{aligned} u^t_{j_x} &= u_{g_x} + u_{0_x} + h_j\theta_y + u_{j_x} \\ u^t_{j_y} &= u_{g_y} + u_{0_y} + h_j\theta_x + u_{j_y} \\ \theta^t_{j_z} &= \theta_{g_z} + \theta_{0_z} + \theta_{j_z} \end{aligned} \tag{3}$$

Substituting Eqs.(3) into Eq.(1) yields

$$\begin{aligned} &[m]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} + [m]\{I\}_x\ddot{u}_{0_x} + [m]\{I\}_y\ddot{u}_{0_y} + \\ &[m]\{I\}_z\ddot{\theta}_{0_z} + [m]\{H\}_x\ddot{\theta}_y + [m]\{H\}_y\ddot{\theta}_x = \\ &-[m]\{I\}_x\ddot{u}_{g_x} - [m]\{I\}_y\ddot{u}_{g_y} - [m]\{I\}_z\ddot{\theta}_{g_z} \end{aligned} \tag{4}$$

where $\{I\}_x, \{I\}_y, \{I\}_z$ are vectors of influence coefficients and $\{H\}_x, \{H\}_y$ are the vectors of heights given as :$\{H\}_x = \{h_1 \ \ 0 \ \ 0, h_2 \ \ 0 \ \ 0, \ldots\ldots, h_n \ \ 0 \ \ 0\}^T$, $\{H\}_y = \{0 \ \ h_1 \ \ 0, 0 \ \ h_2 \ \ 0, \ldots\ldots\ldots, 0 \ \ h_n \ \ 0\}^T$. Also Eqs.(2) can be rewritten as:

$$\begin{aligned} &\{I\}^T_x[m]\{\ddot{u}\} + m_t\ddot{u}_{0_x} + L_{r_0}\ddot{\theta}_y + V_x(t) = -m_t\ddot{u}_{g_x} \\ &\{I\}^T_y[m]\{\ddot{u}\} + m_t\ddot{u}_{0_y} + L_{r_0}\ddot{\theta}_x + V_y(t) = -m_t\ddot{u}_{g_y} \\ &\{H\}^T_y[m]\{\ddot{u}\} + L_{r_0}\ddot{u}_{0_y} + I_{b_x}\ddot{\theta}_x + M_x(t) = -L_{r_0}\ddot{u}_{g_y} \\ &\{H\}^T_x[m]\{\ddot{u}\} + L_{r_0}\ddot{u}_{0_x} + I_{b_y}\ddot{\theta}_y + M_y(t) = -L_{r_0}\ddot{u}_{g_x} \\ &\{I\}^T_z[m]\{\ddot{u}\} + I_{t_z}\ddot{\theta}_{0_z} + T_z(t) = -I_{t_z}\ddot{\theta}_{g_z} \end{aligned} \tag{5}$$

where

$$m_t = \sum_{j=0}^{n} m_j, \ L_{r_0} = \sum_{j=1}^{n} m_j h_j \ , \ I_{b_x} = I_{t_x} + \sum_{j=1}^{n} m_j h_j^2,$$

$$I_{b_y} = I_{t_y} + \sum_{j=1}^{n} m_j h_j^2, \ I_{t_z} = \sum_{j=0}^{n} I_{j_z}$$

For steady state harmonic vibration at frequency ω, the interaction forces acting on the footing can be expressed in complex form as:

$$V_x(t) = \bar{V}_x e^{i\omega t}\ , \ V_y(t) = \bar{V}_y e^{i\omega t}\ , \ M_x(t) = \bar{M}_x e^{i\omega t}\ ,$$
$$M_y(t) = \bar{M}_y e^{i\omega t}\ , \ T_z(t) = \bar{T}_z e^{i\omega t}$$

and the resulting displacements of the footing may be expressed as:

$$u_{0_x}(t) = \bar{u}_{0_x} e^{i\omega t}\ , \ u_{0_y}(t) = \bar{u}_{0_y} e^{i\omega t}\ , \ \theta_x(t) = \bar{\theta}_x e^{i\omega t}\ ,$$
$$\theta_y(t) = \bar{\theta}_y e^{i\omega t}\ , \ \theta_{0_z}(t) = \bar{\theta}_{0_z} e^{i\omega t}$$

As the translational and rocking motions are coupled and the torsional motion is uncoupled,these forces and displacements (for linear model of foundation) may be related as

$$\begin{Bmatrix} \bar{V}_x \\ \bar{V}_y \\ \bar{M}_x \\ \bar{M}_y \\ \bar{T}_z \end{Bmatrix} = \begin{bmatrix} K_{VV}(\omega) & 0 & 0 & K_{VM}(\omega) & 0 \\ 0 & K_{VV}(\omega) & K_{VM}(\omega) & 0 & 0 \\ 0 & K_{MV}(\omega) & K_{MM}(\omega) & 0 & 0 \\ K_{MV}(\omega) & 0 & 0 & K_{MM}(\omega) & 0 \\ 0 & 0 & 0 & 0 & K_{TT}(\omega) \end{bmatrix} \begin{Bmatrix} \bar{u}_{0_x} \\ \bar{u}_{0_y} \\ \bar{\theta}_x \\ \bar{\theta}_y \\ \bar{\theta}_{0_z} \end{Bmatrix} \tag{6}$$

where $K_{VV}(\omega), K_{VM}(\omega), K_{MM}(\omega), K_{TT}(\omega)$ are the impedance functions of the foundation and can be obtained by solving the mixed boundary value problems arising from applying of a harmonic force,moment,or torque separately to the disc footing. Approximate solutions for a circular base on an elastic half space using certain simplifying assumptions regarding the conditions of contact between the disc and foundation surface are available. The impedance functions are expressed as [4,5]

$$\begin{aligned} &K_{VV}(\omega) = (K_{11} + ia_0C_{11})K_x, K_{MM}(\omega) = (K_{22} + ia_0C_{22})K_{\theta_x} \\ &K_{VM}(\omega) = K_{MV}(\omega) = (K_{21} + ia_0C_{21})K_x r_0, K_{TT}(\omega) = (K_{33} + ia_0C_{33})K_{\theta_z} \end{aligned} \tag{7}$$

r_0=radius of footing; K's,C's are dimensionless coefficients depending on Poisson's ratio ν ; $a_0 = \omega r_0/V_s$; $V_s = \sqrt{G/\rho}$ is the shear wave velocity in the half space,G,ρ are the shear modulus and mass density of the soil; and $K_x, K_{\theta x}, K_{\theta z}$ are given as

$$K_x = 8Gr_0/(2-\nu)\ , \ K_{\theta_x} = 8Gr_0^3/3(1-\nu)\ , \ K_{\theta_z} = 16Gr_0^3/3 \tag{8}$$

where

K_x = horizontal static force required to produce a unit horizontal displacement of the disc with no restriction on the value of rocking motion.

$K_{\Theta x}$ = static moment required to rotate the disc footing about x-axis through a unit angle with no restriction on the value of horizontal displacement.

$K_{\Theta z}$ = static torque required to rotate the disc footing (about z-axis) through a unit angle rotation (about z axis) of the footing; θ_x, θ_y are the rotations of the footing due to rocking about x,y axes;and $u^t_{jx}, u^t_{jy}, \theta^t_{jz}$ are the total displacements and rotations of the centre of the mass of j th floor and are given by

$$u^t_{j_x} = u_{g_x} + u_{0_x} + h_j\theta_y + u_{j_x}$$
$$u^t_{j_y} = u_{g_y} + u_{0_y} + h_j\theta_x + u_{j_y} \qquad (3)$$
$$\theta^t_{j_z} = \theta_{g_z} + \theta_{0_z} + \theta_{j_z}$$

Substituting Eqs.(3) into Eq.(1) yields

$$[m](\ddot{u}) + [C]\{\dot{u}\} + [K]\{u\} + [m]\{I\}_x \ddot{u}_{0_x} + [m]\{I\}_y \ddot{u}_{0_y} + [m]\{I\}_z \ddot{\theta}_{0_z} + [m]\{H\}_x \ddot{\theta}_y + [m]\{H\}_y \ddot{\theta}_x = -[m]\{I\}_x \ddot{u}_{g_x} - [m]\{I\}_y \ddot{u}_{g_y} - [m]\{I\}_z \ddot{\theta}_{g_z} \qquad (4)$$

where $\{I\}_x, \{I\}_y, \{I\}_z$ are vectors of influence coefficients and $\{H\}_x, \{H\}_y$ are the vectors of heights given as :$\{H\}_x = \{h_1 \;\; 0 \;\; 0, h_2 \;\; 0 \;\; 0, \ldots\ldots, h_n \;\; 0 \;\; 0\}^T$, $\{H\}_y = \{0 \;\; h_1 \;\; 0, 0 \;\; h_2 \;\; 0, \ldots\ldots, 0 \;\; h_n \;\; 0\}^T$. Also Eqs.(2) can be rewritten as:

$$\begin{aligned}
\{I\}^T_x[m]\{\ddot{u}\} + m_t\ddot{u}_{0_x} + L_{r_0}\ddot{\theta}_y + V_x(t) &= -m_t\ddot{u}_{g_x} \\
\{I\}^T_y[m]\{\ddot{u}\} + m_t\ddot{u}_{0_y} + L_{r_0}\ddot{\theta}_x + V_y(t) &= -m_t\ddot{u}_{g_y} \\
\{H\}^T_y[m]\{\ddot{u}\} + L_{r_0}\ddot{u}_{0_y} + I_{b_x}\ddot{\theta}_x + M_x(t) &= -L_{r_0}\ddot{u}_{g_y} \qquad (5)\\
\{H\}^T_x[m]\{\ddot{u}\} + L_{r_0}\ddot{u}_{0_x} + I_{b_y}\ddot{\theta}_y + M_y(t) &= -L_{r_0}\ddot{u}_{g_x} \\
\{I\}^T_z[m]\{\ddot{u}\} + I_{t_z}\ddot{\theta}_{0_z} + T_z(t) &= -I_{t_z}\ddot{\theta}_{g_z}
\end{aligned}$$

where

$$m_t = \sum_{j=0}^{n} m_j, \; L_{r_0} = \sum_{j=1}^{n} m_j h_j, \; I_{b_x} = I_{t_x} + \sum_{j=1}^{n} m_j h_j^2,$$
$$I_{b_y} = I_{t_y} + \sum_{j=1}^{n} m_j h_j^2, \; I_{t_z} = \sum_{j=0}^{n} I_{j_z}$$

For steady state harmonic vibration at frequency ω, the interaction forces acting on the footing can be expressed in complex form as:

$$V_x(t) = \bar{V}_x e^{i\omega t}, \; V_y(t) = \bar{V}_y e^{i\omega t}, \; M_x(t) = \bar{M}_x e^{i\omega t},$$
$$M_y(t) = \bar{M}_y e^{i\omega t}, \; T_z(t) = \bar{T}_z e^{i\omega t}$$

and the resulting displacements of the footing may be expressed as:

$$u_{0_x}(t) = \bar{u}_{0_x} e^{i\omega t}, \; u_{0_y}(t) = \bar{u}_{0_y} e^{i\omega t}, \; \theta_x(t) = \bar{\theta}_x e^{i\omega t},$$
$$\theta_y(t) = \bar{\theta}_y e^{i\omega t}, \; \theta_{0_z}(t) = \bar{\theta}_{0_z} e^{i\omega t}$$

As the translational and rocking motions are coupled and the torsional motion is uncoupled,these forces and displacements (for linear model of foundation) may be related as

$$\begin{Bmatrix} \bar{V}_x \\ \bar{V}_y \\ \bar{M}_x \\ \bar{M}_y \\ \bar{T}_z \end{Bmatrix} = \begin{bmatrix} K_{VV}(\omega) & 0 & 0 & K_{VM}(\omega) & 0 \\ 0 & K_{VV}(\omega) & K_{VM}(\omega) & 0 & 0 \\ 0 & K_{MV}(\omega) & K_{MM}(\omega) & 0 & 0 \\ K_{MV}(\omega) & 0 & 0 & K_{MM}(\omega) & 0 \\ 0 & 0 & 0 & 0 & K_{TT}(\omega) \end{bmatrix} \begin{Bmatrix} \bar{u}_{0_x} \\ \bar{u}_{0_y} \\ \bar{\theta}_x \\ \bar{\theta}_y \\ \bar{\theta}_{0_z} \end{Bmatrix} \qquad (6)$$

where $K_{VV}(\omega), K_{VM}(\omega), K_{MM}(\omega), K_{TT}(\omega)$ are the impedance functions of the foundation and can be obtained by solving the mixed boundary value problems arising from applying of a harmonic force,moment,or torque separately to the disc footing. Approximate solutions for a circular base on an elastic half space using certain simplifying assumptions regarding the conditions of contact between the disc and foundation surface are available. The impedance functions are expressed as [4,5]

$$K_{VV}(\omega) = (K_{11} + ia_0C_{11})K_x, \; K_{MM}(\omega) = (K_{22} + ia_0C_{22})K_{\theta_x}$$
$$K_{VM}(\omega) = K_{MV}(\omega) = (K_{21} + ia_0C_{21})K_x r_0, \; K_{TT}(\omega) = (K_{33} + ia_0C_{33})K_{\theta_z} \qquad (7)$$

r_0=radius of footing; K's,C's are dimensionless coefficients depending on Poisson's ratio ν ; $a_0 = \omega r_0/V_s$; $V_s = \sqrt{G/\rho}$ is the shear wave velocity in the half space,G,ρ are the shear modulus and mass density of the soil; and $K_x, K_{\theta x}, K_{\theta z}$ are given as

$$K_x = 8Gr_0/(2-\nu), \; K_{\theta_x} = 8Gr_0^3/3(1-\nu), \; K_{\theta_z} = 16Gr_0^3/3 \qquad (8)$$

where

K_x = horizontal static force required to produce a unit horizontal displacement of the disc with no restriction on the value of rocking motion.

$K_{\theta x}$ = static moment required to rotate the disc footing about x-axis through a unit angle with no restriction on the value of horizontal displacement.

$K_{\theta z}$ = static torque required to rotate the disc footing (about z-axis) through a unit angle

The real and imaginary parts of the impedance functions represent the stiffness and damping coefficients,respectively for the foundation. Numerical values of K_{11},C_{11},K_{22},C_{22},K_{21} and C_{21} obtained by Veletsos and Wei [4] are shown in Figure (2). While K_{33},C_{33} are calculated from the compliance functions given by Luco and Westmann [5] and reproduced in Figure (3).

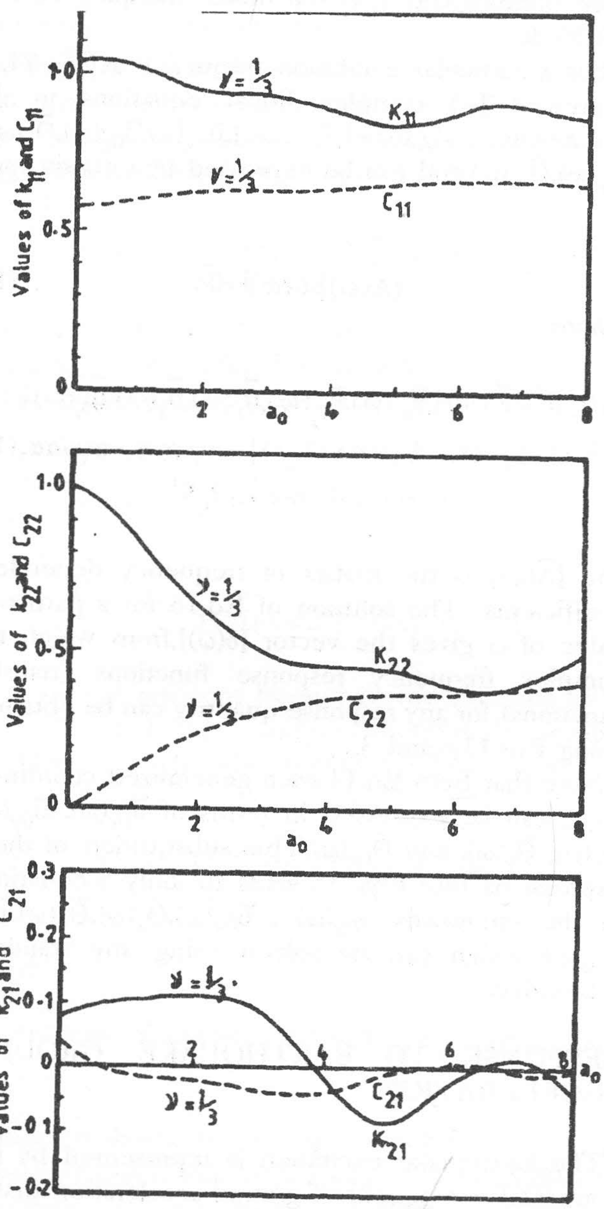

Figure 2. Stiffness and damping coefficients (after veletsos and Wei Ref. [4]).

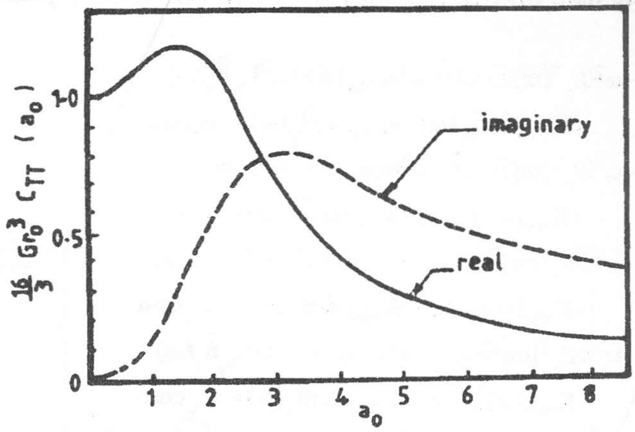

Figure 3. Torsional compliance (after Luco and Westmann Ref. [5]).

RESPONSE TO HARMONIC GROUND EXCITATION

For a harmonic ground acceleration $\ddot{u}_g(t)=e^{i\omega t}$ acting at an angle α (w.r.t x direction) on a linear structure,

$$\ddot{u}_{g_x}(t)=\cos\alpha e^{i\omega t}\ ,\ \ddot{u}_{g_y}(t)=\sin\alpha e^{i\omega t}\ ,\ \ddot{\theta}_{g_z}(t)=Se^{i\omega t} \tag{9}$$

where S=f(sin α.cos α). Any response quantity X(t) of the structure is expressed as $X(t)=\bar{X}(\omega)e^{i\omega t}$, where $\bar{X}(\omega)$ is a complex frequency response function. Thus for the system under consideration,

$$u(t)=\bar{u}(\omega)e^{i\omega t}, u_{0_x}(t)=\bar{u}_{0_x}(\omega)e^{i\omega t}, u_{0_y}(t)=\bar{u}_{0_y}(\omega)e^{i\omega t},$$
$$\theta_x(t)=\bar{\theta}_x(\omega)e^{i\omega t}, \theta_y(t)=\bar{\theta}_y(\omega)e^{i\omega t}, \theta_{0_z}(t)=\bar{\theta}_{0_z}(\omega)e^{i\omega t} \tag{10a}$$

and

$$V_x(t)=\bar{V}_x(\omega)e^{i\omega t}, V_y(t)=\bar{V}_y(\omega)e^{i\omega t}, M_x(t)=\bar{M}_x(\omega)e^{i\omega t}$$
$$M_y(t)=\bar{M}_y(\omega)e^{i\omega t}, T_z(t)=\bar{T}_z(\omega)e^{i\omega t} \tag{10b}$$

where $\bar{u}(\omega)$, $\bar{u}_{0x}(\omega)$, $\bar{u}_{0y}(\omega)$, $\bar{\Theta}_x(\omega)$, $\bar{\Theta}_y(\omega)$, $\bar{\Theta}_{0z}(\omega)$, $\bar{V}_x(\omega)$, $\bar{V}_y(\omega)$, $\bar{M}_x(\omega)$, $\bar{M}_y(\omega)$, and $\bar{T}_z(\omega)$ are complex frequency response functions to be determined. Substitution of Eqs 9 and 10a into Eq. 4 leads to

$$[-\omega^2[m]+i\omega[C]+[K]]\{\bar{u}(\omega)\}-\omega^2[m]\{I\}_x\bar{u}_{0_x}(\omega)-\omega^2[m]\{I\}_y\bar{u}_{0_y}(\omega)$$
$$-\omega^2[m]\{I\}_z\bar{\theta}_{0_z}(\omega)-\omega^2[m]\{H\}_x\bar{\theta}_y(\omega)-\omega^2[m]\{H\}_y\bar{\theta}_x(\omega)= \tag{11}$$
$$-[m]\{I\}_x\cos\alpha-[m]\{I\}_y\sin\alpha-[m]\{I\}_zS$$

As well, substitution of Eqs 6,9,10 into Eqs. 5 yields

$$
\begin{aligned}
&-\omega^2\{I\}_x^T[m]\{\bar{u}(\omega)\}-\omega^2 m_t\bar{u}_{0_x}(\omega)-\omega^2 L_{r_0}\bar{\theta}_y(\omega)\\
&\qquad +K_{VV}(\omega)\bar{u}_{0_x}(\omega)+K_{VM}(\omega)\bar{\theta}_y(\omega)=-m_t\cos\alpha\\
&-\omega^2\{I\}_y^T[m]\{\bar{u}(\omega)\}-\omega^2 m_t\bar{u}_{0_y}(\omega)-\omega^2 L_{r_0}\bar{\theta}_x(\omega)\\
&\qquad +K_{VV}(\omega)\bar{u}_{0_y}(\omega)+K_{VM}(\omega)\bar{\theta}_x(\omega)=-m_t\sin\alpha\\
&-\omega^2\{H\}_y^T[m]\{\bar{u}(\omega)\}-\omega^2 L_{r_0}\bar{u}_{0_y}(\omega)-\omega^2 I_{b_x}\bar{\theta}_x(\omega)\\
&\qquad +K_{VM}(\omega)\bar{u}_{0_y}(\omega)+K_{MM}(\omega)\bar{\theta}_x(\omega)=-L_{r_0}\sin\alpha\\
&-\omega^2\{H\}_x^T[m]\{\bar{u}(\omega)\}-\omega^2 L_{r_0}\bar{u}_{0_x}(\omega)-\omega^2 I_{b_y}\bar{\theta}_y(\omega)\\
&\qquad +K_{VM}(\omega)\bar{u}_{0_x}(\omega)+K_{MM}(\omega)\bar{\theta}_y(\omega)=-L_{r_0}\cos\alpha\\
&-\omega^2\{I\}_z^T[m]\{\bar{u}(\omega)\}-\omega^2 I_{t_z}\bar{\theta}_{0_z}(\omega)+K_{TT}(\omega)\bar{\theta}_{0_z}(\omega)=-I_{t_z}s
\end{aligned}
\tag{12}
$$

Utilizing the mode superposition method with the transformation:

$$\{\bar{u}(\omega)\}=[\phi]\{\bar{Z}(\omega)\} \tag{13}$$

Eqs.11 and 12 are transformed into J+5 equations (J is the number of mode shapes considered in the analysis) as follows:

$$
\begin{aligned}
&(-\omega^2+2i\zeta_i\omega_i\omega+\omega_i^2)\bar{Z}_i(\omega)-\omega^2 L_{x_i}\bar{u}_{0_x}(\omega)-\omega^2 L_{y_i}\bar{u}_{0_y}(\omega)\\
&\qquad -\omega^2 L_{z_i}\bar{\theta}_{0_z}(\omega)-\omega^2 Lr_{x_i}\bar{\theta}_y(\omega)-\omega^2 Lr_{y_i}\bar{\theta}_x(\omega)=\\
&\qquad\qquad -L_{x_i}\cos\alpha-L_{y_i}\sin\alpha-L_{z_i}S \quad i=1,2,\ldots..J
\end{aligned}
\tag{14}
$$

and

$$
\begin{aligned}
&-\omega^2\sum_{i=1}^{J} L_{x_i}\bar{Z}_i(\omega)-\omega^2 m_t\bar{u}_{0_x}(\omega)-\omega^2 L_{r_0}\bar{\theta}_y(\omega)\\
&\qquad +K_{VV}(\omega)\bar{u}_{0_x}(\omega)+K_{VM}(\omega)\bar{\theta}_y(\omega)=-m_t\cos\alpha\\
&-\omega^2\sum_{i=1}^{J} L_{y_i}\bar{Z}_i(\omega)-\omega^2 m_t\bar{u}_{0_y}(\omega)-\omega^2 L_{r_0}\bar{\theta}_x(\omega)\\
&\qquad +K_{VV}(\omega)\bar{u}_{0_y}(\omega)+K_{VM}(\omega)\bar{\theta}_x(\omega)=-m_t\sin\alpha\\
&-\omega^2\sum_{i=1}^{J} Lr_{y_i}\bar{Z}_i(\omega)-\omega^2 L_{r_0}\bar{u}_{0_y}(\omega)-\omega^2 I_{b_x}\bar{\theta}_x(\omega)\\
&\qquad +K_{VM}(\omega)\bar{u}_{0_y}(\omega)+K_{MM}(\omega)\bar{\theta}_x(\omega)=-L_{r_0}\sin\alpha\\
&-\omega^2\sum_{i=1}^{J} Lr_{x_i}\bar{Z}_i(\omega)-\omega^2 L_{r_0}\bar{u}_{0_x}(\omega)-\omega^2 I_{b_y}\bar{\theta}_y(\omega)\\
&\qquad +K_{VM}(\omega)\bar{u}_{0_x}(\omega)+K_{MM}(\omega)\bar{\theta}_y(\omega)=-L_{r_0}\cos\alpha\\
&-\omega^2\sum_{i=1}^{J} L_{z_i}\bar{Z}_i(\omega)-\omega^2 I_{t_z}\bar{\theta}_{0_z}(\omega)+K_{TT}\bar{\theta}_{0_z}(\omega)=-I_{t_z}.S
\end{aligned}
\tag{15}
$$

where

$$L_{x_i}=(\phi_i)^T[m]\{I\}_x,\ L_{y_i}=(\phi_i)^T[m]\{I\}_y,\ L_{z_i}=(\phi_i)^T[m]\{I\}_z,$$
$$Lr_{x_i}=(\phi_i)^T[m]\{H\}_x,\ Lr_{y_i}=(\phi_i)^T[m]\{H\}_y,$$

$\bar{Z}_i(\omega)$ is the ith modal displacement; ω_i, (ϕ_i) are the ith natural frequency and normalized mode shape (i.e $(\phi_i)^T[m](\phi_i)=1$) which are obtained by solving the equation of undamped free vibration of the fixed base building; and ζ_i is the modal damping ratio of ith mode.

For a particular excitation frequency ω, Eqs 14,15 represent J+5 complex linear equations in the unknowns $\bar{Z}_i(\omega), i=1,2\ldots\ldots\ldots.J, \bar{u}_{0x}(\omega), \bar{u}_{0y}(\omega), \bar{\theta}_x(\omega), \bar{\theta}_y(\omega), \bar{\theta}_{0z}(\omega)$, and can be expressed in a matrix form as

$$[\bar{A}(\omega)]\{\delta(\omega)\}=\{\bar{P}\} \tag{16}$$

where

$$
\begin{aligned}
&\{\delta(\omega)\}^T=\lfloor \bar{Z}_i(\omega)\,|\,,\bar{u}_{0_x}(\omega),\bar{u}_{0_y}(\omega),\bar{\theta}_x(\omega),\bar{\theta}_y(\omega),\bar{\theta}_{0_z}(\omega)\rfloor,\\
&\{\bar{P}\}^T=\lfloor(-L_{x_i}\cos\alpha-L_{y_i}\sin\alpha-L_{z_i}s)\,|\,,-m_t\cos\alpha,-m_t\sin\alpha,\\
&\qquad -L_{r_0}\sin\alpha,-L_{r_0}\cos\alpha,-I_{t_z}S\rfloor
\end{aligned}
\tag{17}
$$

and $[\bar{A}(\omega)]$ is the matrix of frequency dependent coefficients. The solution of Eq.16 for a particular value of ω gives the vector $\{\delta(\omega)\}$, from which the complex frequency response functions (transfer functions) for any response quantity can be obtained using Eqs.13,6,and 3.

Note that from Eq.14 each generalized coordinate $\bar{Z}_i(\omega)$ can be expressed in terms of $\bar{u}_{0x}(\omega)$, $\bar{u}_{0y}(\omega)$, $\bar{\theta}_x(\omega)$, $\bar{\theta}_y(\omega)$, and $\bar{\theta}_{0z}(\omega)$. Thus, substitution of these expressions into Eqs. 15 leads to only 5 equations in the unknowns $\bar{u}_{0x}(\omega)$, $\bar{u}_{0y}(\omega), \bar{\theta}_x(\omega), \bar{\theta}_y(\omega)$, and $\bar{\theta}_{0z}(\omega)$ which can be solved using any standard subroutine.

RESPONSE TO EARTHQUAKE GROUND ACCELERATION

The earthquake excitation is represented by the horizontal component of ground acceleration that is incident at an angle α w.r.t x axis. This component is considered as stationary random process with zero mean characterized by the PSDF $S_{\ddot{u}g}(\omega)$ which can be obtained in two different ways:

(i) From the response spectrum using the relationship derived by Kaul [12] as

$$S_{\ddot{u}_g}(\omega)=2\zeta S_a^2(\omega)/\pi\omega[-2\ln\{-(\pi/\omega T)\ln(1-r)\}] \quad (18)$$

ζ=damping ratio; r=exceedance probability which is taken as 0.15; T=earthquake duration taken as 15 sec; and $S_a(\omega)$=ordinate of response spectrum at frequency ω. Figure (4) shows the derived PSDF, $S_{\ddot{u}g}(\omega)$ corresponding to an average acceleration response spectrum for ζ=0.05

(ii) Using the modified Tajimi-Kanai PSDF suggested by Clough and Penzien [13]

$$S_{\ddot{u}_g}(\omega)=S_0|H_1(i\omega)|^2\ |H_2(i\omega)|^2 \quad (19)$$

S_0= spectrum of white noise bed-rock acceleration,and $H_1(i\omega)$,$H_2(i\omega)$are transfer functions of two filters above the bed-rock,where

$$|H_1(i\omega)|^2=\frac{1+(2\zeta_g\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+(2\zeta_g\omega/\omega_g)^2},$$
$$|H_2(i\omega)|^2=\frac{(\omega/\omega_f)^4}{[1-(\omega/\omega_f)^2]^2+(2\zeta_f\omega/\omega_f)^2} \quad (20)$$

ω_g,ζ_g are the resonant frequency and damping ratio of the first soil filter; ω_f,ζ_f are those of the second soil filter. For one sided spectrum,$S_0$is given as

$$S_0=\sigma^2_{\ddot{u}_g}/[\int_0^\infty|H_1(i\omega)|^2.|H_2(i\omega)|^2d\omega \quad (21)$$

where

$$\sigma^2_{\ddot{u}_g}=\text{variance of ground acceleration}=\int_0^\infty S_{\ddot{u}_g}(\omega)d\omega \quad (22)$$

Figure (5) shows two normalized spectra (w.r.t $\sigma_{\ddot{u}g}$) of ground acceleration corresponding to two different conditions of soil;for the first spectrum $\omega_g=\pi,\zeta_g=0.3$, V_s=70 m/sec(representing soft soil medium),and for the second spectrum $\omega_g=5\pi,\zeta_g=0.6$, V_s=600m/sec (representing firm soil medium); and $\omega_f=0.1\omega_g,\zeta_f=\zeta_g$ for both spectra. Since the system considered is linear,the responses will be also stationary random processes with zero means. The PSDF $S_R(\omega)$ of response quantity R(t) may be given as [14]

$$S_R(\omega)=|\bar{R}(\omega)|^2S_{\ddot{u}_g}(\omega) \quad (23)$$

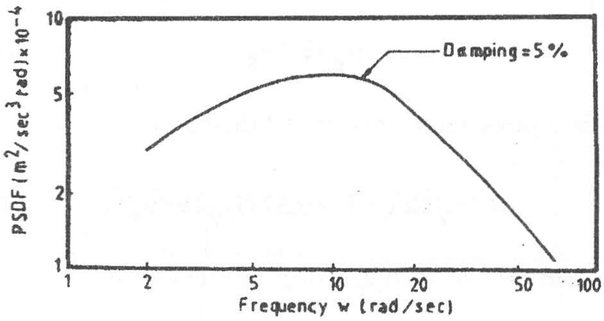

Figure 4. Ground acceleration spectrum derived by Kaul's method.

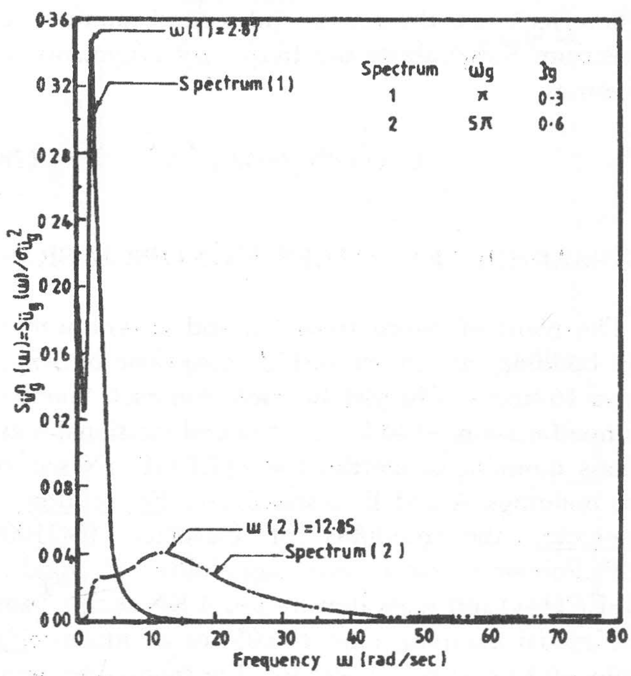

Figure 5. Normalized spectra of ground acceleration.

where $|\bar{R}(\omega)|$ is the magnitude of the complex frequency response function of response quantity R(t) at frequency ω . The root mean square (r.m.s)value of response quantity R(t)(same as the standard deviation σ_R) is obtained as the square root of the area under the PSDF curve i.e

$$\mathrm{r.m.s}(R(t)) = \sigma_R = \sqrt{\int_0^{\infty} S_R(\omega)\,d\omega} \qquad (24)$$

The mean peak value of response quantity R(t),is obtained as

$$\mu_R = K^* \sigma_R \qquad (25)$$

K^* is a peak factor given as [15,16]

$$K^* = \sqrt{2\ln(\nu_0 T)} + 0.5772/\sqrt{2\ln(\nu_0 T)} \qquad (26)$$

$\nu_0$is the rate of zero crossing expressed as

$$\nu_0 = \frac{1}{2\pi}\sqrt{\lambda_2/\lambda_0} \qquad (27)$$

where λ_0,λ_2 are the zeroth and second moments of spectrum $S_R(\omega)$ about the frequency origin,and are given as

$$\lambda_m = \int_0^{\infty} \omega^m S_R(\omega)\,d\omega \qquad (28)$$

NUMERICAL EXAMPLES AND DISCUSSION

The plans of symmetrical (A) and unsymmetrical (B) buildings are shown in Figure(6). The buildings have 16 storys of height 3m each. For each floor,the lumped mass m_j=144 KN.sec^2/m and rotational mass (mass moment of inertia) I_z=7217,7831 KN sec^2.m for buildings A and B, respectively. For reinforced concrete: the modulus of elasticity E=21000 MP$_a$,Poisson's ratio ν=0.15,modulus of rigidity G=E/2(1+ν),and mass density ρ=2.4 KN sec^2/m^4 and the modal damping ratio ζ=0.05 for all modes. For soil:ν_s=0.33,ρ_s=1.6 KN sec^2/m^4. For footing:the mass m_0=5m=720 KN sec^2/m,radius r_0=8.5m which is equivalent to the plan area of the building. Seismic input:the earthquake ground motion is assumed to act horizontally in y direction (i.e the angle of incidence α=90 w.r.t x axis);the ground acceleration is considered as stationary random process with zero mean,characterized by PSDF which is given by Eq. 18 or Eq. 19.

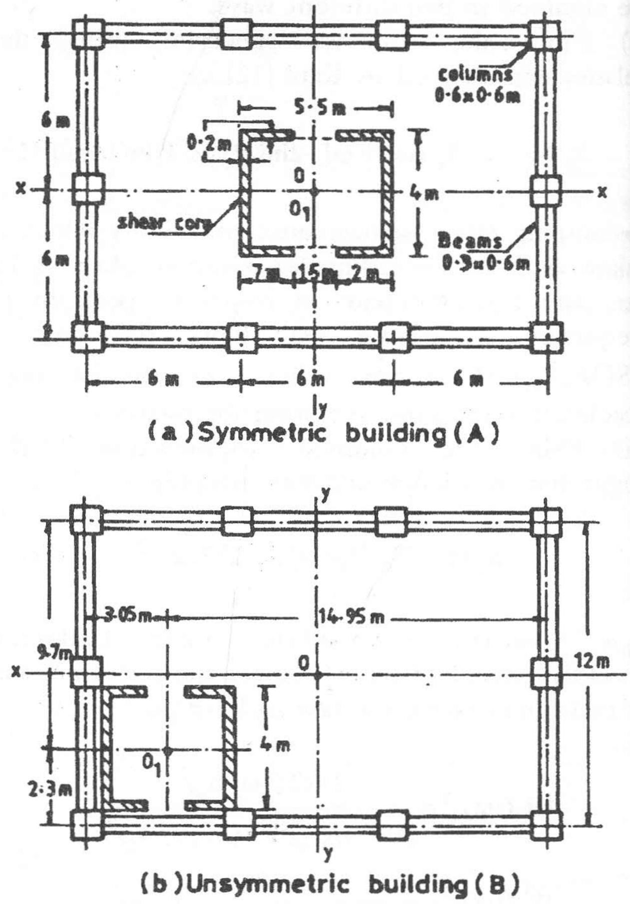

Figure 6. Plans of the buildings A,B.

Effect of Soil Structure Interaction

The effect of soil-structure interaction depends on the relative stiffness of building and its foundation soil (represented by the shear wave velocity of the soil,V_s). The wave parameter γ represents a measure of this relative stiffness where $\gamma=T_1 V_s/H$ ($T_1$is the fundamental period of the building of height H). For the example buildings of height 48^m,the fundamental periods are 1.664,and 1.873 sec for buildings A and B ,respectively. Consequently, γ=0.0347V_s and 0.0390 V_s for buildings A and B. V_s is taken as variable parameter ranges between 70 m/sec (representative to very soft soil) and 600 m/sec (for very stiff soil providing almost a fixed base condition) i.e γ ranges between 2.43 and 20.82 for building A and between 2.73 and 23.4 for building B. The variation of mean peak values of responses with V_s is shown in Figures (7) and (8) for buildings A and B ,respectively. It can be seen that there is no difference between the structural responses for

V_s=600 m/sec and those of fixed base building. In general,the interaction effects are pronounced for low values of V_s. As V_s increases the total displacements decrease while the base shears,moments,and torque increase,ultimately reaching their fixed base values. For very flexible base (V_s=70 m/sec) the displacement in the direction of earthquake (y-direction) is more than twice that of the fixed base. However, the other two displacements (x-direction and rotation) increase only by 78 and 15% ,respectively. On the other hand, the base shears,moments and torque are 12-13%,23-28%,and 43% less than the fixed base values, respectively.

Effect of Torsional Coupling on Response

The symmetric and unsymmetric buildings A and B shown in Figure (6) have nearly the same lateral stiffenesses in x and y directions. Comparing their responses (shown in Figures (7) and (8), respectively),it can be seen that the response quantities in the direction of earthquake wave propagation (y-direction) of case (B) are less than those of case (A). This may be due to the effect of torsional coupling. On the other hand, considerable responses are induced in the other D.O.F's. The responses in the three directions of unsymmetric building (B) may produce more critical stresses in the individual members compared to those developed in symmetric building (A)

Effect of the Nature of the PSDF of Ground Acceleration

The results shown in Figures (7) and (8) are obtained using PSDF, $S_{\ddot{u}g}(\omega)$ given by Eq.18 . The base torque is obtained again using the PSDF given by Eq.19 for firm soil condition and compared with that obtained using Eq.18 with V_s=600 m/sec. The two values are nearly the same indicating that the two spectra have the same energy and the spectrum given by Eq.18 is valid for firm soil condition. However,it is already known that different values of V_s represent different soil conditions,and consequently different dynamic characteristics of filters (ω_g ,ζ_g,ω_f ,and ζ_f) should be assigned for each soil type. Figure (5) shows these values for very soft and firm soils and the corresponding normalized spectra of ground acceleration derived from Eq.19.

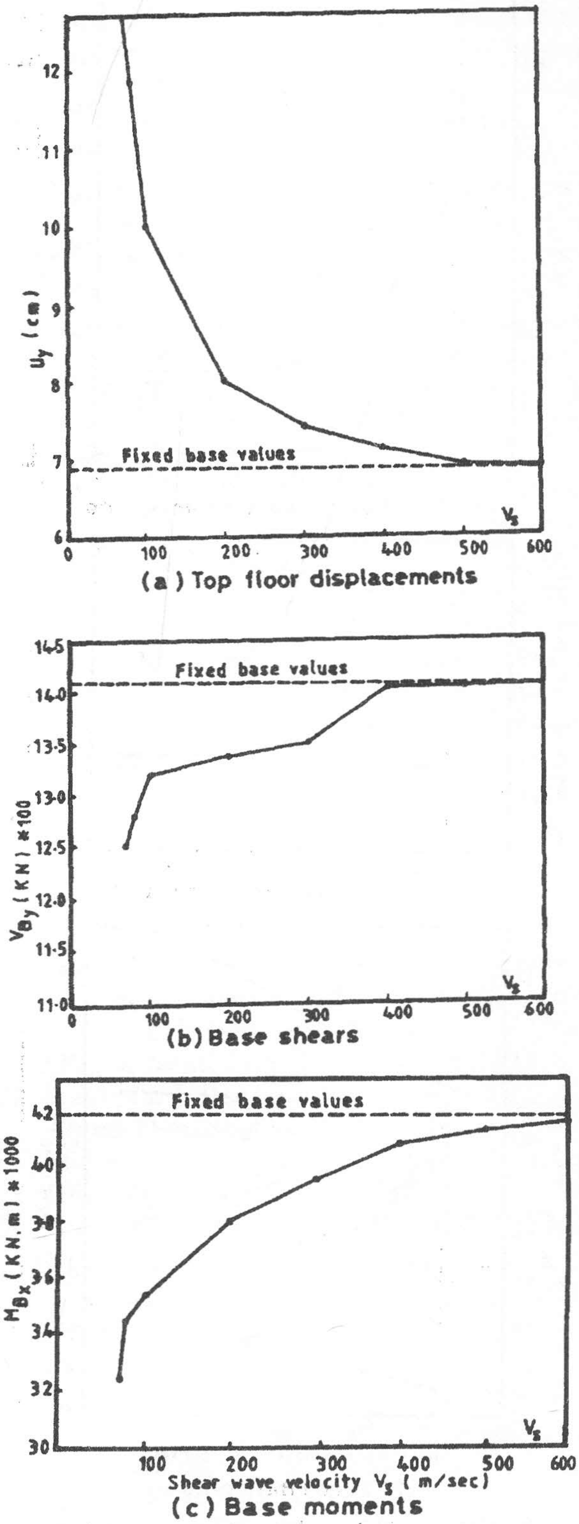

Figure 7. Effect of base flexibility on mean peak responses of symmetric building.

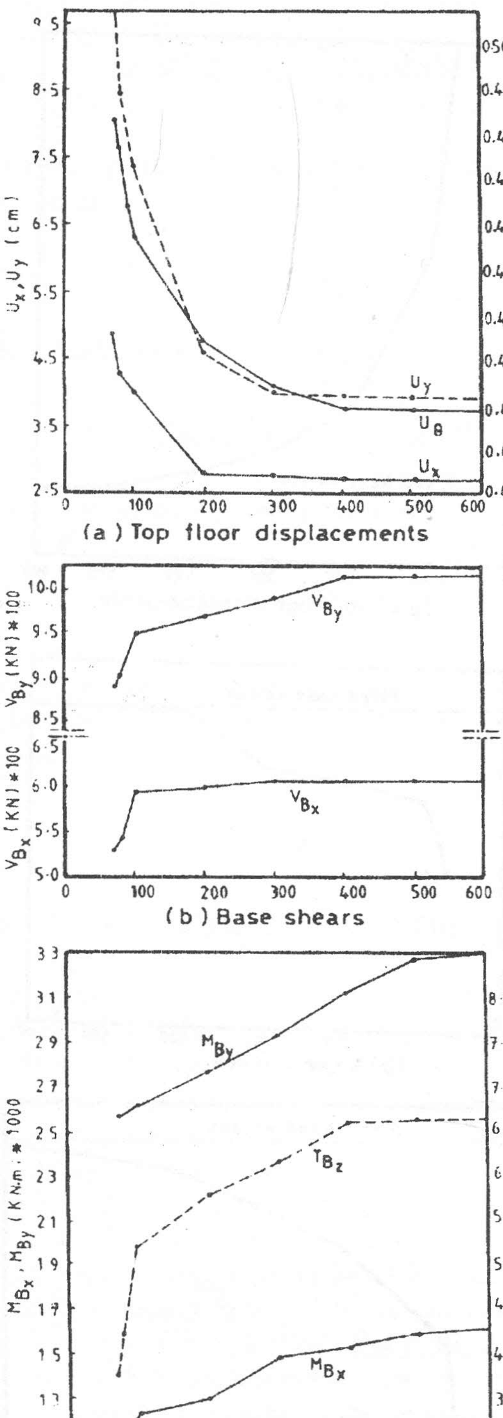

Figure 8. Effect of base flexibility on mean peak responses of unsymmetric building.

Figure (9) shows the PSDF $S_{T_Z}(\omega)$ of the base torque for the very soft,and firm soil conditions. It is clear that the soft soil condition (flexible base) may yield higher peaks (compare figures 9,10) and consequently greater torque than that for firm soil (fixed base) which is contrary to the expected results. This increase depends mainly on the closeness of the resonant frequency of the soil layer (ω_g=3.14) to the fundamental frequency of the building (ω_1=3.355)

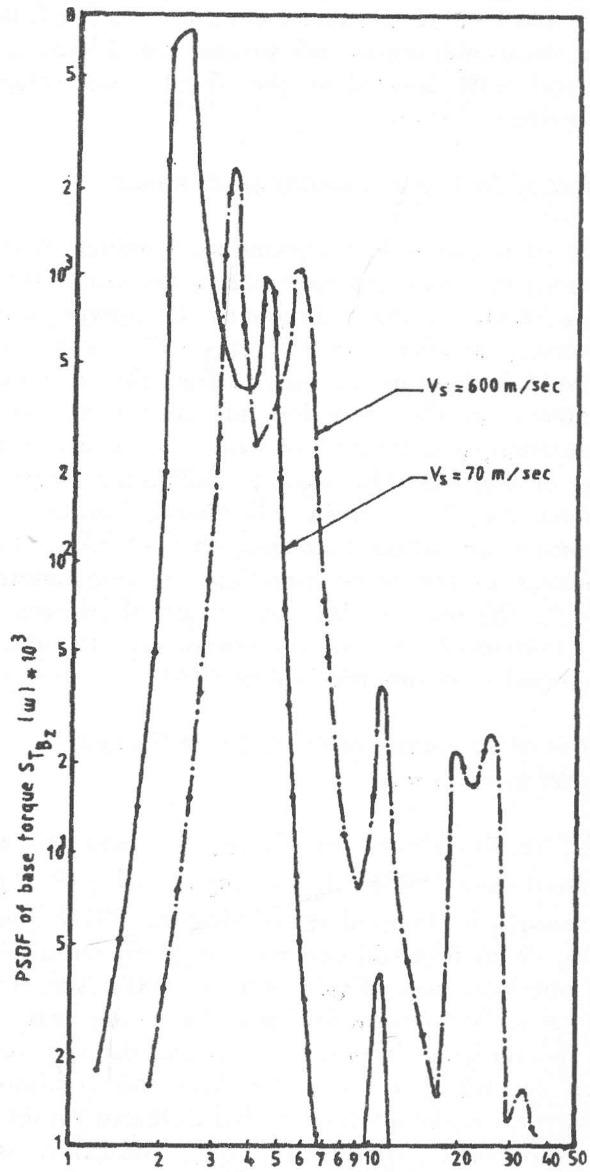

Figure 9. PSDF of base torque for unsymmetric building for different shear wave velocities using corresponding spectra (Eq. (19) and Fig. 5).

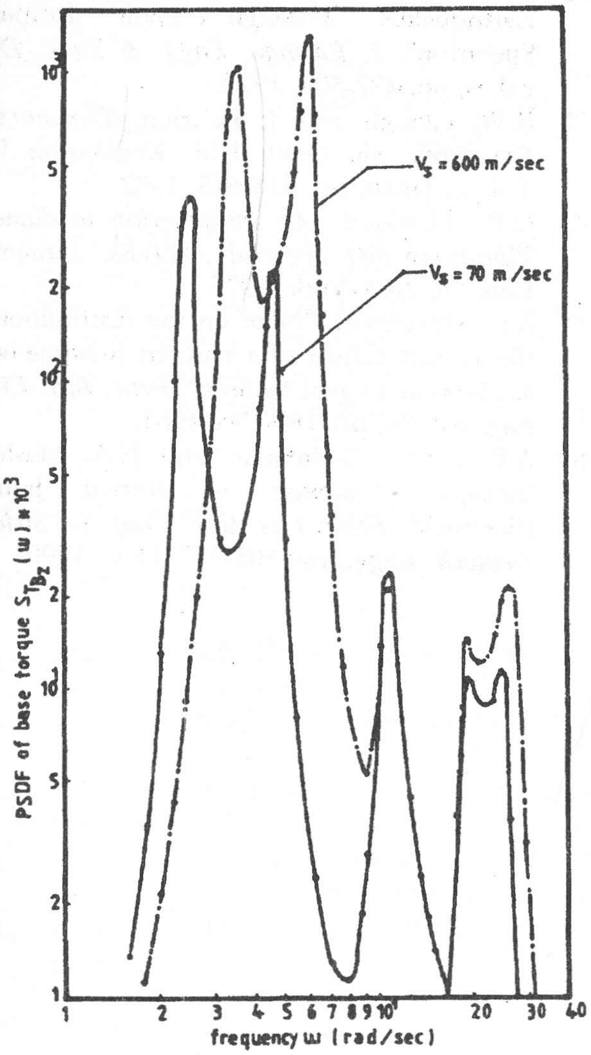

Figure 10. PSDF of base torque for unsymmetric building for different shear wave velocities using spectrum derived by Kaul's method (Eq. (18) and Fig. 4).

CONCLUSION

A method for analyzing the seismic response of 3-D tall buildings including foundation interaction has been presented. The system considered is a multistory shear building on a rigid footing attached to the surface of a linearly elastic half space. Frequency dependent impedance functions representing the foundation stiffness and damping are considered in the analysis. The complex frequency response functions due to unit harmonic ground acceleration (expressed in complex form) are obtained by only solving five complex equations using the normal modes of vibration of the fixed base building. The PSDF of response is obtained from that of earthquake acceleration using the spectral approach in the frequency domain.

The results of the present study show that:

1- With the increase of base flexibility (soft soil condition) the displacement response increases while the base forces are reduced. The increase of displacements may be as high as 100% while the decrease of base forces may be as much as 40%,for very soft soil condition. However,if the input excitation spectrum is also modified for the soil condition this trend does not appear to remain valid.

2- The torsional coupling in general influences considerably the response of tall buildings. The response quantities in the direction of earthquake are reduced due to this effect,while in other D.O.F's considerable responses are introduced.

3- The effect of soil-structure interaction on the torsional response of unsymmetric building is significant at lower frequencies. For flexible base condition the peaks of the PSDF of base torque occur at frequencies less than the fundamental frequency of the fixed base building.

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