MICROBENDING LOSS AND STRESSES INDUCED BY BOTH TEMPERATURE VARIATION AND AXIAL STRAIN IN MULTI-COATED OPTICAL FIBERS

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ABSTRACT

The microbending loss and the stresses induced by both the temperature variation and the axial strain in multi-coated optical fibers are studied. Optical fibers with two up to six coatings are considered. A recurrence relation is obtained for the lateral pressure on the glass fiber, from which the microbending loss and the stresses are calculated. It is found that the microbending loss can be minimized by increasing the number of coatings.

Key words: Microbending loss, Axial strain, Temperature variation, Multi-coated optical fibers.

1. INTRODUCTION

Optical fiber cables must be designed so that transmission characteristics of the fiber are maintained after the cabling process and cable installation. Certain problems may occur either within the cabling process or subsequently which can significantly affect the fiber transmission characteristics. The most important one is the meandering of the fiber axis on a microscopic scale with the cable form. This phenomenon, known as microbending, results from small lateral forces exerting on the fiber. These lateral forces are caused externally by axial stresses and internally by thermal ones. This yields losses due to radiation in both multimode and single-mode fibers.

Mechanical and thermal stresses play an important role in optical fibers, so, they have been studied extensively. Several reports have appeared on the temperature characteristics of jacketed optical fibers. D. Gloge computed the excess transmission loss resulting from statistical surface variations and lateral pressures affecting the fiber [1]. The loss induced in optical fibers by random bends in the fiber axis was studied experimentally by winding fibers under constant tension on to a drum surface that is not perfectly smooth [2]. The shrinkage of a plastic

jacket at low temperature causes microbends in the optical fibers and consequently an excess loss which increases suddenly at low temperature due to the fiber buckling [3]. The thermal stresses and the added transmission loss in both single and double-coated optical fibers at low temperature were calculated by Suhir[4] and Shiue[5]. Shiue evaluated the microbending loss induced by axial strain in double-coated optical fibers as well as the thermal stresses in tightly jacketed double-coated optical fibers at low temperature [6,7]. However, to the best of our knowledge, no work has been done on the problem of microbending loss induced by both external axial strain and thermal stresses in multi-coated optical fibers at low temperature. In this study we derived a recurrence relation for the lateral pressure and, consequently, the microbending loss in a multi-coated optical fibers. The objective of this study is to determine the best number of coatings which gives minimum added microbending loss. This can be accomplished by noting that the lateral pressure should be tensile, i.e., it should have a negative value. As a result, the optical properties of the fiber will not be affected under this condition [6].

2. ANALYSIS

2.1 Lateral pressures

2.1.1 Thermal stresses

Figure (1) illustrates the fiber under consideration which is constructed of a glass (core and cladding) coated by n polymeric layers. The radii of glass fiber cladding, primary coating, and nth coating are r_0, r_1, and r_n, respectively. The optical fiber is assumed infinite along the axial direction. Because the physical properties (e.g., Young's modulus and coefficient of thermal expansion) of the glass and polymeric coatings are different and assuming zero stress in the system at the initial temperature, stress will be built up after the temperature drop. Assuming the glass fiber is rigid compared with the polymeric coatings and the viscoelastic behavior of polymeric coating is not considered, then the thermal stresses in the multi-coated optical fiber can be solved starting from the Lame formulae as follows [8]:
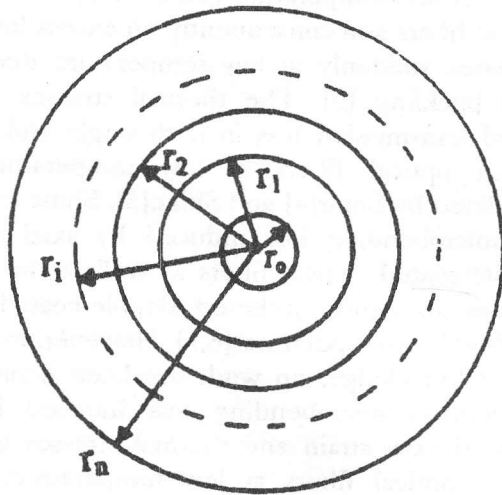

Figure 1. Schematic diagram of a multicoated optical fiber.

The Lame formulae for the radial stress, σ_r, and the tangential stress, σ_θ, in a circular thick-walled tube subjected to internal pressure p_i and external pressure p_e are [8]:

$$\sigma_r = \frac{a^2 b^2 (p_e - p_i)}{r^2 (b^2 - a^2)} + \frac{a^2 p_i - b^2 P_e}{b^2 - a^2}, \qquad (1\text{-}a)$$

and

$$\sigma_\theta = -\frac{a^2 b^2 (p_e - p_i)}{r^2 (b^2 - a^2)} + \frac{a^2 p_i - b^2 P_e}{b^2 - a^2}, \qquad (1\text{-}b)$$

where a and b are, respectively, the inner and outer radii of the tube. The radial (ϵ_r), tangential (ϵ_θ) and axial (ϵ_z) strains are given by [8]:

$$\epsilon_r = -\alpha \Delta T + \frac{1}{E} [\sigma_r - v (\sigma_\theta + \sigma_z)], \qquad (2\text{-}a)$$

$$\epsilon_\theta = -\alpha \Delta T + \frac{1}{E} [\sigma_\theta - v (\sigma_z + \sigma_r)], \qquad (2\text{-}b)$$

and

$$\epsilon_z = -\alpha \Delta T + \frac{1}{E} [\sigma_z - v (\sigma_r + \sigma_\theta)], \qquad (2\text{-}c)$$

where E, α, and v are the Young's modulus, coefficient of thermal expansion, and Poisson's ratio of the material, respectively. Here, the temperature drop ΔT is larger than zero. Because the fiber is assumed infinite and both ends are fastened, the plane strain is adopted, i.e., the axial strain ϵ_z is zero. Due to geometric symmetry, only the radial displacement u is not zero.

The radial and tangential strains are related to the radial displacement u by the following equations [8]:

$$\epsilon_\theta = \frac{u}{r} \qquad (3)$$

Using Eqs.(1), (2) and (3), the radial displacement, u, is obtained as:

$$u = -\alpha \Delta T (1 + v) r + \frac{1+v}{E(1-\gamma^2)} [r(1-2v)(p_i \gamma^2 - p_e) - \frac{a^2}{r}(p_e - p_i)], \qquad (4)$$

where $\gamma = a/b$.
Equation (4) is used to evaluate the thermal radial displacement at each boundary starting from the cladding and first coating interface. We've obtained general forms for the thermal displacements, $u_{i-1,i}$

and u_{ii}, for both of the outer boundary of the coating i-1 and the inner boundary of the coating i as:

$$u_{i-1,i} = -\alpha_{i-1} r_{i-1} \Delta T(1+v_{i-1}) + \frac{r_{i-1}(1+v_{i-1})}{E_{i-1}(1-\gamma^2_{i-1})}$$
$$[2p_{i-1}\gamma^2_{i-1}(1-v_{i-1}) - p_i(1-2v_{i-1}+\gamma^2_{i-1})], \quad (5\text{-}a)$$

and

$$u_{ii} = -\alpha_i \Delta T(1+v_i)r_{i-1} + \frac{r_{i-1}(1+v_i)}{E_i(1-\gamma_{i-1}^2)} [p_i + p_i\gamma_i^2(1-2v_i) -$$
$$2(1-v_i)p_{i+1}], \quad (5\text{-}b)$$

where i = 1,2,..,n. The conditions for compatibility of displacements are [8]:

$$u_{i-1,i} = u_{ii} \quad , \quad (6)$$

Using Eq.(6) and after algebraic manipulation, one can get a recurrence relation for the lateral pressures as:

$$R_{n-j}p_{n-j} = Q_{n-j} + T_{n-j}p_{n-j-1}, \quad (7)$$

where j = 0,1,..,n-1 and the coefficients R_{n-j}, Q_{n-j}, T_{n-j} are functions of the fiber and coatings parameters. Equation (7) can be used to calculate the lateral pressure on the fiber glass for any number of coatings.

2.1.2 Axial strain

In this case, the optical fiber is subjected to an external axial strain ϵ_z. The stress-strain relations are obtained by putting $\Delta T = 0$ in Eq.(2). Eliminating σ_z from these relations, one can get the tangential and radial strains as:

$$\epsilon_\theta = -v\epsilon_z + \frac{1-v^2}{E}(\sigma_\theta - \frac{v\sigma_r}{1-v}), \quad (8\text{-}a)$$

and

$$\epsilon_r = -v\epsilon_z + \frac{1-v^2}{E}(\sigma_r - \frac{v\sigma_\theta}{1-v}) \quad (8\text{-}b)$$

In a similar manner to the thermal stress analysis,

Eqs.(3) and (8-a) can be used to get the radial displacement, u, as:

$$u = -v\epsilon_z r + \frac{r(1-v^2)}{E}(\sigma_\theta - \frac{v\sigma_r}{1-v}) \quad (9)$$

By comparison, Eq.(7) can be used in the calculation of the lateral pressures, however the coefficient Q_{n-j} will be changed.

2.1.3 The general case

Here the optical fiber will subject to both external axial strain and temperature variations. The total axial stress is given directly from Eq.(2-c) as:

$$\sigma_z = \epsilon_z E + \alpha \Delta T E + v(\sigma_r + \sigma_\theta). \quad (10)$$

Following the same procedure previously described, one gets the total radial displacement as:

$$u = -\alpha \Delta T r(1+v) - v\epsilon_z r + \frac{r(1-v^2)}{E}[\sigma_\theta - \frac{v\sigma_r}{1-v}]. \quad (11)$$

Again, changing the coefficient Q_{n-j}, then Eq.(7) is practicable. Hence, using this equation, the lateral pressures can be evaluated as a function of the physical properties of the optical fiber as well as the environmental conditions.

2.2 Microbending loss

The compressive lateral pressure, p_1, in the glass fiber would produce excess microbending loss, so it should be minimized [4]. For j = n-1 in Eq.(7) the lateral pressure, p_1, can be written in the form:

$$p_1 = \frac{Q_1}{R_1} \quad (12)$$

The microbending loss, α, can be evaluated from the lateral pressure, p_1, through the linear relation [6]:

$$\alpha = G p_1, \quad (13)$$

where G is a constant, which has the experimentally determined value of 0.0029 (dB/km)/MPa [5]. Obviously, the developed recurrence relations can be

used to calculate the lateral pressure and consequently the microbending loss induced in multi-coated optical fibers.

2.3 *Normal stresses*

Based on Eq.(1) and putting $a = 0$, $P_i = 0$ and $p_e = p_1$, the radial stress σ_r and the tangential stress σ_θ in the glass fiber will simply equal to $-p_1$. However, the corresponding stresses in the ith coating will have forms:

$$\sigma_r = \frac{r^2_{i-1}r_i^2(p_{i+1}-p_i)}{r^2(r_i^2-r^2_{i-1})} + \frac{p_i r^2_{i-1} - p_{i+1}r_i^2}{r_i^2-r^2_{i-1}}, \qquad (14\text{-a})$$

and

$$\sigma_\theta = -\frac{r^2_{i-1}r_i^2(p_{i+1}-p_i)}{r^2(r_i^2-r^2_{i-1})} + \frac{p_i r^2_{i-1} - p_{i+1}r_i^2}{r_i^2-r^2_{i-1}}, \qquad (14\text{-b})$$

where $r_{i-1} \leq r \leq r_i$.

In the case of axial stress, σ_z, it is desirable to discuss the general case, where thermal stresses are included. According to Eq.(2), σ_z in the glass fiber and in the ith coating are given respectively by:

$$\sigma_z = E_0\alpha_o\Delta T + E_0\epsilon_z - 2\nu_0 p_1 \quad ,0 \leq r \leq r_0 \quad (15)$$

and

$$\sigma_z = 2v_i\frac{p_i r^2_{i-1} - p_{i+1}r_i^2}{r_i^2-r^2_{i-1}} + E_i(\epsilon_z + \alpha_i\Delta T) \quad ,r_{i-1} \leq r \leq r_i \qquad (16)$$

It is clear that σ_z is uniform within each medium. However, both σ_r and σ_θ are dependent on r except for the region $0 \leq r \leq r_0$, Eq.(14). Thus, the tangential and the radial stresses are uniform in the glass fiber. This will not be the case if any lateral pressure on the core-cladding interface is considered. For instance, if p_0 is not equal zero, then p_1 will be given by:

$$p_1 = \frac{Q_1}{R_1} + \frac{p_0 T_1}{R_1}, \qquad (17)$$

and p_0 will be calculated from Eq.(7) by allowing the

value n for j. Hence, the Young's modulus, the coefficient of thermal expansion, and Possion's ratio must be considered separately for both the core and cladding in any calculations. In addition, σ_r and σ_θ are independent on r in the core only.

Equation (14-a) indicates that as r increases, the radial stress changes from the initial value $-p_1$ till the final value p_{n+1} which is assumed to be zero. However, the sign of the lateral pressure depends on the values of E, ν, and γ. Therefore, the radial stress may increase or decrease as $1/r^2$ depending on the sign of the lateral pressures. On the other hand, the dependence of σ_θ on r is the opposite, because of the negative sign in Eq.(14-b).

3. RESULTS AND DISCUSSION

A computer program is developed for the purpose of lateral pressure calculations in optical fibers. For evaluation purposes, this program has been used to calculate the lateral pressure p_1 using the parameters in the first and second designs of Shiue [5,6]. The results are identical to that obtained by in both cases; namely the axial strain and temperature variation. Furthermore, the lateral pressure relations derived by Shiue are obtained analytically [5,6]. This gives us the confidence regarding the wider use of the model in the case of multi-coated optical fibers. The value of p_1 depends on the elastic properties of both the optical fiber and the coatings as well as their radii. In addition, for a real polymeric coating material the Young's modulus, E, and the coefficient of thermal expansion, α, are dependent on each other and could be characterized by a linear relationship [4]:

$$\alpha = \alpha_*(1 - \frac{E}{E_*}), \qquad (18)$$

where α_* is the coefficient of thermal expansion for a polymeric material with a very low Young's modulus, and E_* is the Young's modulus for a polymeric material with a very low coefficient of thermal expansion . The values of α_* and E_* are 6.89×10^{-5} °K^{-1} and 11.35 GPa, respectively [4]. We have considered the general case in which the optical fiber is subjected to a temperature drop $\Delta T = 100$ °K as well as an axial strain $\epsilon_z = 0.001$ [9].

Furthermore, the parameters of the optical fibers under consideration are shown in table 1. For $n = 2$ (double coated optical fiber), these parameters are those considered by Shiue [6]. In addition, the optical fiber is assumed to be rigid so that $E_0 >> E_1$ and $\alpha_0 << \alpha_1$, and the cladding radius $r_0 = 62.5$ μm.

Table 1. The parameters of the optical fibers under consideration.

n	ν	E (MPa)	r (μm)
1	0.35	10	100
2	0.35	1200	125
3	0.35	2755	200
4	0.35	3600	250
5	0.35	4290	400
6	0.35	5790	500

Based on Eq.(18), the effects of thicknesses of the primary and the secondary coatings on the lateral pressure in the glass fiber are shown in Figures (2) and (3), respectively.
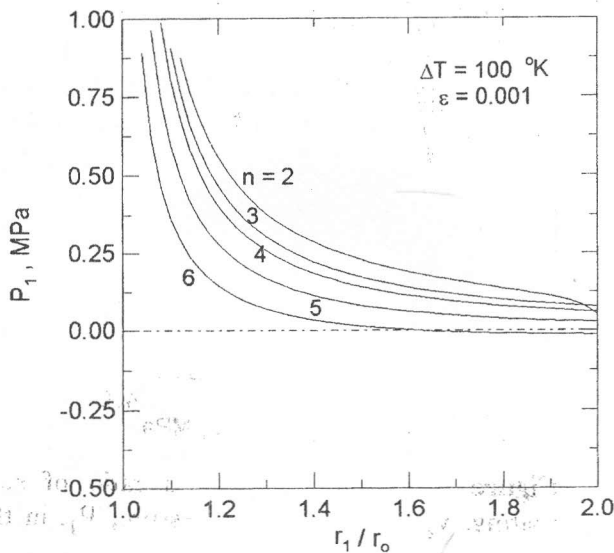

Figure 2. Effect of thickness of primary coating, r_1/r_0, on the lateral pressure, P_1, in the glass fiber.
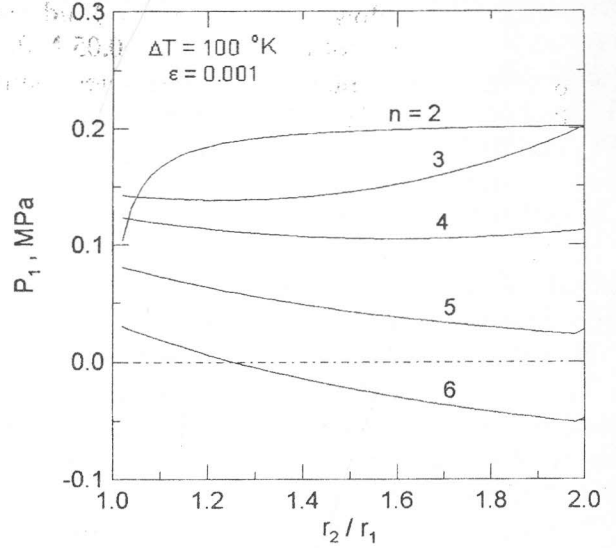

Figure 3. Effect of thickness of secondary coating, r_2/r_1, on the lateral pressure, P_1, in the glass fiber.

It can be seen from Figure (2) that p_1 decreases with the thickness of the primary coating to very small positive values except for the fiber with $n = 6$. In this case, p_1 becomes negative for $r_1/r_0 > 1.62$, and consequently there is no added microbending loss. Therefore, it is recommended that the thickness of the primary coating should be increased. It may be also noted from Figure (3) that p_1 takes small negative values at $n = 6$ provided that $r_2/r_1 > 1.26$. Obviously, as the number of coating increases the curve of p_1 vs r_n/r_{n-1} approaches zero.

The influence of Poison's ratio of the primary coating on the lateral pressure is illustrated in Figure (4). For $n=5$ and $n = 6$ p_1 takes the closest values to the zero, and it turns negative at $\nu_1 \approx 0.45$ and 0.36, respectively. However, as ν_1 increases the lateral pressure, in the former case ($n = 5$), takes higher negative values faster than the later ($n = 6$). Therefore, the fiber with $n = 6$ is more practicable since it maintains the smallest negative values for p_1 when ν_1 changes appreciably. It can be observed from Figure (5) that p_1 is negative for $n = 6$ only at $\nu_2 > 0.36$.

Figure (6) shows the relation between p_1 and E_1. Generally, p_1 increases with E_1 and takes higher positive values, so, there is a microbending loss in this case. Obviously, The Young's modulus of the primary coating must be assigned small values to

So, it is clear that as n increases the lateral pressure on the optical fiber decreases. In addition, the lateral pressure can be kept tensile with negligible values by choosing the suitable coating materials. On the other hand, p_1 should be assigned a suitable negative value to avoid any transition to the compressive region.

4. CONCLUSIONS

The microbending and the stresses induced by both the temperature variation and the external axial strain in multi-coated optical fibers are studied. The microbending due to the temperature variation is additive to that induced by the axial strain. Both the microbending losses and the stresses are proportional to the temperature variation and to the axial strain. In addition, they depend on the physical properties of the coating materials as well as their radii. In order to maintain the optical fiber without excess microbending loss the lateral pressure p_1 must be tensile. The conditions under which p_1 is tensile are determined for the optical fiber with n = 6. It is found that the lateral pressure p_1, and hence the microbending loss, on the glass fiber can be minimized by increasing the number of coatings. The final selection of the most suitable coating design must be based on the material, the optical, the chemical, and the cost consideration.

REFERENCES

[1] D. Gloge, "Optical-Fiber Packaging and Its Influence on Fiber Straightness and Loss," *Bell Syst. Tech. J.*, vol. 54, No. 2, pp. 245-262, 1975.

[2] W.B. Gardner, "Microbending loss in optical fibers," *Bell Syst. Tech. J.*, vol. 54, No. 2, pp. 457-465, 1975.

[3] T. Yabuta, N. Yoshizawa, and K. Ishihara, "Excess loss of single-mode jacketed optical fiber at low temperature," *Appl. Opt.*, vol. 22, No. 15, pp. 2356-2362, 1983.

[4] E. Suhir, "Mechanic Approach to the Evaluation of the Low Temperature Threshold of Added Transmission Losses in Single- Coated Optical Fibers," *IEEE J. Lighwave Technol.*, vol. 8, No. 6, pp. 863-868, 1990.

[5] S.T. Shiue and S. Lee, "Thermal stresses in double-coated optical fibers at low temperature," *J. App. Phys.*, vol.72, No.18, pp. 18-23, 1992.

[6] S.T. Shiue", Axial strain-induced microbending losses in double-coated optical fibers," *J. App. Phys.*, vol. 73, No. 15, pp. 526-529, 1993.

[7] S.T. Shiue, "Thermal stresses in tightly jacketed double- coated optical fibers at low temperature", *J. App. Phys.*, vol. 76, No. 15, pp. 7695-7703, 1994.

[8] S. Timoshenko and J.N. Goodier, *Theory of Elasticity*, 2nd ed., McGraw-Hill Book Company, Inc., 1951.

[9] Morton I. Schwartz, Paul F. Gagen and Manuel R. Santana, "Fiber Cable Design and Characterization," *Proc. IEEE*, vol. 68, No.10, pp. 1214-1219, 1980.