ECONOMIC DESIGN AND ANALYSIS OF TRICKLE IRRIGATION LATERALS

Ahmed I. Al-Amoud, Helmi M. Hathoot
Dept. of Agric. Engrg., Col. of Agric.,
King Saud Univ., Riyadh, Saudi Arabia

and Ahmed S. Al-Mesned
Grad. Stud. Dept. of Agric. Engrg.
Col. of Agric., King Saud Univ.

ABSTRACT

This paper addresses the economic and environmental technical design of trickle irrigation laterals of constant slope and individual equally spaced emitters. The land topography, slope and irrigation water temperature reflect the impact of environment on lateral design. In the present analysis the change of the velocity head and the change of momentum along a lateral are considered. Friction loss is estimated by using the Darcy-Weisbach equation taking in to account the proper choice of the friction coefficient equation according to Reynolds' number. A new computer program for estimating lateral discharge, emitter discharge and pressure head distributions along a lateral is presented. The program provides results close to experimental data (obtained from literature) though it is much simpler than other previously presented programs. The program is developed to be used for designing lateral's economic diameter or length within a high level of distribution uniformity. Two numerical examples for different cases of economic design are provided.

Keywords: Trickle Irrigation, Laterals, Friction loss.

Notations

The following symbols are used in this paper:

A	cross-sectional area of lateral pipe;
a	spacing between emitters;
a_1	distance from main, or submain and the first emitter;
B	quantity defined by Eq.14;
C	emitter constant contained in Eq.1;
C_1	levelized net annual cost of emitter pipes per m ($);
C_2	levelized net annual cost of power per kW ($);
C_p	levelized net annual cost of lateral pipe ($);
C_{pw}	levelized net annual cost of power ($);
D	lateral diameter;
DIF	$Q_{max.} - Q_o$;
d	constant contained in Eq.29;
E	quantity defined by Eq.15;
E_o	quantity defined by Eq.19;
e	constant contained in Eq.29;
f	coefficient of friction;
f_i	coefficient of friction for the lateral reach downstream from emitter i;
g	acceleration due to gravity;
H	pressure head;
H_{av}	average pressure head;
H_i	pressure head at emitter i;
H_n	pressure head at the last downstream emitter;
H_o	pressure head at the main or submain;
h	friction head loss;
h_i	friction head loss at the lateral reach downstream from emitter i;
L	distance between first and last emitters;
L`	distance from main or submain and the last downstream emitter;
n	number of emitters;
PW	power required at the mainline end of lateral;
Q	lateral discharge;
Q_i	lateral discharge at the reach downstream from emitter i;
Q_{max}	maximum lateral discharge;
Q_n	lateral discharge downstream from the last downstream emitter;

Q_o	lateral discharge upstream from the first emitter;
q	emitter discharge;
q_{av}	average emitter discharge;
q_i	discharge from emitter i;
q_n	discharge from the last downstream emitter;
R	Reynolds' number;
S_o	lateral longitudinal slope;
U_c	Christiansen uniformity coefficient;
U_{cc}	the minimum acceptable value of Christiansen uniformity coefficient;
V	average lateral velocity;
y	emitter discharge exponent in Eq.1;
Z	potential head above an arbitrary datum;
Z_i	potential head at emitter i;
γ	specific weight of irrigation water;
Δ	quantity defined by Eq.25;
ΔH_n	quantity defined by Eq.11;
ϵ	small quantity in the order of 0.001;
η	pump efficiency; and
υ	kinematic viscosity of irrigation water.

INTRODUCTION

Trickle irrigation laterals are, generally, polyethylene pipes of constant slope and fitted with similar and equally spaced emitters whose discharge usually decreases in the downstream direction. Many investigators provided approximate solutions for the problem of trickle irrigation lateral design. Among the earlier investigators were Perold (1977); Watters and Keller (1978). Gellespie et al. (1979); Khatri et al. (1979). In their treatments they generally used approximate friction equations such as Hazen-Williams and Scobey, neglected the variation of the velocity head along the lateral and assumed initial uniform emitter flow. Warrick and Yitayew (1988) assumed a lateral with a longitudinal slot and presented design charts based on spatially varied flow. The latter solution has neglected the presence of laminar flow in a considerable length of the downstream part of the lateral. Hathoot et al. (1991) provided a solution based on uniform emitter discharge but took into account the change of velocity head and the variation of Reynolds' number. They used the Darcy-Weisbach friction equation in estimating friction losses. Hathoot et al. (1993) considered individual emitters with variable outflow and presented a step-by step computer program for designing either the diameter or the lateral length. In this paper the authors aim at presenting a solution which accounts for the actual flow nature, variation of Reynolds' number and the change of velocity head. They also aim at presenting a simple computer program for economic lateral designs which satisfies an acceptable level of uniformity.

THEORETICAL DEVELOPMENT

In trickle irrigation laterals emitters are usually installed at equal spacing, a, Figure (1). The first upstream emitter is generally at a different spacing, a_1, from the main or submain. The emitter discharge is a function of the pressure head and the relationship between them may be expressed as

$$q = C\,H^{y} \tag{1}$$

in which q is the emitter discharge, C is a constant, H is the pressure head acting on the emitter and y is a constant which depends on the state of flow and generally ranges between zero and 1.0, Hathoot et al. (1993) and Wu and Yue (1993). Referring to Fig.1, if there are n emitters, the distance between the first and last ones is given by

$$L = (n-1)a \tag{2}$$

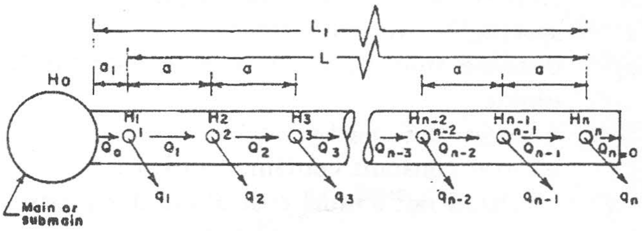

Figure 1. Lateral pipe and emitters.

On the other hand the total length of the lateral is:

$$\begin{aligned} \grave{L} &= a_1 + L \\ &= a_1 + (n-1)a \end{aligned} \tag{3}$$

starting from the last downstream emitter, its discharge is given by:

$$q_n = C\,H_n^y \tag{4}$$

The lateral discharge downstream from the emitter n should be zero and hence:

$$Q_n = 0 \tag{5}$$

On the other hand the lateral discharge upstream from emitter n should equal the emitter discharge:

$$Q_{n-1} = q_n \tag{6}$$

Applying the energy equation between emitters n and n-1:

$$H_{n-1} + Z_{n-1} + \frac{Q_{n-1}^2}{2gA^2} = H_n + Z_n + \frac{Q_n^2}{2gA^2} + h_{n-1} + \Delta H_n \tag{7}$$

in which Z_{n-1} and Z_n are potential heads at emitters n-1 and n, respectively, A is the cross-sectional area of lateral, g is the acceleration due to gravity, h_{n-1} is the friction head loss at the lateral reach downstream from emitter n-1 and Δh_n is the change of pressure lead due to the change of momentum at emitter n, Streeter and Wylie (1979). Solving Eq.7 for H_{n-1}:

$$H_{n-1} = H_n + Z_n - Z_{n-1} + \frac{\left(Q_n^2 - Q_{n-1}^2\right)}{2gA^2} + h_{n-1} + \Delta H_n \tag{8}$$

For a lateral having a longitudinal slope S_o:

$$Z_n - Z_{n-1} = \pm\, aS_o \tag{9}$$

the positive sign corresponds to laterals sloping upwards and vice versa.

For accurate estimation of frictional head loss for smooth irrigation laterals, the Darcy Weisbach equation is the most reliable, Watters and Keller (1978). therefore

$$h_{n-1} = \frac{8 f_{n-1}\, a\, Q_{n-1}^2}{\pi^2\, g\, D^5} \tag{10}$$

in which f_{n-1} is the coefficient of friction for the reach n-1, and D is the lateral diameter.

The change of pressure head due to the change of momentum, Featherstone and Nalluri (1982), is given by:

$$\Delta H_n = \frac{\left(Q_n^2 - Q_{n-1}^2\right)}{gA^2} \tag{11}$$

Substitution from Eqs.9, 10 and 11 into Eq.8 and taking into account that $Q_{n=0}$:

$$H_{n-1} = H_n \pm aS_o - \frac{Q_{n-1}^2}{2gA^2} + \frac{8 f_{n-1} a Q_{n-1}^2}{\pi^2 g D^5} - \frac{Q_{n-1}^2}{gA^2} \tag{12}$$

For convenience, Eq.12 is put in the form:

$$H_{n-1} = H_n - BQ_{n-1}^2 + Ef_{n-1}Q_{n-1}^2 \pm aS_o \tag{13}$$

in which

$$B = \frac{3}{2gA^2} \tag{14}$$

and

$$E = \frac{8a}{\pi^2 g D^5} \tag{15}$$

If two successive intermediate emitter i and i-1 are considered it is easy to apply the above procedure to get the following equation:

$$H_{i-1} = H_i + B\left(Q_i^2 - Q_{i-1}^2\right) + Ef_{i-1}Q_{i-1}^2 \pm aS_o \tag{16}$$

It is worthy to note that emitter and lateral discharges are interrelated by:

$$Q_{i-1} = Q_i + q_i \tag{17}$$

Now the pressure head at the main or submain, H_0 can be evaluated from:

$$H_0 = H_1 + B\left(Q_1^2 - Q_0^2\right) + E_0 f_0 Q_o^2 \pm a_1 S_0 \tag{18}$$

in which H_1 is the pressure head at the first upstream emitter, Q1 is the lateral discharge downstream from the first emitter, Q_0 is the maximum lateral discharge, f_0 is the coefficient of friction for the first upstream reach of the lateral and E_0 is given by:

$$E_0 = E \frac{a_1}{a} \tag{19}$$

THE COEFFICIENT OF FRICTION

Flow of water through smooth trickle irrigation laterals is generally turbulent. In some cases flow is laminar for a considerable length of the downstream reach of the lateral pipe, Hathoot et al. (1993). For laminar flow where R ≤ 2000 the coefficient of friction is given by:

$$f = \frac{64}{R} \tag{20}$$

in which R is Reynolds number given by:

$$R = \frac{VD}{\nu} \tag{21}$$

in which V is the average velocity and υ is the kinematic viscosity of water.
For turbulent flow ($3,000 < R \leq 10^5$) the Blasius equation can be used:

$$f = 0.316\, R^{-0.25} \tag{22}$$

For fully turbulent flow, $10^5 < R < 10^7$, Watters and Keller (1978) recommended the following equation:

$$f = 0.13\, R^{-0.172} \tag{23}$$

THE MAIN COMPUTER PROGRAM

The flow chart of the main computer program is shown in Figure (2). The objective of the program is to provide pressure, lateral discharge and emitter discharge distributions along the lateral. The program is designed to be simple and accurate and the main steps of the program are as follow:

1. The essential data such as the lateral length, diameter, slope, spacing between emitters, the average emitter discharge, the average pressure head, the constant C, the exponent y ... etc. should be known in advance.
2. The maximum lateral discharge at the upstream end of the lateral is to be evaluated from:

$$Q_{max} = q_{av}\,(n) \tag{23}$$

in which q_{av} is the average emitter discharge and n is the number of emitters.

3. Calculations are to be started at the downstream end of the lateral and the last emitter discharge, q_n, is assumed equal to the average discharge, q_{av} as a first approximation of the first trial cycle. The pressure head, H_n is then evaluated by using Eq.4.
4. The pressure head at the emitter n-1 is estimated according to Eq.12 taking into account that the discharge $Q_{n-1} = q_n$, Eq.6.
5. The emitter discharge of the second emitter, q_{n-1}, is estimated according to Eq.1.
6. The pressure head at the third emitter is estimated according to Eq.16 taking into account Eq.17 and then the emitter discharge is evaluated according to Eq.1.
7. Calculations similar to those performed in (6) are repeated from emitter to the next in the upstream direction until the first emitter close to the main or submain is reached.
8. The pressure head at the mainline end of the lateral, H_0 is evaluated by using Eqs.18 and 19.
9. If the assumption $q_n = q_{av}$ is correct, which is not expected, the estimated lateral discharge, Q_o will equal the maximum discharge, $Q_{max.}$, otherwise correction should be made.
10. The difference $Q_{max.} - Q_o$ is estimated and the corrective emitter discharge to be added to the assumed q_n is given by:

$$\Delta = \frac{Q_{max.} - Q_o}{n} \tag{25}$$

11. Steps 3 through 10 are to be repeated with the new emitter discharge given by:

$$q_n = q_n + \Delta \tag{26}$$

12. Trial cycles are continued until the difference $DIF = Q_{max.} - Q_o$ becomes practically small such that:

$$\frac{|100 \times DIF|}{Q_{max}} \leq \epsilon \tag{27}$$

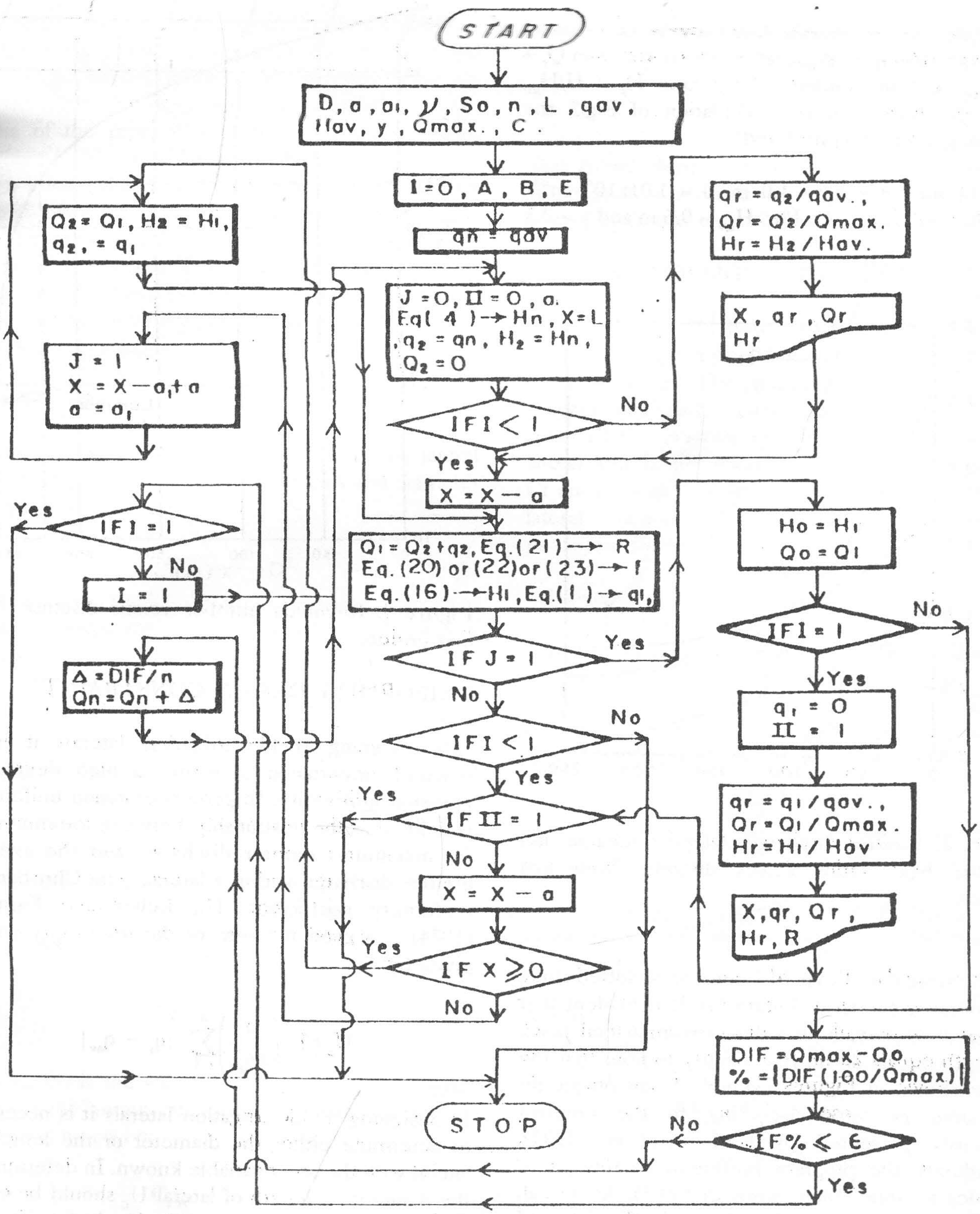

Figure 2. Flow chart for the main program.

in which ϵ is a small quantity in the order of 0.001.

In Figure (3) are shown distributions of emitter discharge ratio $q_r = q/q_{av}$, lateral discharge ratio $Q_r = Q/Q_{max.}$ and the pressure head ratio $H_r = H/H_{av}$ along the lateral. In the calculation of Fig.3 the following data are considered:

$D = 14$ mm, $a = a_1 = 1.0$ m, $\upsilon = 1.01\text{x}10^{-6}$ m^2/s. $S_o=0.0$, $n=251$, $q_{av}=4.0$ l/hr, $H_{av} = 9.6$ m and $y = 0.5$.

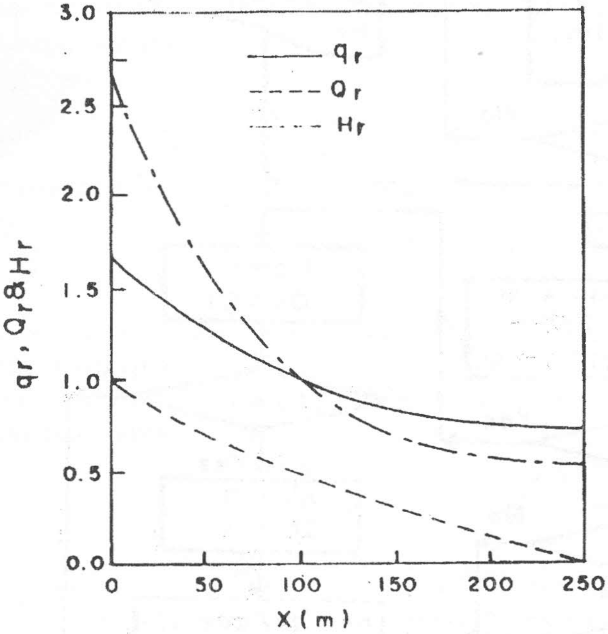

Figure 3. Emitter discharge, lateral discharge and pressure head ratios versus distance from first emitter.

For the same data Reynolds' number is plotted along the lateral as shown in Figure (4). It is evident that laminar flow prevails in a downstream lateral reach of length equals 28 m. It is worthy to note that the results shown in Figures (3) and (4) are practically the same as those according to the program previously presented by Hathoot et al. (1993). Accordingly the program presented in this paper provides accurate results since that of Hathoot et al. (1993) was experimentally checked, Warrick and Yitayew (1988) and Yitayew and Warrick (1988).

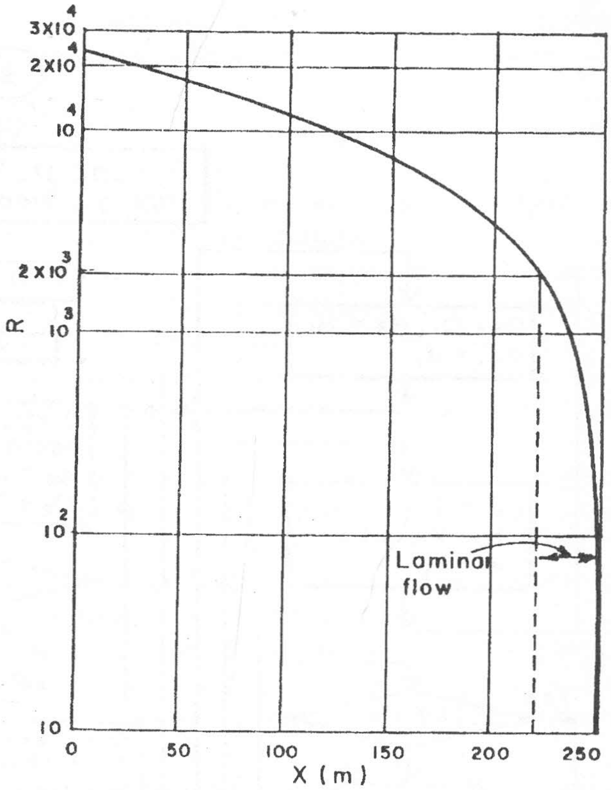

Figure 4. Reynolds number versus distance from first emitter.

UNIFORMITY DESIGN CONSTRAINT

In designing trickle irrigation laterals it is of practical importance to assure a high degree of emission uniformity. In general emission uniformity is defined as the relationship between the minimum (or maximum) emitter discharge and the average emitter discharge within a lateral. The Christiansen uniformity coefficient, U_c, Keller and Karmelli (1974) is a good measure of the uniformity and is given by:

$$U_c = 1 - \left(\frac{1}{nq_{av}}\right)\sum_{i=1}^{n} |q_i - q_{av}| \tag{28}$$

In designing trickle irrigation laterals it is necessary to determine either the diameter or the length of lateral with the other variable known. In determining the diameter or length of lateral U_c should be equal to or greater than an acceptable level of uniformity U_{cc} which is taken herein 0.95.

ECONOMIC DESIGN CONSIDERATIONS

The main factors affecting the economic design of a pipe are the initial investment cost of pipes and pumps, the annual operating and maintenance costs for the life of the pipes and pumps, and the salvage value of the pipe, Albertson and Simons (1960), Hathoot (1980) and Hathoot (1984).

It is known that a pipe of large diameter produces a small head loss against which pumps should act. On the other hand, through a pipe of a smaller diameter is cheaper, it produces a greater friction head loss, Hathoot (1986). Therefore an economic design is that in which the total annual cost of pipes and power is a minimum.

Cost of Pipes

Investigation of the cost of trickle irrigation lateral polyethylene pipes shows that the cost and diameter are interrelated by:

$$C_1 = d + eD^2 \tag{29}$$

in which C_1 is the levelized net annual cost per m of lateral, d and e are constants. Therefore the cost of pipe is

$$C_p = C\grave{L} \tag{30}$$

in which L` is given by Eqn.3.

Cost of Power

For a pressure head H_o at the mainline end of the lateral and a maximum discharge $Q_{max} = Q_o$ the required power of pump is

$$PW = \frac{\gamma Q_o H_o}{\eta} \tag{31}$$

in which γ is the specific weight of irrigation water and η is the pump efficiency. Therefore the cost of energy is given by:

$$C_{PW} = C_2\, PW \tag{32}$$

in which C_2 is the levelized net annual cost of power per k.W.

The Total Cost

The sum of the costs of pipe and power represents the total cost given by:

$$C_t = C_p + C_{pw} \tag{33}$$

The lateral design which satisfies the uniformity constraint with a minimum value of the total cost, C_t is the required economic design.

ECONOMIC DESIGN COMPUTER PROGRAM

In Figure (5) is shown the flow chart of the computer program provided to design the economic lateral diameter. The program is based on the main program of Figure (2). The program is designed to cover all the practical lateral diameters for a given lateral length. For each diameter Uc, Cp, Cpw and Ct are evaluated to help in selecting the economic lateral diameter. In example 1 is shown the procedure followed in lateral design.

Example 1

It is required to find the economic lateral diameter for the following data: $q_{av} = 4.0$ l/hr, $H_{av} = 9.6$ m, $a = a_1 = 1.0$ m, $S_o = 0.001$, $\upsilon = 1.01\text{x}10^{-6}$ m^2/S, y = 0.5, n = 251, $\eta = 0.66$, $\gamma = 9.81$ kN/m^3 and the costs of power and pipes are as given in Table 1.

Table 1.

Levelized net annual cost of power per kW ($)	Levelized net annual cost of emitter pipes per m ($)					
	D=13mm	D=16mm	D=19mm	D=25mm	D=32mm	D=40 mm
170	0.0173	0.0193	0.0230	0.0267	0.0410	0.0567

Solution

The annual cost per unit length of emitter pipes is successfully fitted by the following equation:

$$C_1 = 0.0126 + 27.533\, D^2 \tag{34}$$

in which C_1 is in \$/m and D in m.

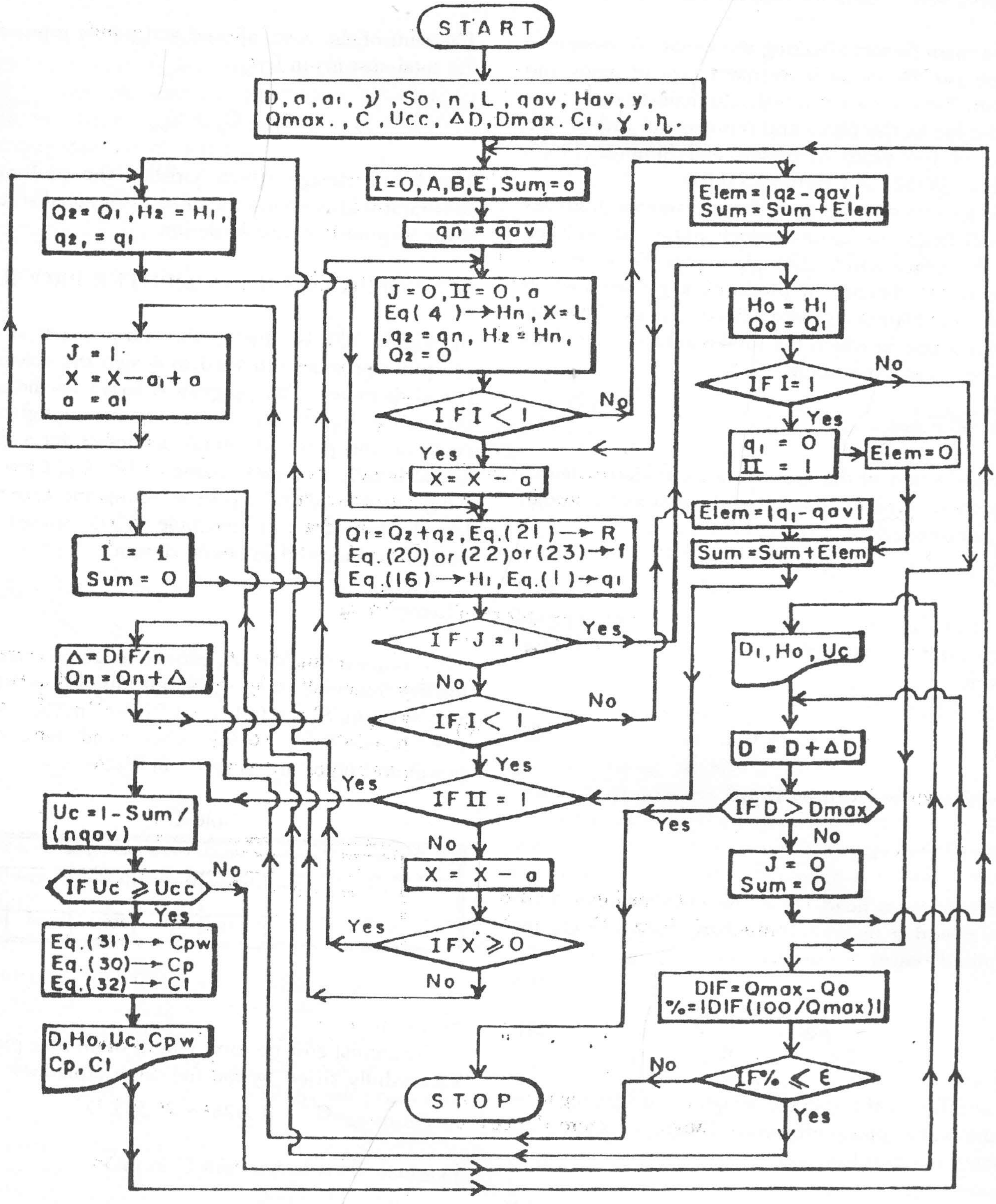

Figure 5. Flow chart for the optimal diameter program.

Figure (6) is plotted on the basis of the design program of Figure (5). It is evident from Figure (6) that the minimum lateral diameter which fulfills the uniformity constraint, Uc ≥ 0.95 is D = 22 mm. Although the cost of power decreases as the pipe diameter increases, the cost of lateral increases at a higher rate such that the total cost increases on increasing the pipe diameter. Therefore the economic lateral diameter in this example is D = 22 mm.

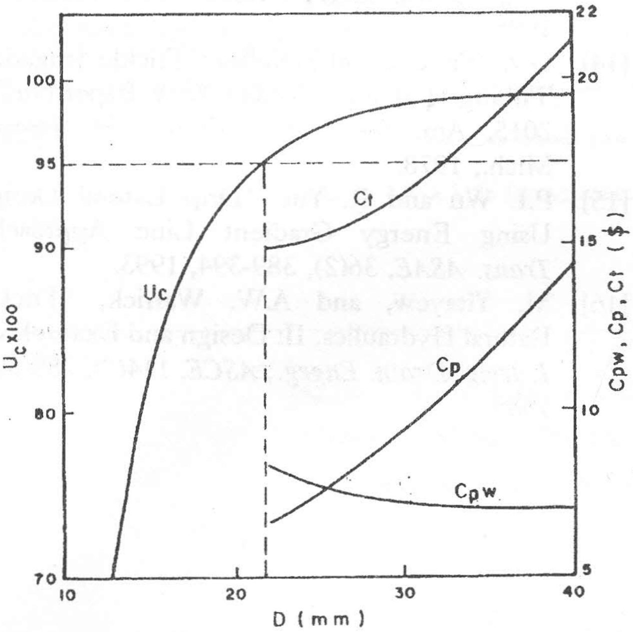

Figure 6. Design results of example 1.

As mentioned before for a given lateral diameter the pipe length which satisfies the design criteria may be required. In such cases a computer program similar to that of Figure (5), but with variable length of lateral and constant diameter may be used.

Example 2

For the same data of example 1 it is required to find the lateral length provided that the diameter = 16 mm and So = -0.002.

Solution

The solution of example 2 is provided in Figure (7). The Christiansen uniformity coefficient, U_c, decreases as the lateral length increases. This is because as the lateral length increases friction losses increase and hence the difference between operating pressure heads increases. In Figure (7) it is evident that the maximum allowable lateral length is 167.0 m. Lateral lengths less than 167.0 m correspond to more economic designs and hence the optimal lateral length is chosen according to practical requirements.

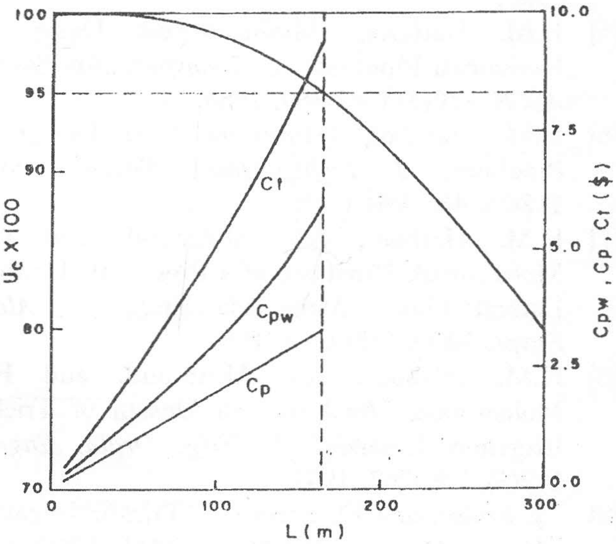

Figure 7. Design results of example 2.

CONCLUSION

Although the computer program, presented in this paper, is simple and logic it provides accurate results which are close to results of other recognized programs and to experimental data. Two numerical examples are given for designing either the lateral diameter or the lateral length for a certain diameter. In the first example the economic lateral diameter which satisfies a high level of uniformity is determined. In the second example the maximum economic lateral length is evaluated and it is found that practical requirements are the criteria for choosing the design length of lateral.

REFERENCES

[1] M.B. Albertson, and D. Simons, *Fluid Mechanics for Engineers*, Prentice Hall, New York, N.Y., 1960.

[2] R.E. Featherstone, and C. Nalluri, *Civil*

Engineering Hydraulics, Essential Theory, Worked Examples Granda Publishing, New York, N.Y. 1982.

[3] V.A. Gellespie, A.L. Phillips, and I.P. Wu, "Drip Irrigation Design Equations," *J. Irrig. Drain. Div., ASCE*, 105(3), 247-257, 1979.

[4] H.M. Hathoot, "Optimum Design of Irrigation Pipelines," Bulletin, International Commission on Irrigation and Drainage (ICID), 29(2), 73-76, 1980.

[5] H.M. Hathoot, "Minimum-cost Design of Horizontal Pipelines," *J. Transportation Engrg., ASCE*, 110(3), 382-389, 1984.

[6] H.M. Hathoot, "Minimum Cost Design of Pipelines," *J. Transportation Engrg., ASCE*, 112(5), 465-480, 1986.

[7] H.M. Hathoot, A.I. Al-Amoud, and F.S. Mohammad, "Analysis of a Pipe with Uniform Lateral Flow," Alexandria Engrg. *J., Alex., Egypt*, 30(1), C49-C54, 1991.

[8] H.M. Hathoot, A.I. Al-Amoud, and F.S. Mohammad, "Analysis and Design of Trickle Irrigation Laterals," *J. Irrig. Drain. Engrg.*, 119(5), 756-767, 1993.

[9] J. Keller, and D. Karmelli, "Trickle Irrigation Design Parameters," *Trans. ASAE*, 17(4), 678-684, 1974.

[10] K.C. Khatri, I.P. Wu, H.M. Gitlin, and A.L. Phillips, "Hydraulics of Microtube Emitters," *J. Irrig. Drain. Div., ASCE*, 105(2), 163-173, 1979.

[11] R.P. Perold, "Design of Irrigation Pipe Laterals," *J. Irrig. Drain. Div., ASCE*, 103(2), 179-195, 1977.

[12] V.I. Streeter and B.E. Wylie, *Fluid Mechanics*, McGraw-Hill Co., New York, N.Y., 1979.

[13] A.W. Warrick, and M. Yitayew, "Trickle Irrigation Hydraulics I: Analytical Solution," *J. Irrig. Drain. Engrg., ASCE*, 114(2) 281-288, 1988.

[14] G.Z. Watters, and J. Keller, "Trickle Irrigation Tubing Hydraulics," *ASAE Tech.* Paper No.78-2015, Am. Soc. Agric. Engrs., St. Joseph, Mich., 1978.

[15] P.I. Wu and R. Yue, "Drip Lateral Design Using Energy Gradient Line Approach," *Trans. ASAE*, 36(2), 389-394, 1993.

[16] M. Yitayew, and A.W. Warrick, "Trickle Lateral Hydraulics, II: Design and Examples,", *J. Irrig. Drain. Engrg., ASCE*, 114(2), 289-300, 1988.