DETACHMENT OF A TURBULENT PLANE JET FROM AN INCLINED PLATE

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ABSTRACT

The detachment of a plane turbulent jet from an offset-circular edged-inclined plate of definite length has been studied theoretically and experimentally. An integral approach, which includes the entrainment concept and variation of both base pressure and jet radius of curvature, is adapted. Theoretical predictions are correlated and a formula for determining the detachment angle, in terms of the system geometrical parameters, has been obtained. This formula may be of great help for the design of many fluidic systems. Both theoretical predictions and derived correlations were compared with experimental results where a reasonable agreement has been observed.

Keywords: Detachment of Jet, Jet-Plate interaction.

Nomenclature

a1, a2, ... a5	: Coefficients in the equation of jet center line trajectory.
AR	: Nozzle aspect ratio.
b	: Dimensionless measure of jet spread, b^*/w^*.
BR	: Dimensionless distance between the inflection point and the plate measured along the x_1 direction, BR^*/w^*.
H	: The Dimensionless y-coordinate of the lower edge of the jet at exit from nozzle, H^*/w^*.
J	: Momentum flux = $\int_{-w}^{w} u^2 dx_1$
l_d	: The dimensionless reattachment length, i.e., the distance between the plate edge and the point of attachment, l_{d*}/w^*.
P	: Dimensionless pressure, $P/\rho u_d^{*2}$
P_b	: Dimensionless base pressure.
Q_e	: Dimensionless rate of mass entrainment, $Qe^*/u_d^* w^*$.
r	: Dimensionless radius of curvature r^*/w^*.
Re	: Reynolds number at nozzle exit, $w^*.u_d^*/\upsilon$
T	: Dimensionless plate edge radius, T^*/w^*
u	: Dimensionless axial velocity, u^*/u_d^*
u_m	: Dimensionless maximum velocity u_m^*/u_d^*.
w	: Discharge width.
x, y	: Dimensionless cartesian coordinates, x^*/w^*, y^*/w^*.
x_1, x_2, x_3	: Dimensionless jet coordinates.
X_o	: Dimensionless x-direction offset of plate edge, x_o^*/w^*.
y_o	: Dimensionless y-direction offset of plate edge, y_o^*/w^*.
α	: Entertainment coefficient
θ_d	: Plate inclination angle,
θ_{12}	: Plate inclination angle corresponding to l_d = 12.
υ	: Kinematic viscosity of fluid.

Subscripts:

d : discharge.
i : point of inflection.
ϕ : evaluated at end of zone of flow establishment.

INTRODUCTION

The study of the flow of a turbulent jet issuing off parallel to or inclined on a wall boundary is important in many engineering applications. Among these applications there are the flow conditions in fluidics, a sudden expansion path of various equipment and a combustion furnace. Three types of jet-boundary interaction problems are commonly encountered : (1) wall jet where the fluid is discharged at the boundary,

(2) impinging jets where the discharge is aimed towards the boundary, and (3) offset jets where the fluid is discharged at some distance from the boundary and attaches to it due to lateral pressure forces. These forces are created due to the reduced fluid entrainment from the boundary side of the jet. As a result, the jet bends towards the boundary and attaches to it. This phenomenon is known as Coanda effect.

Review of previous work on free jets and jet-boundary interaction is of great importance for the present work. Free jets have been studied by many authors [e.g. 1-3]. Experimental study of flow and diffusion of a circular turbulent free jet was given by Wygnanski, I. et al., [1]. Loony, M.K. et al., [2] presented a detailed study of plane turbulent free jet. They gave numerical solutions of the governing equations and discussed the mean flow properties and turbulence characteristics. S.B. Pope [3] solved a modeled joint probability density function equation to calculate the one point statistical properties of a self-similar plane jet. Mean velocity profiles and Reynolds stresses were predicted and confirmed by comparison with experimental results.

Studies on plane turbulent wall jets were carried out by many authors [e.g. 4-6]. Myers, G.E. et al., [4] investigated the mean flow properties and jet diffusion characteristics. Y. Katz et al. [5] studied theoretically and experimentally the forced turbulent wall jet. They showed that the jet spread rate and the mean velocity distribution are not affected by external excitation. Wygnaski, I. et al. [6] studied experimentally the applicability of various scaling laws to the turbulent plane wall jet. They showed that the "law of the wall" applies only to the viscous sublayer.

Studies on jet-parallel boundary interaction are numerous [e.g. 7-15]. Recently D.T. Walker et al., [7], C.M. Madina et al. [8] and D.G. Anthony et al. [9] studied experimentally the turbulent structure due to the interaction of a turbulent round jet with a free parallel surface. Davis M.R. et al. [10] studied experimentally the effects of the circular jet nozzle transverse displacement on the mean flow and turbulence characteristics. Tsutomu et. al. [11] applied the approximate solutions of a two dimensional free jet on a model of reattachment jet. Their results shows that the behavior of the reattachment jet is independent of Reynolds number for high aspect ratios of jet nozzle. J. Hoch et al. [12] studied the reattachment jet in the presence of a secondary free stream. Tsutomu Nozaki et al. [13] studied theoretically and experimentally the effects of initial turbulence intensity, at jet exit, on the reattachment jet. They showed that the jet behavior dependency on turbulence intensity diminishes as it becomes large. Tsutomu Nozaki [14] studied experimentally the effect of jet nozzle aspect ratio on the reattachment jet. He found that earlier results of a two-dimensional reattachment flow are applicable to a flow having aspect ratio larger than 3. For smaller aspect ratios, correction factors were introduced as function of aspect ratio, offset ratio and Reynolds number. Studies on jet reattaching to an offset inclined wall seem to be little. However Toshihiko Shakouchi et al. [15] studied theoretically and experimentally the influence of the jet passage with existing opposite wall. The switching action, caused by traversing the opposite wall quasi-statistically, was studied. They presented an analysis, assuming constant jet radius of curvature and base pressure, for small inclination angles of the side wall.

The above review shows that the interaction between a jet and an offset inclined boundary of definite length has not yet been studied enough. The present work is a contribution to this area. For an offset jet, which is initially attaching to an inclined boundary, as the inclination angle increases the jet detaches at a certain angle and becomes a free jet. This angle is called the detachment angle. The following analysis includes the determination of the detachment angle under the effects of jet-boundary geometrical conditions.

ANALYSIS

Figure (1) depicts the flow regions for a reattaching jet. Analysis will be restricted on the reattachment region to get the reattachment length "l_d" corresponding to an inclination angle θ_d.

Assuming incompressible, two dimensional flow with constant fluid properties, the integral conservation equations are formulated in a dimensionless form as follows [12]:

Using the jet coordinate system shown in Figure (1), normalizing, using the discharge velocity u_d^* and nozzle width W^*, conservation of mass gives:

$$\frac{d}{dx_3}\int_{-w}^{w} u\, dx_1 = \frac{dQ_e}{dx_3} \tag{1}$$

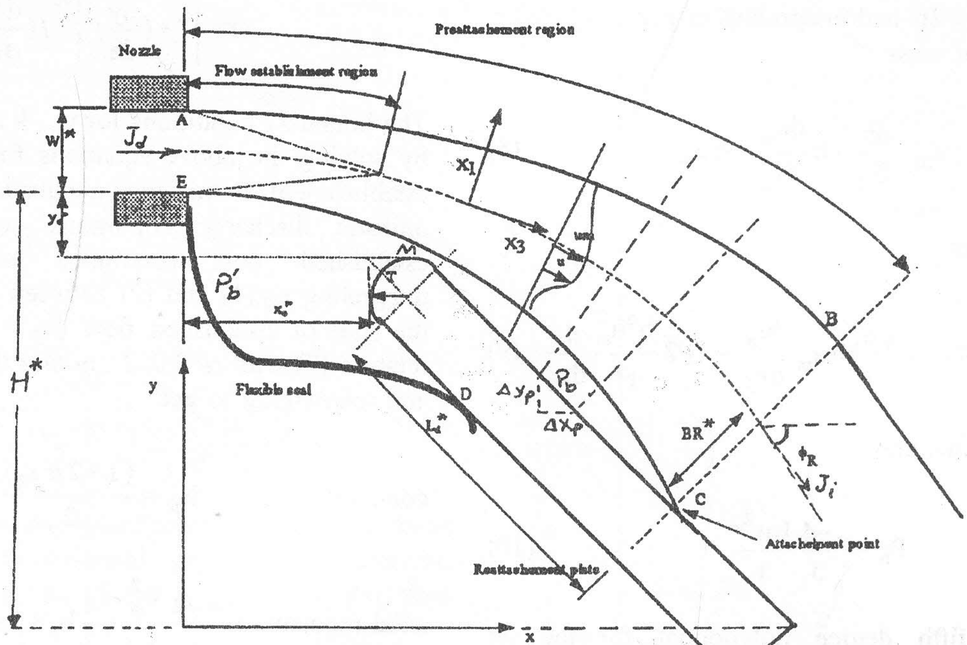

Figure 1. Schematic of the two-dimensional offset jet.

Conservation of axial and transverse momentum gives:

$$\frac{d}{dx_3}\int_{-w}^{w} u^2 dx_1 = \frac{d}{dx_3}\int_{-w}^{w} P dx_1 \quad (2)$$

and

$$\frac{1}{r}\int_{-w}^{w} u^2 dx_1 = -P_b(x_3) \quad (3)$$

Where x_1 and x_3 are the transverse and axial coordinates respectively, u the x_3-direction velocity, w the jet's outer limit, Qe entertainment mass rate, P the local static pressure and P_b the base pressure. The base pressure is obtained by evaluating P at x_1 =-w. Thus:

$$P_b\,(x_3) = P\,(-w, x_3) \quad (4)$$

Solutions of equations (1) through (4) are obtained by assuming the velocity profiles along the flow. Following previous work [11-14] by assuming the reattaching jet velocity profiles to be similar to its free jet counterpart, the velocity profile may be given by [17]:

$$u = u_m \operatorname{sech}^2 (x_1/b) \quad (5)$$

Where b is a measure of jet spread and u_m is the centerline velocity.

The static pressure P is assumed to vary linearly across the jet as follows:

$$P(x_1, x_3) = \frac{P_b(x_3)}{2}\left[1 - \frac{x_1}{w(x_3)}\right] \quad (6)$$

Which satisfies the conditions P = o at x_1 = w and $P=P_b$ at x_1 =-w.

Using the entrainment rate closure assumption [12] it follows that:

$$\frac{dQe}{dx_3} = 2\,\alpha\, u_m \quad (7)$$

Where α is the entrainment coefficient which varies between the free jet value of 0.069 near the discharge and the wall jet value of 0.036 near the point of attachment. In the present analysis it is taken as the average value of 0.052 in the zone of flow establishment and then it is allowed to decrease linearly to the value of 0.036 at the point of attachment. Substituting from (5) through (7) into eqs (1) through

(3), using $w = \sqrt{2}b$ and integrating to get:
Conservation of mass:

$$u_m \frac{db}{dx_3} + b\frac{du_m}{dx_3} = \alpha u_m \quad (8)$$

axial momentum:

$$u_m^2 \frac{db}{dx_3} + 2(1-\sqrt{2}\frac{b}{r}) b u_m \frac{du_m}{dx_3} = -\sqrt{2}\frac{b^2 u_m^2}{r^2}\frac{dr}{dx_3} \quad (9)$$

Transverse momentum:

$$P_b = \frac{-4}{3}\frac{b u^{2}_{m}}{r} \quad (10)$$

Assuming a fifth degree polynomial for the jet centerline trajectory as [12]:

$$y = a_o + a_1x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \quad (11)$$

Noting that the jet is discharged in x-direction (i.e. $\frac{dy}{dx} = 0$ and hence $a_1 = 0$ at $x = 0$), neglecting the second and third order derivatives (i.e. $a_2 = a_3 = 0$ at $x=0$) and with known discharge location (i.e. $y = H + 0.5$ at $x = 0$) then eq. (11) reduces to :

$$y = (H+0.5) + a_4 x^4 + a_5 x^5 \quad (12)$$

At the point of attachment, an inflection point (x_i,y_i) in the trajectory exists where the following conditions apply:

$$\text{at } x=x_i;\ y=y_i\ ;\ \text{and } \frac{d^2y}{dx^2} = 0 \quad (13)$$

using these conditions, eq. (12) yields :

$$y = (H+0.5) - \frac{5}{2}(y+0.5-y_i)\left[\left(\frac{x}{x_i}\right)^4 - \frac{3}{5}\left(\frac{x}{x_i}\right)^5\right] \quad (14)$$

and the centerline radius of curvature is determined from:

$$r = \left[1 + (\frac{dy}{dx})^2\right]^{3/2} / (\frac{d^2y}{dx^2}) \quad (15)$$

The boundary conditions for u_m, b and P_b are obtained by solving the above equations for the zone of flow establishment. In this zone a transition from a relatively uniform discharge conditions, at jet exit, to an established flow conditions takes place. Thus integrating eq.(1) and (2) between jet exit ($x_3=0$) and the start of established flow ($x_3 = x\phi$) and using the relation (7) with $\alpha=0.052$, noting that $u_m = 1$ at $x = 0$ and rearranging to get:

$$\text{continuity:} \quad b_\phi = \frac{(1+2\alpha x_\phi)}{2} \quad (16.a)$$

$$\text{axial momentum:} \frac{4\sqrt{2}}{3}\frac{b_\phi^2}{r_\phi} - \frac{4}{3}b_\phi + 1 = 0 \quad (16\text{-}b)$$

An iterative solution of the above equations gives the values of x_ϕ and b_ϕ for a jet trajectory having the inflection point (x_i, y_i). Then a marching step-by-step solution gives the flow parameters u_m, P_b and r along the remaining jet trajectory. The adequate inflection point should satisfy the following two conditions. The first is that at point of attachment, the width w is equal the distance "BR" between the inflection point and the inclined boundary. The second condition is obtained by applying conservation of momentum in both x and y directions on the control volume "ABCDE" enclosing the reattachment zone as shown in Figure (1). At x = 0, the momentum flux $J_d = 1$ and at the point of attachment the momentum flux J_i is assumed to make an angle ϕ_R with x-direction. Referring to the components of the pressure forces in x and y directions as F_H and F_V respectively, then momentum balance gives:

in x-direction: $J_i \cos\phi = F_H + 1$

and

in y-direction: $J_i \sin\phi = F_V$

As a result, the following condition is obtained:

$$\tan \phi_R = \frac{F_v}{F_H + 1} \qquad (17)$$

where:

$$F_v = \sum_M^C P_b . \Delta x_p \qquad (18)$$

and

$$F_H = P'_b (y_o + 0.5) + \sum_M^C P_b . |\Delta y_p| \qquad (19)$$

where x_p, y_p are the coordinates of the intersection point between the normal (x_1-direction) and the boundary, P_b' is the base pressure in the flow establishment zone.

Solution procedure:

The x_i-coordinate is first assumed and the y_i coordinate is determined by iteration, so that the first boundary condition is satisfied. At this value of (x_i, y_i) the boundary condition (17) is checked, then using the half-interval search method the adequate (x_i, y_i) values are obtained. The implicit linear differential equations are solved simultaneously using simple Euler's formula [16] with a step size of 0.1 which gave acceptable accuracy of the results.

RESULTS AND DISCUSSION

Predictions of pressure distribution, jet radius of curvature and plate reattachment length "l_d" under the effects of different geometrical parameters are obtained. These parameters are the plate edge lateral offset "y_o",(ii) axial offset "x_o" ; (iii) the plate thickness "T" and (iv), the inclination "θ_d". The results are shown in Figures (2) through (12). Results for inclination angles larger than 90° are extrapolated safely for very limited cases where the assumed trajectory did not satisfy the prevailing geometrical conditions.

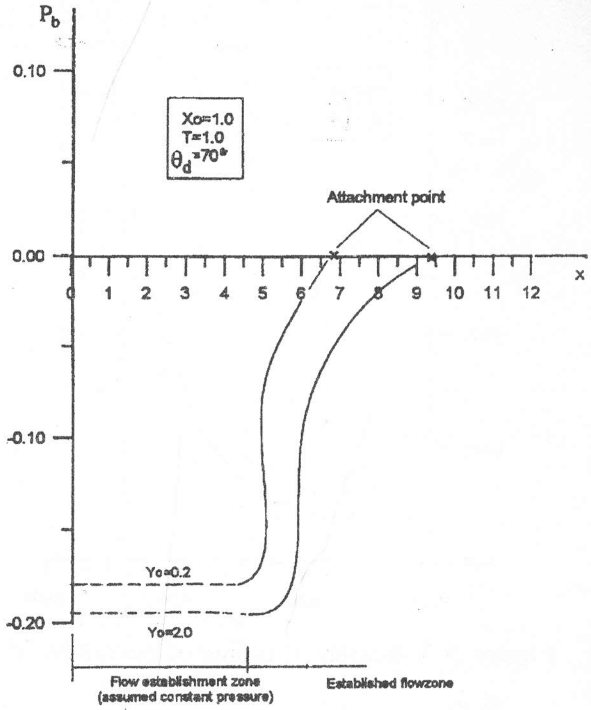

Figure 2. Predicted pressure distribution.

Base pressure distribution

A typical wall pressure distribution along the blade is shown in Figure (2). The same trend was reported [12] by early studies for the case of parallel offset wall. It is seen that an adverse pressure gradient prevails upstream the attachment point. The pressure in the flow establishment zone is assumed to be constant as shown by the dotted line.

Jet radius of curvature "r":

Typical variation of the radius of curvature is shown in Figure (3). Figure (4) shows the corresponding jet centerline trajectories. It is seen that the assumption of a circular arc centerline adapted by early studies [13-15] is not valid especially at low transverse offsets.

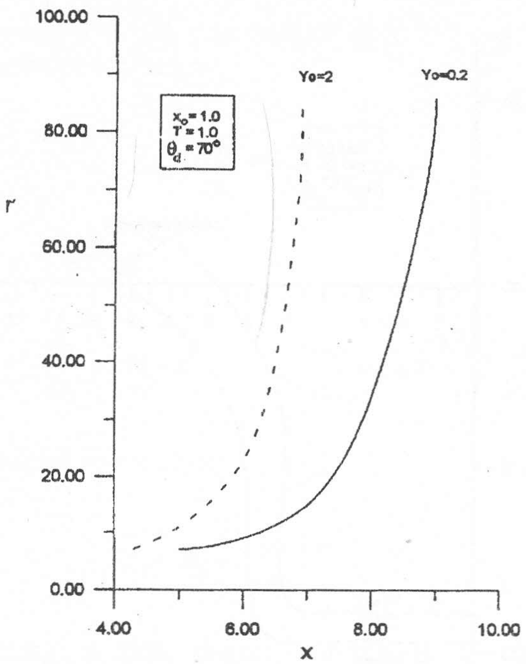

Figure 3. Variation of radius of curvature "r"

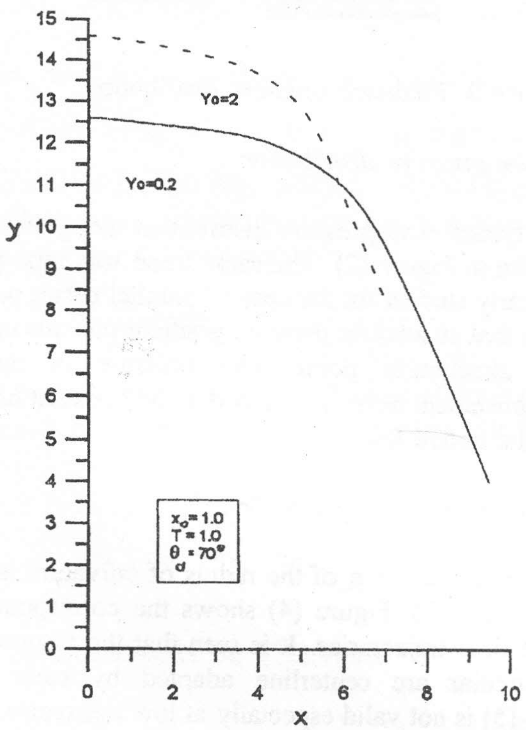

Figure 4. Jet centerline trajectory.

Reattachment length "l_d"

The distance, along the plate, of the reattachment point "l_d" [see Figure (1)] is plotted in Figures (5) through (9). The effect of the axial offset x_o, lateral offset y_o and plate inclination θ_d are indicated as follows. Figures (5) through (7) show that as θ_d increases l_d increases due to jet passage divergence. Also, these figures show that, with other parameters kept constant, l_d decreases as y_o increases. This is a direct result of the reattaching jet momentum balance where the horizontal vacuum becomes higher for higher y_o causing earlier jet reattachment. Figures (8) and (9) show that l_d decreases-as physically expected-with the increase of either x_o or T where the plate surface becomes nearer to the reattaching stream.

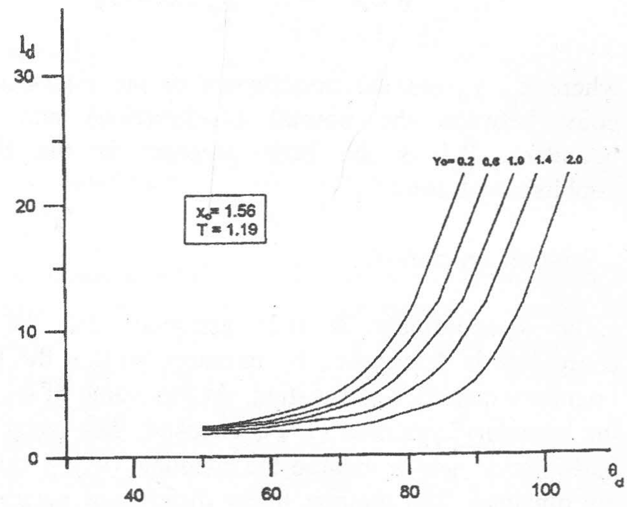

Figure 5. Variation of reattachment length (l_d) with inclination angle (θ_d), [X_o=1.56, T=1.19].

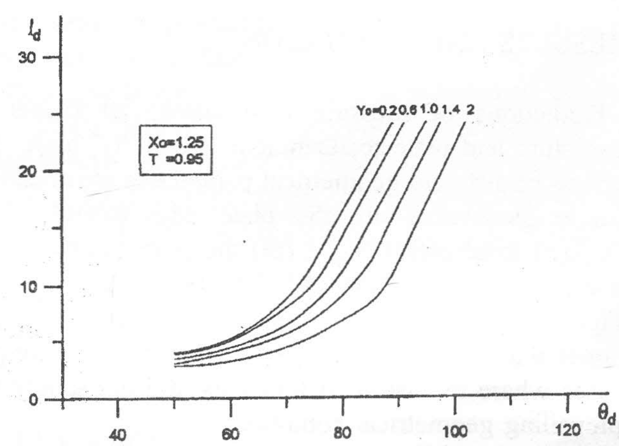

Figure 6. Contd. [X_o = 1.25, T=0.95].

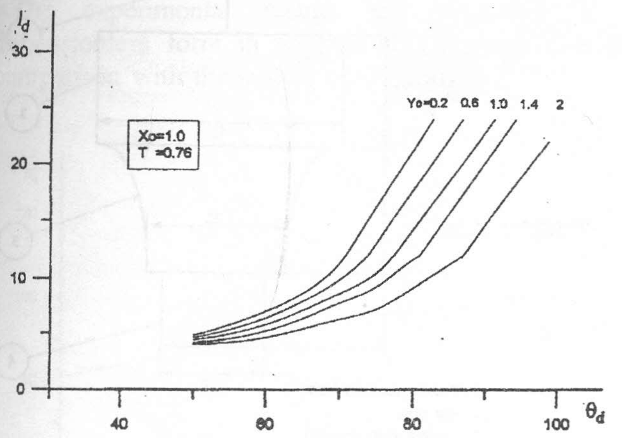

Figure 7. Contd. [X_o = 1.0, T=0.76].

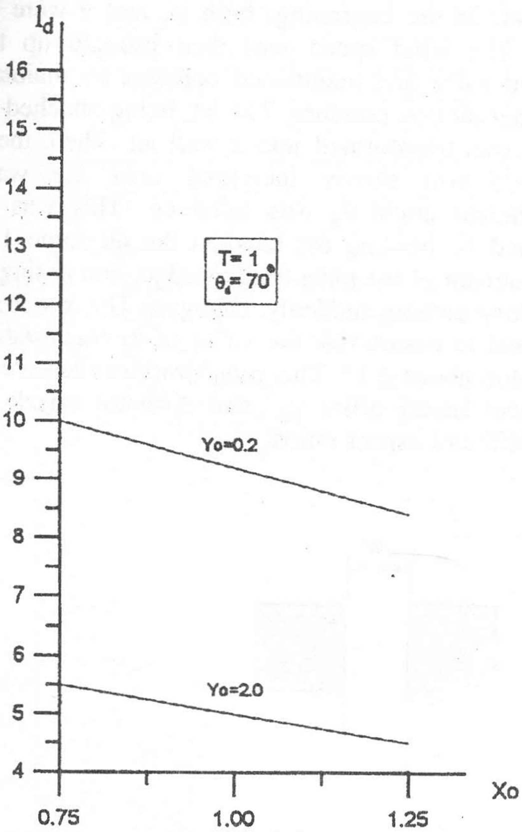

Figure 8. Effect of X_o on "l_d".

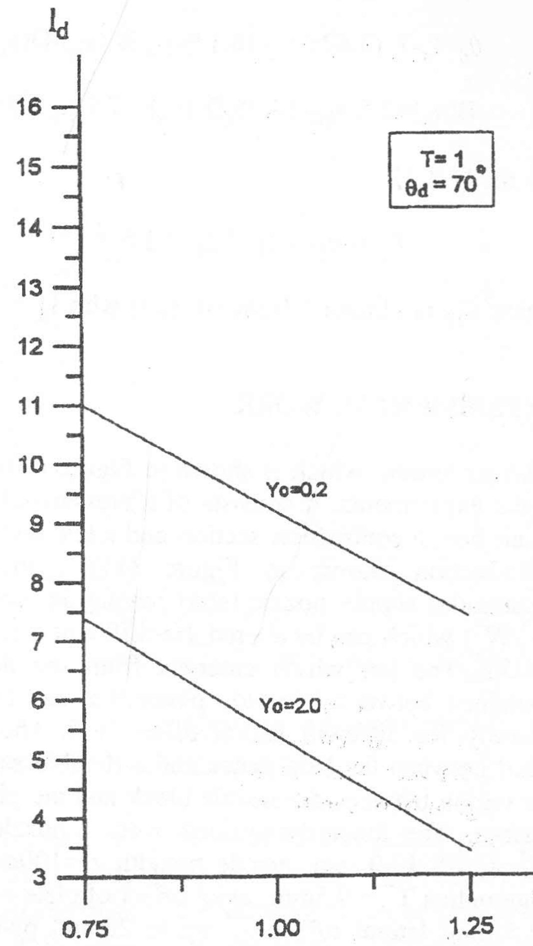

Figure 9. Effect of T on "l_d".

Jet detachment correlations:

For a jet initially attaching to a finite length plate, as the inclination angle increases the reattachment length increases. At a certain angle "θ_d" the reattachment length "l_d" exceeds the plate length and the jet detaches from the plate. Thus the predicted parameters l_d and θ_d represent the limits for a reattaching jet. Hence for a plate length "l_d" the detachment angle is θ_d. On the other hand, for a plate inclination angle "θ_d" a minimum length of "l_d" is required to keep jet attachment. Predictions were obtained for the following ranges: $0.75 \leq x_o \leq 1.25, 0.75 \leq T \leq 1.25, 0.2 \leq y_o \leq 2.0$ Correlations between l_d and other parameters resulted in the following formula:

(i) for $l_d \geq 12$

$$\theta_d = l_d - T\,(3.625\,y_o - 18.13) - y_o\,B\,(x_o) - D(x_o) \quad (20)$$

$$B(x_o) = 2.5\,x_o - 14.25, D\,(x_o) = -7.5\,x_o - 35 \quad (21)$$

(ii) for $l_d < 12$

$$\theta_d = \theta_{12} - [(12 - l_d) / 1.5\,]^{2.14} \quad (22)$$

where θ_{12} is obtained from eq. (20) with l_d = 12.

EXPERIMENTAL WORK

An air circuit, which is shown in Figure (10), is used in the experiments. It consists of a pressurized air line, an air box, a contraction section and a test section. The test section shown in Figure (11) , includes a rectangular supply nozzle (slot) having an aspect ratio (Z^*/W^*) which can be altered via different sized nozzle blocks. The jet, which emerges from the nozzle, is contained between two side plates that can be moved laterally for different lateral offset "y_o". The plate is fitted between the side plates and a flexible seal closes the region between the nozzle block and the plate outer surface. The main dimensions were : nozzle widths W^*=8, 10, 12.0 mm; nozzle breadth z^*=100mm; plate edge radius T^* = 9.5mm; axial offset of plate edge x_o^*= 12.5 mm; lateral offset y_o^* up to 20mm, plate length l_d^*=100mm and average jet exit velocity u_d^* =32 m/s.

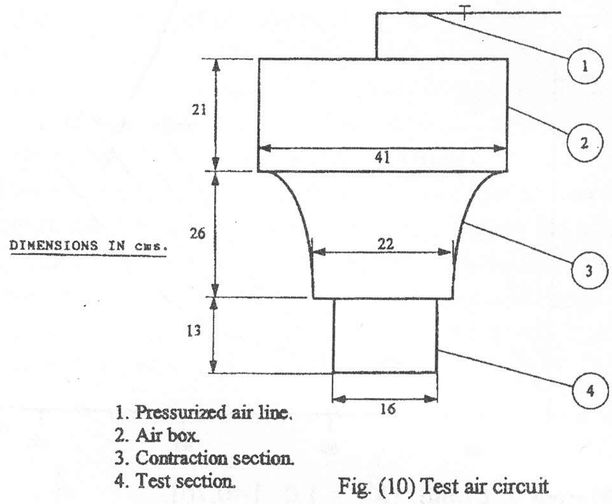

Figure 10. Test air circuit.

The plate detachment angle θ_d was determined as follows. In the beginning, both y_o and θ were set to zero. The wind speed was then brought up to the desired value and maintained constant by maintaining constant air box pressure. The jet, being attached to the plate, was transformed into a wall jet. Then, the plate angle θ was slowly increased until the wall jet detachment angle θ_d was achieved. This was easily detected by holding the hand in the jet some 15 cms downstream of the plate trailing edge, and noting when the flow pattern suddenly changes. The process was repeated to ensure that the value of θ_d was established to within about $\pm 1°$. This procedure was repeated with different lateral offset y_o^* and different nozzle sizes, i.e., different aspect ratios.

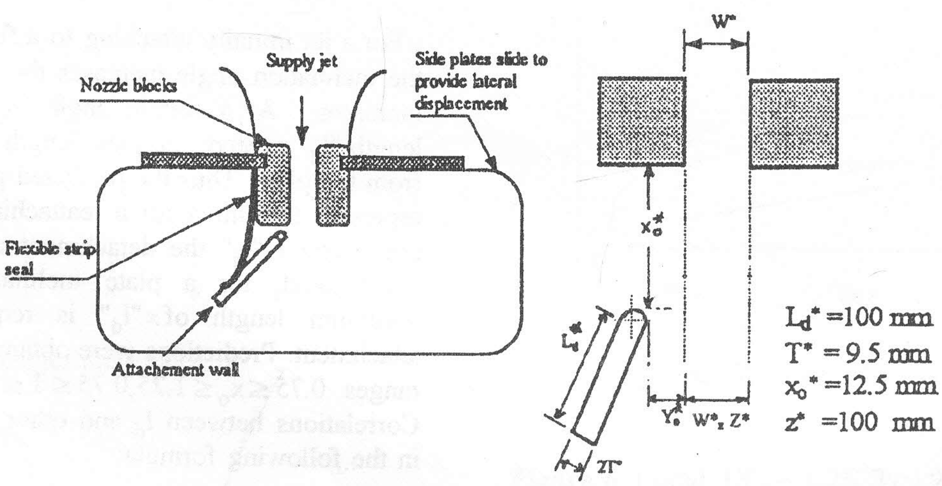

Figure 11. The test section.

The experimental results are presented in a dimensionless form in Figures (12) through (14) for comparison with theoretical predictions.

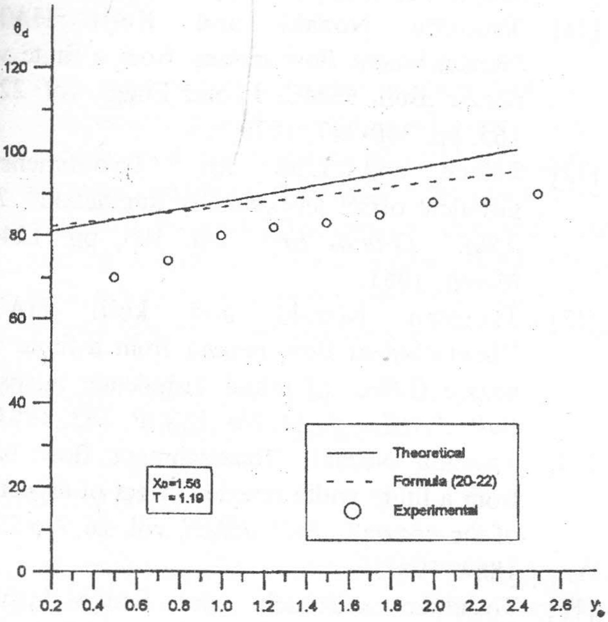

Figure 12. Variation of detachment angle (θ_d) with plade edge offset (y_o) (W^*=8mm, AR = 12.5, Re=17100).

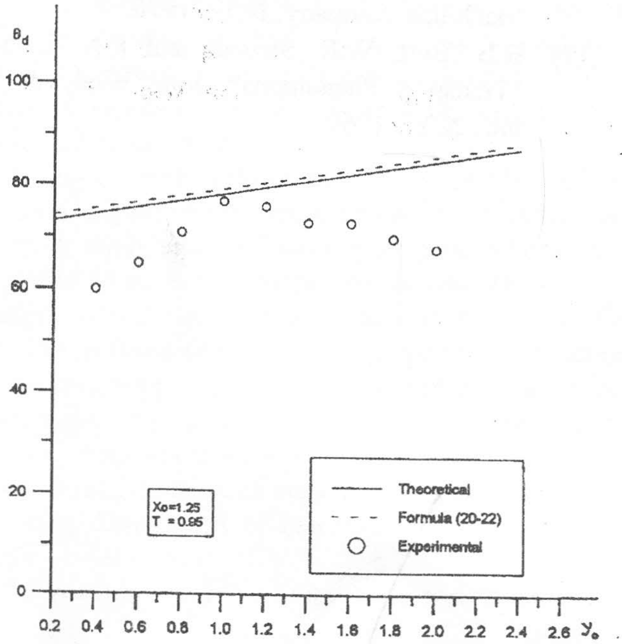

Figure 13. Contd. (w^*=10mm, AR=10, Re=21300).

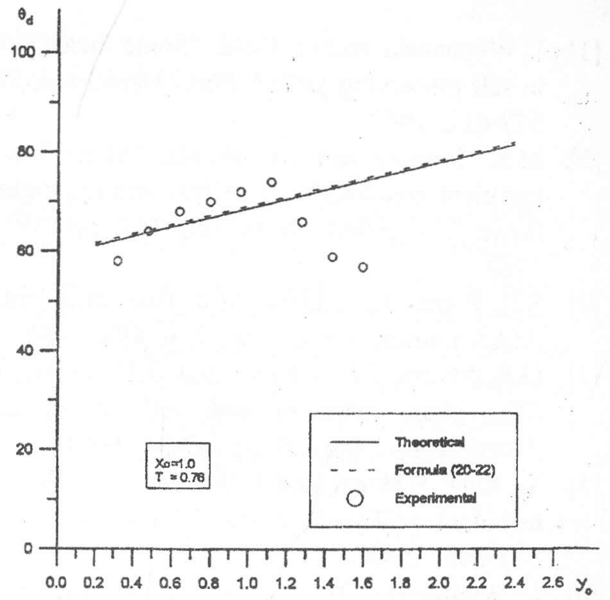

Figure 14. Contd. (w^*=12.5 mm, AR=8, Re=26700).

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS.

Comparison between each of theoretical predictions and the consequent formula [(20)-(22)] and experimental results are shown in Figures (12) through (14). It is seen that reasonable agreement exists especially for higher aspect ratio jet nozzles, where the flow approaches the two dimensional one. Also, it is seen that the derived formula [(20)-(22)] fits well the theoretical predictions.

CONCLUSIONS

The detachment of a plane turbulent jet from an offset-circular edged-inclined plate of definite length has been studied theoretically and experimentally. A formula which determines the detachment plate inclination angle, for a jet-plate system, has been presented. This formula applies in the following ranges: lateral plate edge offset $0.2 \leq y_o \leq 2$, axial offset $1.25 \geq x_o \geq 0.75$ and plate edge radius $1.25 \geq T \geq 0.75$. This formula is confirmed by comparison with experimental work for a nozzle aspect ratio "AR" range of $12.5 \geq AR \geq 8$ and a Reynolds number Range of $7100 \leq Re \leq 26700$.

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