CONDUCTION OF HEAT THROUGH MULTI-PHASE SYSTEMS

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ABSTRACT

An experimental technique was devised to measure the thermal conductivity of sands and various fine-particle granular matrices of glass beads. The study covers a fairly wide ranges of practical and experimental combinations of grain size, porosity, water saturation and temperature, under atmospheric pressure conditions. An effective thermal conductivity for such multiphase porous systems was accordingly estimated for each set of physical conditions experimented with. Experimental results were then assessed and hence correlated in terms of the thermophysical properties of the individual components involved and their relative composition. One of the main conclusions of the study is that the effective conductivity of porous systems is substantially enhanced by water saturation, especially for the Athabasca sands.

Keywords: Heat Conduction, Effective Thermal Conductivity, Porous Media, Thermophysical Properties, Granular Materials.

INTRODUCTION

The thermal response and thermophysical properties of granular materials are of great and increasing importance to so many practical applications. In fields such as construction, insulation, packing, heating, chemical and oil industries etc.., the thermal conductivity of porous granular beds plays vital roles in process design, optimization and implementation. In an in-situ thermal oil recovery process, for instance, an effective thermal conductivity data for a particular reservoir matrix is an essential parameter for the effective simulation of the thermal process, whether analytical or physical.

Although some information is available in the literature regarding thermal conductivities of Athabasca oil sand, sandy soils and some other porous beds, (e.g. Cervenan et al., 1987; Hanafi and Karim, 1986), there appears a lack of a fundamental treatment of the problem in terms of an effective property as a function of the relevant properties of the individual components comprising the granular system. Moreover, some effective parameters such as porosity, temperature and water or oil saturation of the granular beds are found to be either disregarded or inaccurately considered in a rather global fashion.

The present study is meant, however, to be a fundamental analysis of the problem through experimental testing under practical ranges of working physical and geometrical conditions.

Various experimental techniques have been employed for determining thermal conductivities of beds of oil sands, porous rocks and granular materials, (Krupiczka, 1967; Somerton et al., 1974; Karim and Hanafi, 1986). These techniques are basically either of the steady-state type or the transient heating method. The steady-state technique involves essentially the measurement of the steady-state temperatures at two or more points along the direction of heat flow near a constant thermal energy source. In the transient heating method, temperature measurements are recorded continuously with time. The accuracy of the results of a transient technique is highly dependent on the response characteristics and accuracy of the temperature monitoring instrumentation, as well as on the reliability of the associated theoretical models adapted. Hence, the steady-state technique, though tedious and time consuming, was preferred in the present investigation.

The effective thermal conductivity of clean Athabasca reservoir sand was determined experimentally over the temperature range 25 to 450°C. Also, to demonstrate the effect of the grain size, the thermal conductivity of

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various controllable uniform diameter spherical glass beads matrices was established over virtually the same temperature range. Moreover, tests were carried out on sands and glass beads under wet conditions ranging from 0 to 100 percent water saturation. Results were then analyzed, interpreted and hence correlated in terms of the relevant properties of the solid, gas and liquid components involved as well as the relative proportions of the different phases comprising the porous system.

EXPERIMENTAL APPARATUS

The clean Athabasca reservoir sand used in this experimental study was obtained by extracting the bitumen from oil sand deposits by means of toluene. The extracted sand was then heated to 600°C for 24 hours to remove any residues of cooked oil, and the sand finally became fairly white clean. Typical average grain size distribution of the sands is shown in Figure (1), (Mehta, 1990; Hanafi, 1979).

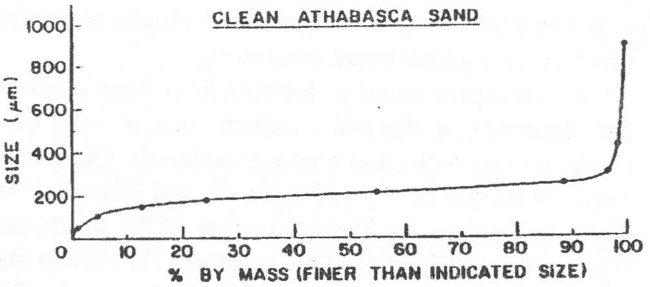

Figure 1. Particle size distribution of clean washed Athabasce sand.

Five different uniform size spherical glass beads were provided by Fisher Scientific, Pittsburgh, U.S.A., with diameter ranging from 89 μm up to 3 mm. The chemical composition of the glass beads as reported by the manufacturer was given as 72.5% SiO_2, 13.7% Na_2O, 9.75% CaO, 3.3% MgO, 0.4% Al_2O_3, 0.1% K_2O, and traces of some other mineral oxides, mass basis.

The average density of the dry sand was measured to be 1.61 g/cm^3 while the bulk (no-gap) solid density was found to be 2.62 g/cm^3, which was obtained by means of a water displacement technique. Similarly, the glass beads were found to have a bulk solid density of 2.45 g/cm^3, while the dry glass beads' density was observed to range from 1.46 up to 1.59 g/cm^3, depending on the grain size. Comparing the bulk and dry densities of a particular bed of granular material, the average porosity or voidage volumetric fraction of the bed could be accordingly determined.

The apparatus employed, which is shown schematically in Figure (2), consisted essentially of a cylindrical container with a central co-axial rod-type electric heater. The figure displays the locations of the various thermocouples and the associated instrumentation involved. All the thermocouples used were grounded, chromel-alumel ISA, 'K' type. Three vertically traversing thermocouples of 1.59 mm outside diameter were fitted with Inconel 600 sheath. The traversing mechanism was properly designed so that it positioned the probes precisely at the desired locations. Nine thermocouples were surface-flush welded onto the heater surface so as to monitor the uniformity of the temperature over its entire length. They also provided the necessary data for occasional minor corrections to the power density of the heater, especially at elevated temperatures.

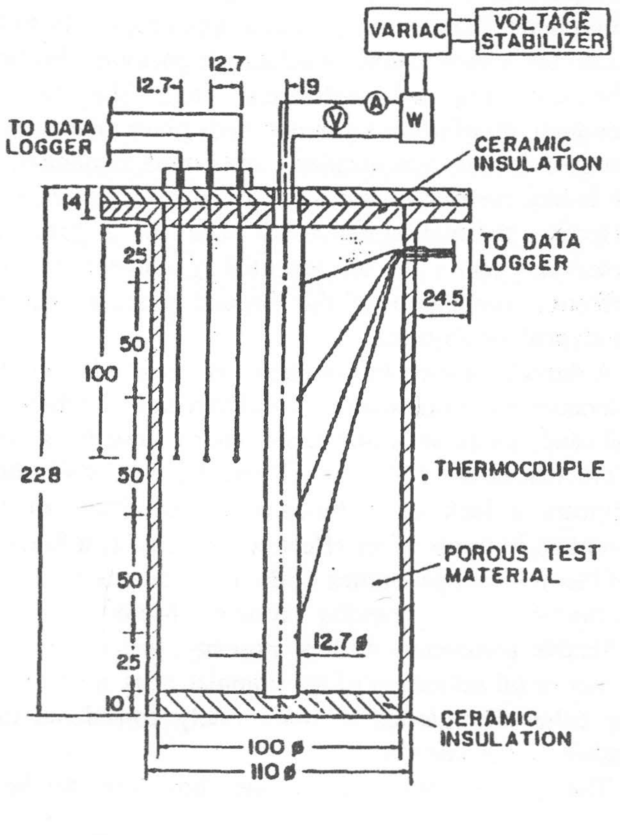

Figure 2. Schematic view of the apparatus employed.

With the heating rod element in place, the cylindrical container was filled with the unconsolidated test material, which was then subjected to ultrasonic vibrations, long enough to achieve uniform compactness. The traversing thermocouples were then carefully lowered with a properly designed positioning glass spacer through the closed lid into the fully packed vessel. The vessel was subsequently well insulated at its top and bottom ends, and the instrumentations were carefully connected. The heater is now powered at a prescribed setting and the apparatus is left for 12 to 16 hours to attain a steady state condition. Once all the necessary data were collected, the procedure was repeated for another power setting.

The respective traversing probe temperatures were found to be practically uniform over the middle 100 mm length, confirming the uniform heat generation rate by the central heating element. Care was taken to avoid water evaporation within water-saturated beds by proper setting of the heater power such that its surface was maintained below 30°C throughout the wet grains' tests.

THE RADIAL HEAT FLOW MODEL

In an axially symmetrical homogeneous system, if the rate of thermal energy generated by a long central cylindrical uniform heater embodied within a packed bed of uniform granular materials is known and the steady-state temperatures at two typical redial locations within the bed are also known, the effective thermal conductivity of the bed can be established. This is obtained on the basis that the heat generated by the axial heater is transferred uniformly in the radial direction by the mechanism of conduction while neglecting any possible convection effect within the pores of the bed. Accordingly, a simple dimensional conduction can be considered, and the corresponding effective thermal conductivity may be expressed as:

$$k_e = \left[\frac{\ln(r_2/r_1)}{2\pi L}\right]\frac{Q}{(T_1 - T_2)} \qquad (1)$$

where

k_e = effective thermal conductivity, W/m.K

Q = steady rate of thermal energy generated by the heater, W

L = length of the heater, m

T_1 & T_2 = steady-state temperatures at radii r_1 and r_2, respectively, °C or K

Thus, an appropriate measurement of Q with T_1 and T_2 at r_1 and r_2, respectively under steady-state condition, would, in principle, permit an estimate of the effective thermal conductivity of the material under consideration, averaged over the temperature range T_1 to T_2. This average effective thermal conductivity is considered as a representative property for the multiphase (solid/gas, solid/liquid, or solid/gas/liquid) porous system under the prevailing physical and geometrical conditions.

RESULTS AND DISCUSSIONS

The conduction of heat through any heterogeneous media is known to be strongly dependent on the structure of the matrix and the thermal conductivity of each phase. The most difficult aspect of the analysis of heat conduction through a porous medium is the structural modelling. This is because the representative elementary volumes are three-dimensional and have complicated structures that vary greatly among different porous media, (Kaviany, 1991).

For the analysis of the macroscopic heat flow through a multi-phase granular medium, volume-averaged (or effective) properties such as the effective thermal conductivity are used. The present study aims, therefore, at having insight to identify the most significant parameters and their interrelated correlations regarding the effective thermal conductivity of porous beds of unconsolidated granular materials through direct measurements.

The steady- state temperature at various locations along with the corresponding electric power inputs to the heater, Figure (2), were measured for the test sample of granular materials under consideration, Experimental runs were conducted on porous beds of unconsolidated sands and of various uniform-size spherical glass beads of different porosities. The experimental program also considered beds of wet (three-component) as well as dry (two-component) particles at different levels of temperature.

Based on the experimental data, which were closely monitored and continuously recorded, averaged effective thermal conductivity values could hence be estimated for each set of prescribed working conditions.

Figures (3) and (4) display the experimental effective thermal conductivity values for the reservoir clean sands and the various spherical glass beads tested in dry as well as in wet bed conditions. It can be observed generally that the effective conductivity increases with temperature for both sands and glass beads. This result is likely anticipated since, over the experimental range of temperature, each of the individual components comprising the bed is known to be more heat conductive as the level of temperature is raised. Bulk silica and glasses, for instance, are quoted to have their conductivities improving with temperature in a nearly linear fashion, (Weast, 1987). Similarly, for the fluid components air and water, the standard published data lists increasing conductivities with higher temperatures, (Vargaftik, 1975; Green, 1984).

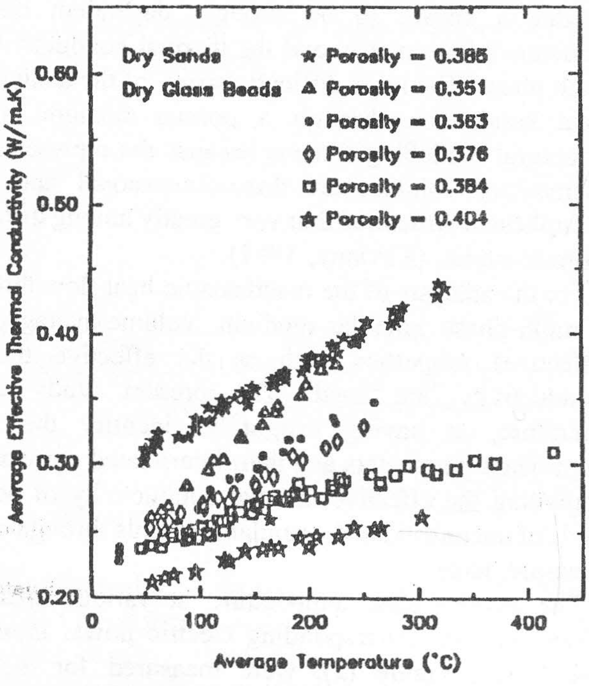

Figure 3. Average effective thermal conductivity of various dry granular beds, measured at different average temperatures.

The experimental results exhibit also the effect of water saturation on the effective conductivity of porous beds, where the conductivity is shown to be consistently improving by water saturation. This phenomenal effect could be attributed to the fact that the water conductivity is known to be substantially higher than that of the air and therefore, the addition of water to a granular bed to replace some of the air entrapped within its voidage, would necessarily expedite the effective heat flow rate through the combined multi-phase system. On the other hand, this water saturation role is observed, however, to be more dramatically functioning in the case of sand beds versus that of glass beads. This could be interpreted by the naturally distinguished matrix structures of these two types of granular beds. Sand grains are abrasive randomly distributed non-uniform particles, whereas the glass beads are smooth spheres of uniform diameter. Thus, the surface wettability of the sand grains is greater than the smooth glass beads. Consequently, the water tends to adhere to the sand grains than does with the glass ones in which case the water may find its way to seep down off the glass beads due to gravitational effects. Moreover, the inherent irregularity of the sand grains' surfaces and the associated non-uniform size and porosity distribution are likely to support probabilities of small scattered cavities to form within the sand bed matrix, in which fluid convection may have become significantly vigorous. This may also justify the general superiority of the effective conductivity of dry sand beds over the glass beads ones under virtually the same working conditions.

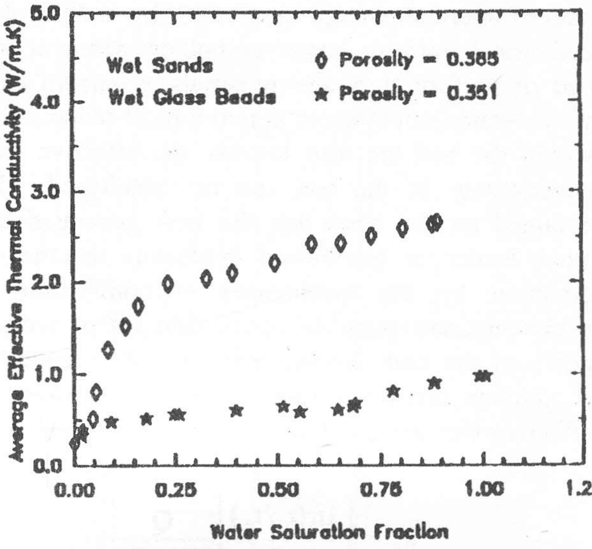

Figure 4. Sample thermal conductivity measured at 25 °C for water saturated granular beds.

A fairly thorough review was made of both analytical and empirical correlations published in the open literature. (e.g. Carbonell and Whitaker, 1984; Slattery, 1981; Nozad et al., 1985; Shonnard and Whitaker, 1989; Prasad et al., 1989), in order that a representative mathematical expression may be adapted to correlate the present experimental results reasonably. None of these appeared to work well with the present data, and even the agreement between them tends to be inconsistent throughout. Moreover, they consider only two component systems of granular materials, which when partly water-saturated they necessarily comprise three phase media, in which case the analyses tend to ignore one of the fluid phases. However, the present study aimed at approaching the problem in a rather simple yet adequate fundamental way.

Since the thermal conductivity of the solid phase is generally larger than that of the fluids, the contact between the solid particles influences the conduction significantly. It is therefore, to be expected, however, that the effective thermal conductivity of a multi-phase granular system would be essentially dependent on the following main parameters:

(a) the relative composition of the different phases involved,
(b) the thermal conductivity of each individual component under the same prevailing temperature,
(c) the structure of the matrix and the extent of the continuity of each phase.

On these bases, several attempts were exercised to fundamentally correlate the present experimental results in terms of the main parameters identified and significantly involved. The relative volumetric composition of the different phases could be represented by the average porosity or voidage fraction. The extent of water saturation, is also expressed as volumetric fractions of the entire granular bed.

The dry granular beds are basically two-phase systems composed of solid particles and air. A volumetric composition averaging technique was found to be reasonably successful in correlating the present experimental data. Both types of granular materials, sand grains and glass beads, could be simply described by the same empirical formula derived for dry beds tested at different temperatures. Average effective thermal conductivities of dry granular beds could, therefore be expressed in terms of the thermal conductivity and relative volumetric composition of each phase at the average temperature prevailing in the system. Accordingly, for a dry granular bed, the average effective thermal conductivity k_e may be predicted by the following simple mathematical expression:

$$k_e = k_s^{\Psi} k_g^{\phi} \qquad (2)$$

where

k_e = average effective thermal conductivity of the combined multi-phase system, (W/m.K)
ks = thermal conductivity of the bulk solid component, (W/m.K)
k_g = thermal conductivity of the gaseous phase, (W/m.K)
ϕ = average porosity (volumetric voidage fraction) of the granular bed.
Ψ = $(1-\phi)^2$

The least squares method was employed for scatter minimization of the experimental points. To check the validity and applicability of this experimentally-based correlation, Equation 2 was used to predict values of thermal conductivity of dry granular beds under the same experimental conditions. Comparison was then made between these predicted values and the corresponding average effective conductivities measured. Figure (5) exhibits such a comparative plot for dry beds of glass beads and of sand grains over the temperature range 50 to 400°C. As can be seen, both predicted and measured values of average effective thermal conductivity compare quite well over the range of temperature and porosity experimented with. The ultimate goal of such empirical correlation is represented by the solid line extended with a slope of unity over the entire domain of comparison. The off-line points and the extent of scatter could be interpreted as a reflection of some experimental errors combined with a lack of comprehension of some effects such as fluid convection inside the pores of the granular bed. Such convective role, if considered, would be very much complicating the treatment. The assumption of one dimensional conduction and the neglect of the end effects of the experimental heater would overestimate slightly the rate of heat flow. Moreover, the inevitable non-uniformity of the natural sand grains in terms of size and geometry would also add to the uncertainty.

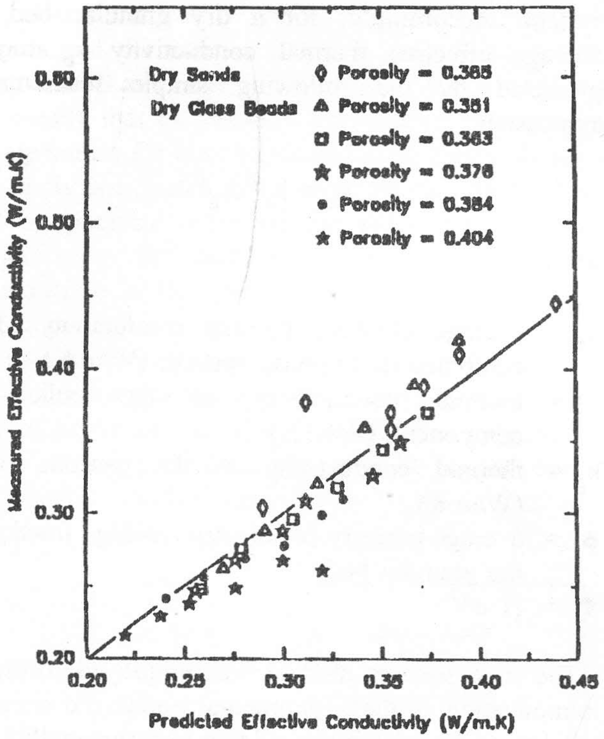

Figure 5. Measured versus predicted values of effective thermal conductivity for various dry granular beds at different temperatures (50-400°C).

The correlation of Equation 2 could be considered, nevertheless, for predicting average values of effective conductivity of dry granular materials within the prescribed range of physical and working conditions.

Unlike the dry test results, where the effective thermal conductivity of the two types of granular materials could be described by the same equation, it was not found feasible to produce a single formula to account for the extent of water saturation of the wet beds. Instead, two separate formulations were needed for each of the two types of beds involved. The extent of water saturation was represented by the average volumetric fraction occupied by the liquid, relative to the total voidage fraction of the porous bed. Accordingly, the experimentally measured average effective thermal conductivity of water-saturated granular beds could be fitted according to the following formulas:

(i) For the Athabasca wet sands,

$$k_e = k_s^{\Psi} k_g^{\phi} \left[1 + 14 \sqrt{\omega}\, k_l^{\omega} \right] \tag{3}$$

(ii) For the uniform-size wet glass beads,

$$k_e = k_s^{\Psi} \left[\omega\, k_l^{\omega\phi} + (1-\omega) k_g^{(1-\omega)\phi} \right] \tag{4}$$

where

k_l = thermal conductivity of the wetting liquid component, (W/m.K)

ω = water-saturated volumetric fraction of the granular bed.

Equation 3 of the wet sand reveals that the water saturation effect is directly predictable through the bracketed factor multiplier of the dry-bed conductivity of Equation 2. Similarly in a somewhat modified fit, is Equation (4) for the wet glass beads. Both equations 3 and 4 are reduced to the same dry-bed formula given by Equation 2, when setting the water saturation fraction w to zero.

Equations 3 and 4 were then used to predict values of the effective thermal conductivity of water-saturated beds of sand grains and of glass beads with different porosities. The results were subsequently compared against the corresponding experimentally-measured average conductivities obtained under the same working conditions. Figure (6) displays such comparative plot over the range of water saturation from dry to fully saturated bed conditions for both types of the granular materials examined. The comparison exhibits a reasonable extent of scatter. Once more, the nature of the sand grains, surface texture, wettability, irregularity, random size variation and the associated non-uniformly distributed porosity, versus the uniform size smooth spherical glass beads may justify the need for the two separate correlation Equations 3 and 4. However, since the thermal conductivity of liquid water is more than twenty times that of air, the role of the liquid component is significantly more influential than that of the gaseous component.

CONCLUSIONS

The simple formulation made for the effective thermal conductivity of dry and wet granular systems showed good agreement with the corresponding experimental values.

There is still however, a need for further work to cover wider variations in the physical and geometrical parameters and to consider further varieties of porous media of practical implications.

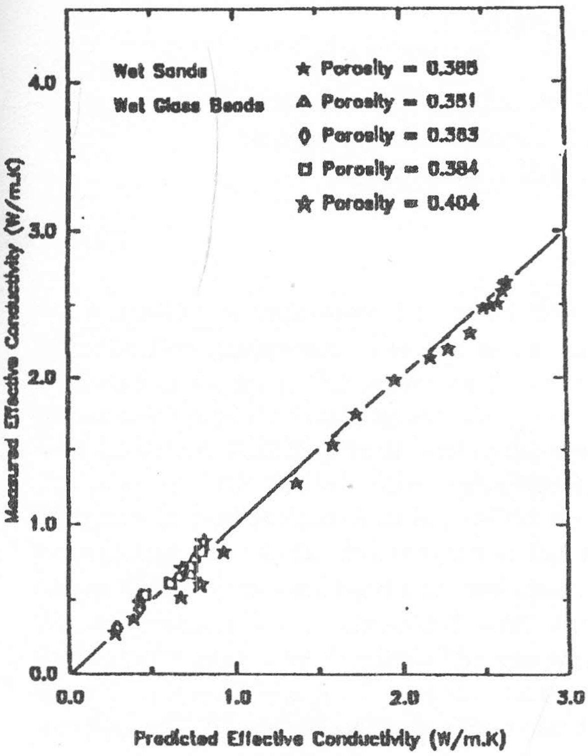

Figure 6. Measured and predicted average effective thermal conductivity values for water saturated beds of various granular materials.

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