A QUADRATIC PROGRAMMING METHOD FOR OPTIMAL CAPACITOR ALLOCATION IN DISTRIBUTION SYSTEMS

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ABSTRACT

The paper presents an accurate method for optimal solution of capacitor allocation problem using quadratic programming technique. The developed model takes into consideration the scattered nature of the load on distribution feeders as well as the actual impedances of different feeder segments. The actual feasible positions for capacitor allocation have also been considered while the capacitor cost has been accurately represented by a second order function. The voltage profile constraint at both light and peak load conditions has been included as the basic constraint governing the selection of the optimal number, sizes and locations of the added capacitors. The derivation of the model has been made to solve the general configuration of radial feeders having a main and a number of laterals. Verification of the model has been made using a test example.

Keywords: Distribution systems, Capacitor allocation, Quadratic programming.

LIST OF MAIN SYMBOLS

- M number of nodes or branches
- N number of terminal points
- C1 cost of unit peak power loss in LE/KW/year
- C2 cost of unit energy loss in LE/KWh
- F reactive load factor
- T operation time of fixed capacitors in hr./year
- R_i resistance of branch i
- X_i reactance of branch i
- ic_i capacitive current injected at node i due to the installation of fixed capacitors
- icl_i capacitive current injected at node i due to the installation of first type switched capacitors
- $ic2_i$ capacitive current injected at node i due to the installation of second type switched capacitors
- ir_i reactive load current in ampere found at node i
- Ic_i capacitive current in ampere flowing in branch i due to the existing of fixed capacitors alone.
- Icl_i capacitive current in ampere flowing in branch i due to the existing of fixed and first type capacitors
- $Ic2_i$ capacitive current in ampere flowing in branch i due to the existing of all capacitors
- Ir_i reactive load current in ampere flowing in branch i
- Ia_i active load current in ampere flowing in branch i
- $PL1_i$ peak power loss in kw on branch i before capacitor installation
- $PL2_i$ peak power loss in kw on branch i after capacitor installation
- CP_i yearly cost in LE of peak power loss reduction on branch i
- CP'_i cost of peak power loss reduction in LE in branch i due to existing fixed and switched capacitors
- CE_i yearly cost of energy loss reduction on branch i in LE
- VD_i voltage drop on branch i in volt after capacitor installation
- VT_i voltage drop up to terminal point i in volt after capacitor installation
- H_i voltage drop on branch i in volt due to the flow of active and reactive load current
- $H1_i$ voltage drop on branch i in volt due to the flow of active and reactive load current of light load

$H2_i$ voltage drop on branch i in volt due to the flow of active and reactive load current of peak load

f_i yearly cost in LE of fixed capacitor installed at node i

IM_i maximum capacitor size in ampere permissible at node i

K1(i) set of nodes whose currents flow in branch i

K2(i) set of branches found on the path from the source up to terminal point i

V_{min} minimum permissible value of voltage drop in volt

V_{max} maximum permissible value of voltage drop in volt

1. INTRODUCTION

Shunt capacitors are commonly applied in distribution systems for reactive power compensation. In addition to peak power and energy loss reduction and capacity release of both feeder and feeding substation, revenue increase due to voltage improvement, capacitors are mainly used for voltage control to provide a feeder voltage within the prescribed maximum and minimum permissible values at both light and peak load conditions. As these benefits depend largely on both capacitor sizes and locations, the fixed and switched general capacitor problem searches mainly for optimizing the number, locations and sizes of the installed capacitors which maximize an objective function while realizing the voltage profile constraint. In order to develop acceptable and practical solution methods, the general feeder configuration (a main and laterals), capacitor costs, the random scattering of load along the feeder, the feasible locations for capacitor installation as well as the voltage profile constraint must be taken into consideration.

The investigation of the previously developed work has revealed that both analytical methods [1-8] and optimization techniques based methods [9-15] have been developed. For mathematical programming based methods, the discrete programming technique [9], the dynamic programming technique [10], the local variable method [11], the mixed programming method [12,13] and a general purpose optimization technique called "simulated annealing" [14,15] have been utilized. Also, the fixed capacitor allocation problem [1-7] as well as the general fixed and switched capacitor problem have been studied.

In analytical solution methods, the actual feeder has been normally approximated by an equivalent uniformly loaded feeder, uniformly loaded feeder with an end load and a feeder with a uniformly and concentrated loads. In these methods, rounding off procedure is a common practice with respect to capacitor sizes and locations and the assumption of equal size capacitors as well as a pre-determined number of capacitors are also common in most of these methods. In feeders with voltage problems, analytical methods, in general, can not consider accurately the voltage drop constraint.

In this paper, a quadratic programming based simulation model has been developed which takes into consideration the main features of the capacitor allocation problem. The capacitor cost has been accurately represented by a quadratic function to simulate the fact that the cost per unit kva decreases as the capacitor size increases.

2. GENERAL CAPACITOR ALLOCATION PROBLEM

The general problem of optimal control of losses and voltage profile on radial distribution feeders using fixed and/or switched shunt capacitors for a certain feeder can be given as follows:

a- Reactive load duration curve
b- Physical configuration and feeder data such as wire sizes, section lengths, electrical parameters and spatial distribution of kvar load along the feeder.
c- Feasible points at which capacitors can be installed.
d- Available capacitor sizes and their costs
e- Different design parameters such as cost of peak power loss, cost of unit energy loss, interest rate, capacitor life time, ... etc.

Determine the optimum number, sizes and locations of fixed and/or switched capacitors, as well as the in service time of switched capacitors if any, which maximize the saving from the peak power and energy loss reduction while considering the voltage profile constraint at both light and peak load conditions.

In solving the above problem, the following points can be considered:

a- The selection of the capacitor type (fixed or switched or both) depends mainly on the reactive load curve. From the reactive load curve shown in Figure (1), many utilities prefer the installation of fixed capacitors for the light or fixed part of the load curve and switched capacitors for the variable or peak load part.

b- The number of capacitor to be installed may be fixed as determined by the utility, or a certain maximum number is not to be exceeded.

c- If fixed capacitor problem is only studied, reactive load curve is not required where only peak and light loads as well as reactive load factor are used. In this paper, only fixed capacitor allocation problem is considered.

3. PROBLEM FORMULATION

3.1 *Feeder Description*

The presented model has been developed to solve the general feeder configuration having a main and laterals. To describe the feeder, a number is assigned to each node and branch. A node is defined as a point on the feeder at which a load is existing and/or a capacitor can be installed. From the feeder configuration, the reactive current flowing in each branch is to be obtained. Also, it is assumed that a capacitor is installed at each given node such that each branch will carry a capacitive current according to its location in the feeder.
From the given configuration, branch currents can be obtained as follows:

$$Ic_i = \sum_{j \in k1(i)} ic_j \qquad (1)$$

$$Ir_i = \sum_{j \in k1(i)} ir_j \qquad (2)$$

3.2 *Power and Energy Loss Reduction*

The peak power losses in KW before and after capacitor installation are given on branch i by:

$$PL1 = 3\ R_i\ Ir_i^2/1000 \qquad (3)$$

$$PL2 = 3\ R_i\ (Ir_i - Ic_i)^2/1000 \qquad (4)$$

The cost of peak power loss reduction on branch i is

$$CP_i = 0.003\ R_i\ C1\ (2Ir_iIc_i - Ic_i^2) \qquad (5)$$

Similarly, the cost of energy loss reduction on branch i is given by:

$$CE_i = 0.003\ R_i\ C2\ T\ (2FIr_i\ Ic_i - Ic_i^2) \qquad (6)$$

The total cost to be saved due to capacitors installation is given by:

$$S = \sum_{i=1}^{M} (Z1_i Ic_i - Z2_i Ic_i^2) \qquad (7)$$

where

$Z1_i = 0.006\ R_i\ (C1+C2\ T\ F)\ Ir_i$ and

$Z2_i = 0.003\ R_i\ (C1+C2\ T)$

3.3 *Cost of Installed Capacitors*

The actual cost function of a capacitor installed at the ith node has a discrete nature. The difference between consecutive levels may be taken as the current corresponding to the smallest capacitor size considered. The exact simulation of this cost can only be made if integer programming techniques are used.

Due to the fact that the cost per kva decreases as the capacitor size increases, accurate representation of this cost can only be made using second order or generally nonlinear approximate functions. Hence, assuming second order function, the yearly cost of a capacitor unit located at node i can be given as:

$$f_i = W1_i\ ic_i + W2_i\ \ ic_i^2\ \ \text{L.E.} \qquad (8)$$

Where $W1_i$ and $W2_i$ are fixed coefficients of capacitor cost if located at node i. Firstly, the yearly capital cost of each capacitor size is to be drawn against capacitor size assuming capacitor life time and interest rate to be known. Then, the least square error criteria is used to get the above coefficients assuming a quadratic function.

3.4 *Voltage Drop Constraint*

The determination of the optimum capacitors to be installed must be made while satisfying the voltage profile at both light and peak load conditions.
The voltage drop on branch i with the capacitive current flowing is given by:

$$VD_i = Ia_i\ R_i + (Ir_i - Ic_i)\ X_i \tag{9}$$

$$= H_i - Ic_i\ X_i$$

The voltage drop up to terminal point i is

$$VT_i = \sum_{j \in k2(i)} (H_j - Ic_j\ X_j) \tag{10}$$

The voltage profile constraint is made such that the voltage drop at all terminal points must be within the permissible values at light and peak loads.

4. MATHEMATICAL PROGRAMMING TECHNIQUES

The form of the optimization or mathematical programming based problems can be generally given as follows:

$$\text{optimize} \quad F(y_1, \ldots\ldots, y_n) \tag{11}$$

subjected to the constraints

$$G_k(y_1, \ldots\ldots, y_n) \geq = \leq b_k, \ k=1,..,M \tag{12}$$

Based on the type of the variable y_i (continuous or integer or both) and the type of the cost function F and constraint equations G, the above problem has been generally classified as linear programming problem, quadratic programming problem, non-linear programming problem, dynamic programming problem, zero one programming problem, mixed programming problem,.. etc [16]. For each type of the above problems, standard solution techniques are generally available.

As the cost of loss reduction has a second order function the capacitor problem can be easily solved using quadratic programming technique if capacitor costs are approximated by a second order function. This function can accurately represent the fact that capacitor cost per kva decreases as capacitor size increases. Due to the assumption of continuous change of capacitor size, rounding off procedure is to be used to determine the nearest integer size required at each selected node from the available set.

5. MATHEMATICAL MODEL FORMULATION

The mathematical simulation model to be solved by quadratic programming technique is given as follows:
Maximize

$$F = \sum_{i=1}^{M} (Z1_i\ Ic_i - Z2_i\ Ic_i^2) - \sum_{i=1}^{M} (W1_i\ ic_i + W2_i ic_i^2) \tag{13}$$

subject to:

a. logical constraint for branch i

$$Ic_i = \sum_{j \in k1(i)} ic_j \qquad i=1,2,..,M \tag{14}$$

b. voltage drop constraint at terminal point i for both light and peak loads

$$V_{min} \leq \sum_{j \in k2(i)} (H1_j - Ic_j\ X_j) \leq V_{max} \tag{15}$$

$$V_{min} \leq \sum_{j \in k2(i)} (H2_j - Ic_j\ X_j) \leq V_{max} \tag{16}$$

$$i=1,2,..,N$$

c. capacitor size constraint at node i

$$ic_i \leq IM_i \tag{17}$$

$$i=1,2,..,M$$

The capacitor size constraint can be used, if required, to take into consideration the available area permissible for capacitor installation. Also, this constraint can be easily modified to prevent over compensation in all feeder branches and to prevent the installation of a capacitor at any location by assigning a zero value to IM of that location. The solution of the model can be made using standard quadratic programming techniques [16].

As it is assumed that the capacitor size changes in a continuous manner, the quadratic model will normally assign capacitive currents at most locations. Hence, using the above model in an iterative manner, where in each new time the location of minimum injection is to be deleted, can give a number of sub-optimal solutions with different number of capacitors and the optimal or more economical solution can be easily picked up.

6. EXTENSION OF THE MODEL TO SOLVE THE FIXED AND SWITCHED GENERAL CAPACITOR PROBLEM

The developed model in the last section can be easily extended to study the general capacitor allocation problem. Before developing the quadratic programming model, the following assumptions are made (refer to Figure (1)):

a. A fixed capacitor is assumed to be installed at each feasible point with operation period T.
b. Two switched capacitor types are assumed to be used. The first type installed at feasible point i has a current of $ic1_i$ and operation time T1 and the second one has a current $ic2_i$ and a time T2.
c. The cost function is obtained by dividing the whole period into three intervals as follows: first interval tf=T-T1 (only fixed capacitor are in operation) second interval ts=T1-T2 (fixed and first type switched capacitors are only on) third interval th=T2 (all capacitors are on).
   The modification made in the cost function can be stated as follows:

i) Cost of peak power loss reduction is derived when all the capacitors are existing and it is independent of the operation period.
ii) Cost of energy loss reduction is developed for each period separately.
   As an example, the cost of power loss reduction at branch i is given by:

$$CP'_i = 0.003\ R_i\ C1\ (2.\ Ir_i\ Ic'_i - (Ic'_i)^2) \qquad (18)$$

where:

$$Ic'_i = Ic_i + Ic1_i + Ic2_i,$$

$$Ic_i = \sum_{j \in kl(i)} ic_j,\quad Ic1_i = \sum_{j \in kl(i)} ic1_j$$

and

$$Ic2_i = \sum_{j \in kl(i)} ic2_j$$

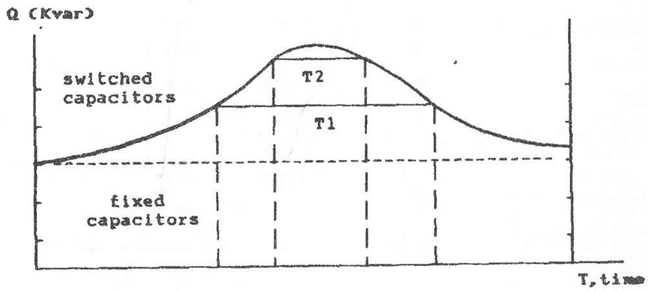

Figure 1. Reactive load curve.

With respect to the constraint equations, the voltage profile constraint at the different nodes are to be developed for the three intervals. The light and peak load of each interval can be easily obtained from the load curve.

The above analysis, although made assuming two switched capacitor types, can be easily extended to any number of switched capacitors with different operation times.

7.TEST EXAMPLE

The developed quadratic model has been verified through its application to the distribution feeder shown in Figure (2). The feeder segment impedances and load data are given in Tables (1) and (2) respectively. The different parameters used for model construction can be summarized as follows:

a. C1 120 LE/Kw/year , T=8760 hr.
b. C2 0.06 LE/Kwh , F=0.56
c. The actual capacitor costs and sizes used are:

Table (1)
Feeder impedance

seg. no	R ohm	X ohm
1	.095	.017
2	.125	.022
3	.09	.016
4	.29	.051
5	.185	.032
6	.12	.021
7	.24	.058
8	.186	.045
9	.86	.032
10	.108	.019
11	.205	.06
12	.18	.052

Table (2)
Load Data

node no.	peak current real amp.	peak current imag. amp.	light current real amp.	light current imag. amp.
1	12.5	9.3	5.1	3.06
2	9.35	7.	4.	2.4
3	18.	13.5	9.	5.4
4	18.	13.5	9.	5.4
5	16.2	12.14	7.2	4.8
6	4.9	3.6	1.8	1.1
7	13.8	10.3	5.2	3.2
8	14.3	10.8	7.1	4.2
9	10.5	7.8	3.9	2.3
10	8.2	6.2	3.2	1.9
11	6.5	4.8	2.5	1.5
12	4.5	3.4	1.8	1.1

size (Kva)	size (ampere)	cost (LE/year)
200	10.5	120
400	21	200
600	31.5	280
800	42	360

d. W1=11.484 LE/ampere and W2=-0.0645 LE/sq. ampere

e. Maximum voltage drop or rise at all load levels is 3% (190.53 volt)

f. Line voltage is 11 Kv

g. Over compensation constraint is not considered.

h. The voltage drops up to terminal points are given as:

terminal point	voltage drop at peak load (volt)	voltage drop at light load (volt)
1	124.86	53.396
2	107.01	45.297
3	75.3	32.111

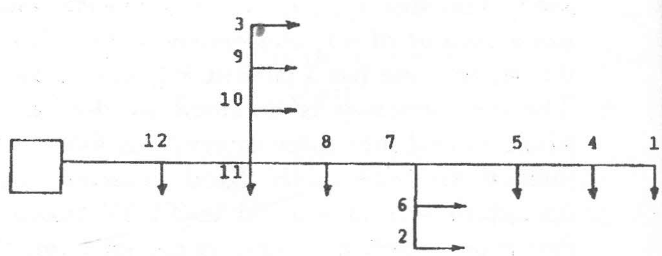

Figure 2. 12 Nodes test example.

The solution results have been obtained iteratively as shown in Table (3) where, at any iteration, the location of minimum injected power is to be deleted in the following iterations.

If rounding off procedure is made, for each iteration, the actual installed capacitors and injected capacitive current at each node are as shown in Table (4).

The investigation of these tables reveals the followings:

a. The total injected capacitive current, shown in Table (3), is nearly constant around 66 ampere in most iterations.
b. The annual benefits obtained from the continuous capacitive current, Table (3), decrease with progress of iteration counter while the total annual benefits from actual capacitor installation, Table (4),increase up to certain point where start to decrease again.
c. The optimum case obtained from Table (4) is found at iteration number 7 with three installed capacitors.
d. The new terminal voltages obtained with the installed capacitors are given as:

terminal point	voltage drop at peak load (volt)	voltage drop at light load (volt)
1	109.477	38.01
2	93.37	31.65
3	65.66	22.47

8.EFFECT OF VOLTAGE DROP CONSTRAINT

The previous study case has been made for a feeder having no voltage drop problems. To show the importance of applying mathematical programming techniques for feeders having voltage problems, the length of segments has been assumed to be increased three times as the old lengths. In this case, the voltage drops at terminal points are obtained as:

terminal point	1	2	3
peak load voltage drop (volt)	216.26	185.35	129.27
light load voltage drop (volt)	92.49	72.69	55.6

The application of the presented model for the above case, using the iterative process discussed above, has resulted into the following optimum solution:

optimum number of added capacitors 3
optimum locations (node number) 1 7 10
optimum size at each node
(respectively) 31.5 31.5 31.5
net profile (LE/year) 9894.25

The new voltage drops at terminal points are:

terminal point	voltage drop at peak load (volt)	voltage drop at light load (volt)
1	181.89	58.11
2	155.78	43.12
3	110	36.23

If this case is resolved neglecting the voltage drop constraint, by increasing the voltage drop limit, the optimum solution has been obtained as before for the normal length feeder, but with a net return calculated as 13570.5 LE/year.

9. EFFECT OF ENERGY AND PEAK LOSS COSTS

The presented model has been applied for the above case for the following costs:

a. Peak power loss cost is kept constant while energy loss cost is increased in steps up to three times the old value.
b. Energy loss cost is kept constant while peak loss cost value is increased in steps up to three times the old value. In all these cases, the optimum solution is the same while only the net return is increased.

10. CONCLUSIONS

The paper presents a mathematical programming based model for solving the capacitor allocation problem utilizing a quadratic programming technique. The developed model realizes the different factors and features governing the distribution feeders in addition to its ability to solve the general radial configuration having a main and laterals. The model can be easily utilized to study the effect of changing the different problem parameters and can be very important for feeders having voltage problems. Although model verification has been made for fixed capacitor allocation problem, it can be easily extended to study the general fixed and switched capacitor problem.

Table (3)
Optimal injected Capacitive Currents as obtained from the Quadratic Model

node no.	iteration no. 1	2	3	4	5	6	7	8
1	3.994	3.994	0.0	0.0	0.0	0.0	0.0	0.0
2	5.913	5.911	5.887	5.947	0.0	0.0	0.0	0.0
3	13.118	14.447	14.447	16.787	16.843	16.866	18.354	0.0
4	12.375	12.376	17.43	17.42	17.633	19.704	19.36	18.733
5	7.696	7.693	6.997	7.093	5.13	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	9.789	9.803	9.987	9.524	19.004	23.12	28.52	38.45
8	7.281	7.213	7.185	9.41	8.022	7.42	0.0	0.0
9	1.363	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	3.889	4.503	4.506	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Σ ic amp.	65.42	65.94	66.44	66.18	66.63	67.11	66.23	57.18
yearly cost of saved loss in LE	6745	6747	6740	6727	6702	6684	6641	6136
annual benefit in LE	6033	6031	6027	6022	6005	5995	5979	5598

Table (4)
Actual Injected Capacitive Currents at Different Nodes

node no.	iteration no. 1	2	3	4	5	6	7	8
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.
2	10.5	10.5	10.5	10.	0.0	0.0	0.0	0.
3	10.5	10.5	10.5	21.	21.	21.	21.	0.
4	10.5	10.5	21.	21.	21.	21.	21.	21.
5	10.5	10.5	10.5	10.5	10.5	0.	0.	0.
6	0.	0.	0.	0.	0.	0.	0.	0.
7	10.5	10.5	10.5	10.5	21.	21.	31.5	42.
8	10.5	10.5	10.5	10.5	10.5	10.5	0.	0.
9	0.	0.	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.	0.	0.
Σ ic (amp.)	63	63	73.5	84	84	73.5	73.5	73.[illegible]

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