AN APPROACH FOR SEDIMENT TRANSPORT ESTIMATES IN WATER WAYS

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ABSTRACT

Empirical formulas were developed to determine the total rate of sediment transport in Alluvial channels in term of hydraulic parameters, sediment properties and the bed undulation. Based on the data collected by Brownlie[2], a multiple regression analyses were made to develop the equations for predicting the rate of sediment transport. A comparison study has been made between the developed equations and the formulas suggested by DuBoy[6], Shields[15], Mayer - Peter - Muller[10], Kalinske[8], Ackers and White[1], Englund and Hansen[7], Parker[13], Karim and Kenndy[9], Yang - Molinas[18], Swamee[17] and Brownlie[3]. The analysis showed that the obtained equations in the present study give more accurate results for the cases of bed covered by ripples, plane bed and antidunes.

Keywords: Rate of sediment transport, Bed Load, Suspended load, ripples - dunes - transition - plane bed - antidunes.

Notations

B	bed width (m)
C_s	sediment concentration
d_{50}	median diameter (mm)
d_g	dimensionless grain diameter (mm)
F_g	mobility number
Fr	Froude number
g	gravitational acceleration (m/sec^2)
L_B	meander width (m)
L_M	meander length (m)
M	Karmer's uniformity coefficient
n	Manning's roughness coefficient (m$^{1/6}$.sec)
n_s	Strickler's coefficient for grain roughness
q	unit discharge (m^3/sec/m)
q_b	bed load discharge in weight per unit time and unit width (Kg/sec/m)
q_{ss}	suspended load discharge in per weight unit time and unit width (Kg/sec/m)
q_s	total sediment load in weight per unit time and unit width (Kg/sec/m)
q_{s*}	dimensionless total sediment load
R	hydraulic radius (m)
Re_*	grain Reynolds number
S	energy slope
S.F.	shape factor
T_*	dimensionless bed shear stress
T_{*c}	dimensionless critical shear stress
V	average flow velocity (m/sec)
u_*	shear velocity (m/sec)
u_{*c}	critical shear velocity (m/sec)
Y	average depth of flow (m)
γ	specific weight of fluid (t/m^3)
γ_s	specific weight of sediment (t/m^3)
ν	kinematic viscosity (cm^2/sec)
σ_g	geometric standard deviation of bed mixture
τ	average shear stress (t/m^2)
τ_c	critical shear stress (t/m^2)

INTRODUCTION

One of the most important aspect of fluvial processes is the motion of sediment in rivers, to which river bed degradation and agradation are closely related. This problem has attracted many investigators in the last five decades who developed numerous equations to predict the rate of sediment transport. There are big discrepancies between the results of the most of the well known equations. Therefore, the selection of the appropriate sediment transport formula is very difficult. The suitability of a formula must be judged by the assumption used in the development of the formula and the range of data collected for calibration and verification.

The equations predicting the rate of sediment transport were based on three approaches:

i- Shear stress approach:- In this approach the rate of sediment transport is mainly a function of the bed shear stress "τ_b" and the critical shear stress of the bed sediment "τ_c".DuBoy[6], Shields[15], Mayer Peter-Muller[10],Parker[13] and Swamee[17] used this approach to develop their formulas.

ii- Power approach:- The rate of sediment transport is a function of energy expenditure per unit weight of water which is defined as the product of average velocity "V" by the energy slope "S". Englund-Hansen[7] and Ackers-White[1] used this approach to form their equations.

iii- Parametric approach:- For which the rate of sediment transport is a function of flow parameters and sediment parameters such as mean velocity, depth of flow, mean sediment size, water temperature and geometric standard deviation. Colby produced graphs to compute the sediment discharge using this approach.

Nakato[11] tested several formulas used for predicting the rate of sediment transport and he demonstrated the difficulty of using such formulation predict sediment discharges in natural rivers. He found that the prediction of bed load would have been much worse in most cases had depth predictors been used instead of the measured values.

Swamee and Ojho[17] developed empirical equations for bed load and suspended load transport rate taking into their consideration the effect of sediment mixture nonuniformity which was neglected by most of popular sediment formulas. Their formula for total load is illustrated in appendix I and the accuracy of this formula is tested in the present study.

Ceballos [5] studied bed load equations and he rearranged these formulas so that it can take the following general form:

$$q_b = c_1 c_2 (T_* - T_{*c})^{c_3} d_{50} \qquad (1)$$

Where q_b is the rate of sediment transported as bed load ,T_* and T_{*c} are the dimensionless Shields diameter and the dimensionless critical shear stress, d_{50} is the median diameter of the bed mixtures, and c_1, c_2 and c_3 are constants vary according to the formula used. He concluded that Smart[16] coefficients for bed load formula provide good estimates For Gilbert and Mayer-Peter-Muller data.

The aim of this study is to obtain equations to determine the total rate of sediment transport covering a wide range of sediment data and flow conditions.

DIMENSIONAL ANALYSIS

The rate of sediment transport is affected by sediment properties, flow conditions and the geometry of the alluvial stream. The sediment properties are the median diameter of the bed mixture "d_{50}", the uniformity of the grain size distribution which can be expressed by the geometric standard deviation of bed mixture σ_g and is defined as $\sqrt{d_{84.1}/d_{15.9}}$, the submerged specific gravity of sediment "$(\gamma_s$-$\gamma)$" , the fall velocity of sediment "ω", critical shear stress of bed mixture "τ_c" and the shape of sediment particles which is normally expressed by the shape factor "S.F.". The shape factor is defined as $a/\sqrt{bc}$ where "a" is the shortest diameter of the grain particles, "b" is the longest diameter of the particle and "c" is the diameter of the particle perpendicular to "b". The rate of sediment transport increases with the decrease of sediment diameter, the submerged specific weight of sediments, critical shear stress and the fall velocity of sediment particles, while it is proportion to the shape factor and the geometric standard deviation. The flow conditions affecting the rate of sediment transport are characterized by the depth of flow "Y", the longitudinal channel slope "S", channel width "B", the kinematic viscosity "ν", the average velocity of flow and the resistance of flow. Since the rate of sediment transport is normally defined as the dry weight or volume of sediment passing a certain section per unit time and unit width (i.e. the average rate of sediment transport per unit width), the effect of bed width will not be considered in this study. However Carson and

Griffith[4] found that the rate of sediment transport decreases with the increase of bed width. Osman and Thorne [12] came to the same conclusion for the effect of bed width on the rate of sediment transport because bank erosion is a major source of supply of bed material.

The flow resistance is divided into skin friction due to the roughness of sediment particles forming the bed and the friction due to the bed undulation. The skin friction due to grains can be characterized by any representative size of bed mixture particles for example the size of "d_{90}" as proposed by Mayer Peter and Muller[10] or "d_{60}" according to Einstein. In this study the median diameter "d_{50}" is taken to characterize the grain friction.

The way of which sediments are transported is greatly affected by the type of bed forms. For bed covered with ripples the transported sediment are mainly bed load, while for transition and bed covered with antidunes most of the transported sediments are in suspension. Therefore, the equations for sediment transport differs according to the type of bed form. So it is very essential to develop an equation for sediment transport for each type of bed forms.

The rate of sediment transport is also affected by channel shape in plane such as channel bends or meanders. The existence of meanders decreases the rate of sediment transport. Meanders can be characterized by the meander length "L_M", which denotes the longitudinal distance between corresponding points on two consecutive loops of fully developed meanders, and meander belt "L_B", which is defined as the distance between lines drawn tangential to the extreme limits of fully developed meander loop.

According to the foregoing analysis the rate of sediment transport "q_s" can be expressed as follows:

$$q_s = f(d_g, (\gamma_s - \gamma), \sigma_g, \omega, S.F., \tau_c, V, Y, B, S, \nu, L_M, L_B, g) \qquad (2)$$

The application of the π - theorem on equation (2), leads to the following equation:

$$q_{s*} = f\left(Re_*, T_*, T_{*c}, \frac{Y}{d_{50}}, \sigma_g, \frac{\omega d}{\nu}, S.F., Fr, \frac{L_B}{L_M}\right) \qquad (3)$$

Where $q_{s*} = \dfrac{q_s}{\gamma_s V^2 d_{50}^2}$ is the dimensionless rate of sediment transport, $Re_* = \dfrac{u_* d_{50}}{\nu}$ is the grain Reynolds number, T_* , $T_{*c} = \dfrac{\tau\ ,\ \tau_c}{(\gamma_s - \gamma) d_{50}}$ is the dimensionless average shear stress and the critical shear stress of the median diameter, $u_* = \sqrt{gRS}$ is the shear velocity, $\tau = \gamma RS$ is the average shear stress and $Fr = \dfrac{V}{\sqrt{gY}}$ is channel Froude number.

The ratio of L_B to L_M express the curvature of the meander which equals zero for straight channel and greater than zero for meanders. The Function of equation (3) will be determined for each type of channel bed forms.

SEDIMENT TRANSPORT DATA

A complete reliable data for rate of sediment transport and other dimensionless parameters given in equation (3) is very important to determine its function. Unfortunately, there is no data relating the rate of sediment transport with the shape factor of grains and with the curvature of alluvial channels. The data used in the present study was collected by Brownlie[2] it consists of 7027 records (5263 laboratory records and 1764 field records). Each record consists of 10 basic parameters. These parameters are discharge "Q" in liter/sec, channel width "B" in meters, average depth "Y" in meters, energy slope "S", median particle size "d_{50}" in millimeter, geometric standard deviation "σ_g" grain specific gravity "γ_s" in ton/m^3, the concentration of sediment load "C_s" in part per million (ppm) and the condition of bed. Not all observation parameters are available to Brownlie and he was not certain of some of them. In the present study only records containing complete and reliable data were used. The sum of the utilized records is 975 records. The value of the fall velocity for each record is calculated using Rubey's [14] formula which is expressed as follows:

$$\omega = \sqrt{\frac{36\nu^2}{d^2} + \frac{2(\gamma_s-\gamma)d}{3\rho}} - \frac{6\nu}{d} \qquad (4)$$

The value of the critical shear stress for each record is computed using Shields diagram for incipient motion. The collected data was divided according to the condition of bed form into bed covered with:- *i- ripples, ii- dunes, iii- transition, iv- plane bed and v- antidunes*. The number of complete records and the range of each parameter is given in Table (I).

Table I. Ranges of the collected data for each type of bed undulations.

Bed condition	No. of records	Median diameter (mm)		Depth of water (m)		Energy slope	
		d_{min}	d_{max}	Y_{min}	Y_{max}	S_{min}	S_{max}
Ripple	137	0.110	0.620	0.152	0.853	1.400×10^{-4}	1.52×10^{-3}
Dunes	586	0.100	4.376	0.151	4.260	5.830×10^{-5}	5.77×10^{-3}
Transition	71	0.150	1.349	0.151	3.627	1.058×10^{-4}	14.6×10^{-3}
Plane bed	115	0.083	1.349	0.155	5.290	6.300×10^{-5}	7.9×10^{-3}
Antidunes	66	0.100	1.350	0.150	0.457	1.100×10^{-3}	23.8×10^{-3}

EQUATIONS FOR SEDIMENT TRANSPORT

The way in which sediments is transported differs according to the condition of bed undulation. For alluvial channel bed covered with ripples sediment are transported by rolling over the upstream face of the ripple and then falls over the downstream face. Once the particle lyes on the downstream face it does not move till it exposed in the upstream part of the ripple. Suspended load in rippled bed is rarely found in nature. Applying a multiple regression analysis on the data collected for ripples, the following equation is obtained:-

$$q_{s*} = 0.313 \frac{(T_* - T_{*c})^{0.668} Fr^{5.603}}{\sigma_g^{0.476} Re_*^{0.627}} \left(\frac{Y}{d_{50}}\right)^{2.25} \qquad (5)$$

The correlation coefficient of equation (5) is 0.878 and the average standard error of estimate equals 0.695. The plot of actual records for dimensionless rate of sediment transport "q_{s*}" versus the estimated dimensionless rate of sediment transport is given by Figure (1). According to equation (5) the dimensionless rate of sediment transport increases with the increase of Froude number, the difference between the dimensionless average bed shear stress and the dimensionless critical shear and also increases with the increase of the ratio of the water depth to the median diameter of the bed mixture. While it decreases with the increase of grain Reynolds number and the geometric standard deviation of bed mixture. The relationships between both the grain Reynolds number and the difference between dimensionless average and critical shear stress versus the dimensionless records of the rate of sediment transport are given by Figures (2) and (3), respectively. The effect fall velocity is disappeared from equation (5) because no sediment will be transported in suspension.

For the case of bed covered with dunes, the height of bed undulation exceeds the laminar sublayer, some of sediments will be transported in suspension but most of the transported sediments are bed load. Based on the collected data for bed covered with dunes, a multiple regression analysis has been made to derive the following empirical equation:-

$$q_{s*} = 14.03 (T_* - T_{*c})^{0.317} Fr^{1.93} \sigma_g^{0.461} Re_*^{0.847} \left(\frac{Y}{d_{50}}\right)^{0.946} \left(\frac{\nu}{\omega d_{50}}\right)^{1.246} \qquad (6)$$

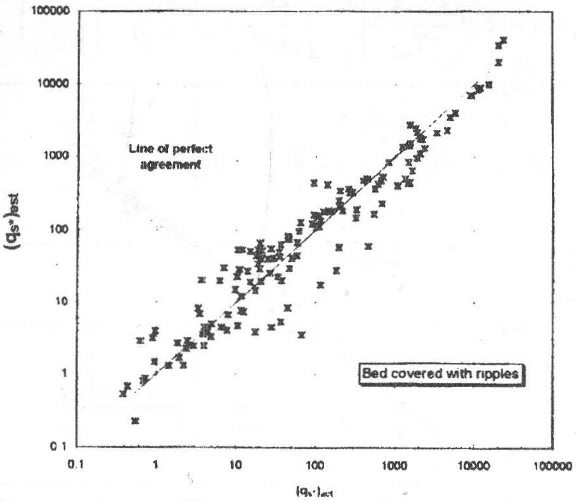

Figure 1. Plot of estimated of dimensionless total rate of sediment transport obtained from equation (5) versus the actual values for case of rippled bed.

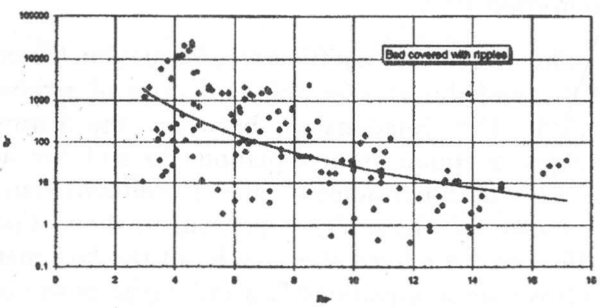

Figure 2. Variation of grain reynolds number with the dimensionless total rate of sediment transport in case of bed covered ripples.

The correlation coefficient of equation (6) equals 0.912 and the average standard error of estimate equals 0.466. Figure (4) shows a plot of estimated dimensionless rate of sediment transport versus the actual records. Equation (6) shows that for case of dunned bed the dimensionless rate of sediment transport increases with the increase of dimensionless bed shear stress, grain Reynolds number, geometric standard deviation, Froude number and the ratio of depth of flow to the median diameter. While it decreases with the increase of grains fall velocity. Since part of the shear stress is required to overcome the resistance of bed undulation which is greater for dunned bed than bed covered with ripples, therefore the power of the $(T_* - T_{*c})$ is less in case of dunned bed than in rippled bed. The variation of the difference between dimensionless bed shear stress and the critical shear stress of bed mixture versus the dimensionless rate of sediment transport is given by Figure (5).

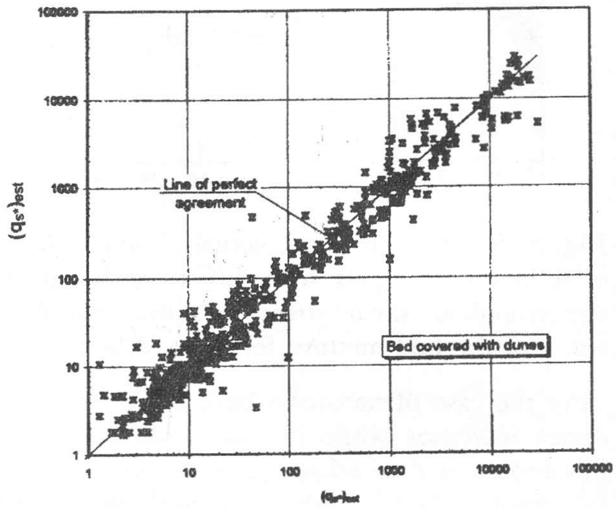

Figure 3. Relationship between actual dimensionless total rate of sediment transport and the difference between the dimensionless average shear stress and the dimensionless critical shear stress for rippled bed.

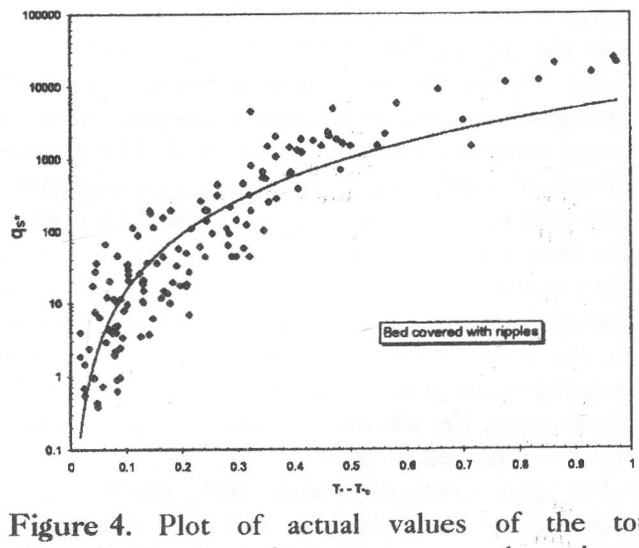

Figure 4. Plot of actual values of the total dimensionless rate of transport versus the estimated total rate of sediment transport determined from equation (6).

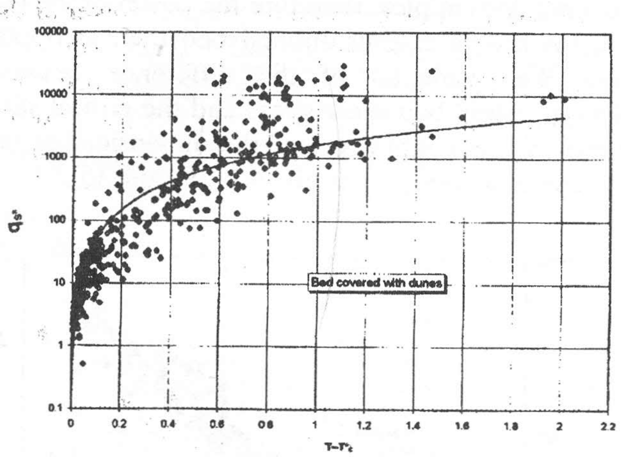

Figure 5. Variation of the actual dimensionless rate of sediment transport with difference between the dimensionless shear stress and the critical shear stress of the bed mixture for dunned bed.

For the case of transition bed, the wave length of dunes increases while its amplitude decreases. In this condition the bed roughness is less than in case for dunned bed. Applying a multiple regression analysis on the collected data to determine the function of equation (3), the following equation is obtained:

$$q_{s*} = 5.193 \frac{(T_* - T_{*c})^{0.865} Fr^{2.58} \sigma_g^{1.26}}{Re_*^{0.294}} \left(\frac{Y}{d_{50}}\right)^{0.946} \left(\frac{\nu}{\omega d_{50}}\right)^{0.2} \quad (7)$$

The correlation coefficient of equation (7) is 0.987 and the average standard error of estimate equals 0.397. Figure (6) shows a plot between estimated dimensionless rate of sediment transport versus the actual records for bed transition bed. The exponent of the value $(T_* - T_{*c})$ is greater in case of transition bed than in case of dunned and rippled bed because the flow resistance in case of transition bed is less than in the other two cases. Equation (7) shows that the rate of sediment transport is inversely proportion to the grain Reynolds number and the grain fall velocity while it increases with the increase of bed shear stress, Froude number and geometric standard deviation of bed mixture.

For the case of plane bed where all bed undulations are washed out and most of transported sediment are suspended load, the best empirical equation that fit the collected data for plane bed is expressed as follows:-

$$q_{s*} = 586.653 \left(\frac{T_* - T_{*c}}{T_{*c}}\right)^{0.096} \frac{Re_*^{2.492}}{Fr^{0.088} \sigma_g^{0.29}} \left(\frac{d_{50}}{Y}\right)^{0.047} \left(\frac{\nu}{\omega d_{50}}\right)^{2.37} \quad (8)$$

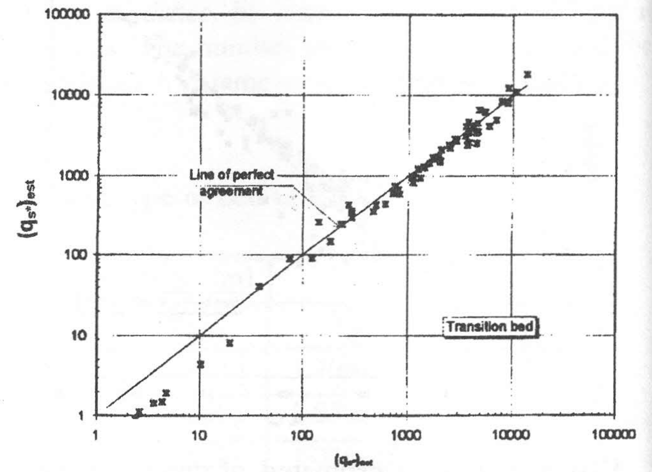

Figure 6. Plot of estimated values of the total dimensionless rate off sediment transport determined from equation (7) versus the actual records for transition bed.

The correlation coefficient of equation (8) equals 0.812 and the average standard error of estimate is 0.635. The relationship between the estimated values obtained from equation (8) and the actual records of dimensionless rate of sediment transport is shown by Figure (7). Equation (8) shows that the effect of the critical shear stress of the bed material is less than in equations 5, 6 and 7 (the power of the term T_*-T_{*c} is minimum) because most of sediments are gone to suspension. The relationship between the term "$(T_*-T_{*c})/T_{*c}$" and the dimensionless rate of sediment transport is shown in Figure (8). It is obvious from Figure (8) that the effect of T_* is insignificant when the value of "$(T_*-T_{*c})/T_{*c}$" is greater than 60.

For the case of bed covered with antidunes which probably occurs at Froude number greater than 0.8 (i.e. high flow velocity) most of the transported sediments are transported as suspended load. Using the collected data for this condition and applying the a multiple regression analysis the following empirical equation is obtained:

$$q_{s*} = 0.00015 \frac{Fr^{1.33} Re_*^{2.203}}{(T_* - T_{*c})^{0.317} \sigma_g^{0.2}} \left(\frac{Y}{d_{50}}\right)^{2.221} \left(\frac{\nu}{\omega d_{50}}\right)^{1.524} \quad (9)$$

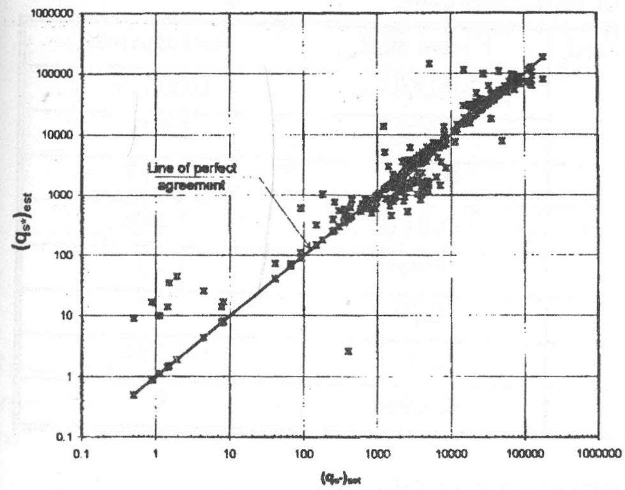

Figure 7. Plot of estimated values of the total dimensionless rate of sediment transport determined from equation (8) versus the actual records for plane bed.

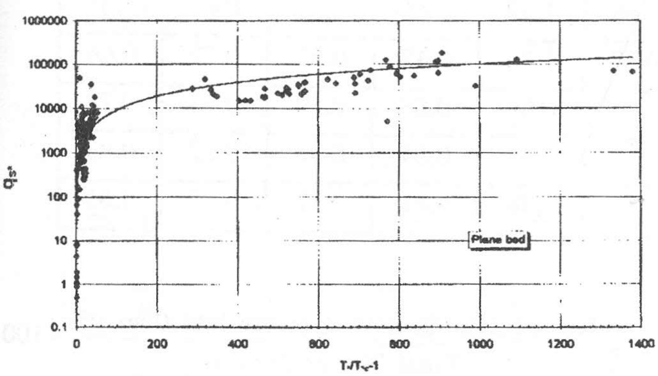

Figure 8. Variation of the relative dimensionless shear stress T_*/T_{*c}-1 versus the actual records dimensionless rate of sediment transport for the case of plane bed.

The correlation coefficient of equation (9) equals 0.91 and the average standard error of estimate equals 0.474. The plot of estimated dimensionless rate of sediment transport obtained from equation (9) versus the actual records of the dimensionless rate of sediment transport is shown in Figure (9). In this condition the dimensionless rate of sediment transport is inversely proportion with the difference between the dimensionless bed and critical shear stress. The value of the dimensionless rate of sediment transport increases with the increase of grain Reynolds number and Froude number.

According to the foregoing analysis, the equations expressing the rate of sediment transport can be written in general form as follow:

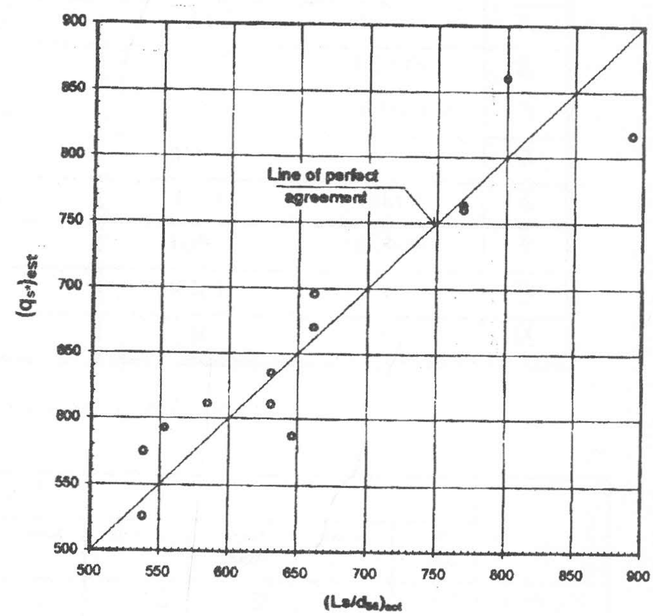

Figure 9. Plot of the estimated values of the total dimensionless rate of sediment transport determined from equation (9) versus the actual records for the condition of bed covered with antidunes.

$$q_{s*} = A\left[Re_*^{B}\left(T_* - T_{*c}\right)^{C}\left(\frac{y}{d_{50}}\right)^{D} Fr^{E}\,\sigma_g^{F}\left(\frac{\omega d}{\nu}\right)^{G} T_{*c}^{H}\right] \qquad (10)$$

Where *A, B, C, D, E, F, G* and *H* are constants their values depend on the bed form conditions.

The values of the constants is illustrated in Table II.

COMPARATIVE STUDY BETWEEN SEDIMENT TRANSPORT FORMULAS

A comparison between the equations predicting the rate of sediment transport obtained in the present study and equations of sediment transport developed by DuBoy [6], Shields [15], Mayer - Peter - Muller [10], Kalinske[8], Acker and White [1], Englund and Hansen [7], Parker [13], Karim and Kenndy [9], Yang - Molinas [18], Swamee [17] and Brownlie [3] has been made. The equations presented by these authors are illustrated in appendix (I). The comparative study has been made by applying the equations deduced in the present study and the equations developed by the afore-mentioned authors on the data collected by Brownlie[2].

Table II. Constants of equations (10).

	Ripples	Dunes	Transition Bed	Plane bed	Antidunes
A	0.313	14.03	5.193	586.653	0.00015
B	-0.627	0.847	-0.294	2.492	2.203
C	0.668	0.317	0.865	0.096	-0.317
D	2.25	0.946	0.946	-0.047	2.221
E	5.603	1.93	2.58	-0.088	1.33
F	-0.476	0.461	1.26	-0.29	-0.2
G	0	-1.246	-0.2	-2.37	-1.524
H	0	0	0	-0.096	0

Table III. Values of the average error of estimate.

	Average error of estimate										
	Pres. study	Duboy	Mayer Peter	Shields	Kalinske	Englund	Ackers	Brownli	Yang	Kennedy	Swamee
Ripples	0.69	1.73	0.75	24.49	26.14	1.69	1.38	1.86	2.28	0.85	1.01
Dunes	0.47	0.76	0.88	5.01	3.39	0.57	1.23	0.42	0.63	0.79	0.66
Transition	0.38	0.84	0.90	5.12	5.44	0.44	0.89	0.58	0.69	0.73	0.78
Plane	0.37	0.90	0.95	2.84	1.13	0.56	0.98	0.34	0.30	0.67	0.83
Antidune	0.37	0.92	0.89	4.13	0.82	0.52	1.46	0.41	0.77	0.69	0.61

The percentage average standard error of estimate for each type of bed condition are calculated and presented in Table (III). It is found that the equations developed in the present study produce minimum values of the average standard error of estimate except for the case of dunned and plane bed where Brownlie's formula gives almost the same values (as shown by Figure (10) and Table III). Yang formula gives the best estimate of the total rate of sediment transport for the case of plane bed.

In order to compare the accuracy of the equations deduced in the present study with the equations developed by the afore-mentioned authors, the percentage number of records having values of estimated error less than 5%, 10% , 25% and 50% is introduced, which is defined as follows:

% No. of records having error less than p% =

$$= \frac{\text{No. of records having errors less than p\%}}{\text{Total No. of records}} \times 100$$

Where the error is calculated as follows:

$$\frac{|(q_s)_{actual} - (q_s)_{estimated}}{(q_s)_{actual}}$$

Figures (11) and (12) show a comparison between the present study and the other equations in respect to the percentage number of records having error less than 5, 10, 25 and 50%. It has been found that the equations developed in the present study give higher number of accurate records than the other equations. This means that these equations can be fairly used to determine a better rough estimate of the rate of sediment transport than the other formulas.

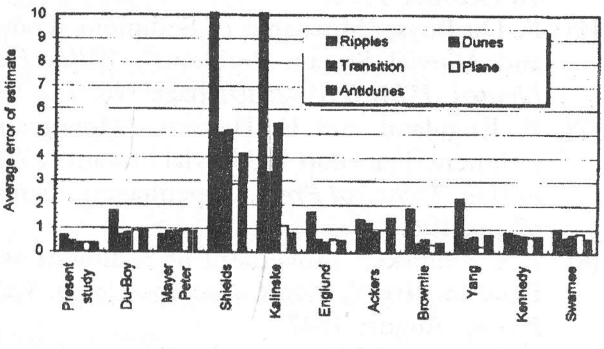

Figure 10. Comparison between the percentage error of estimate calculated using different formulas and that computed by the equations 5,6,7,8,9.

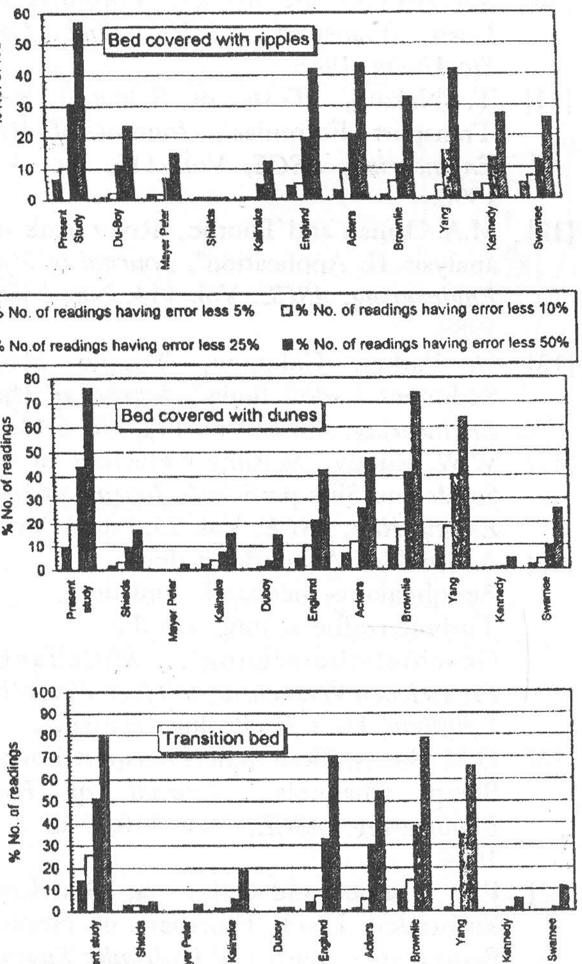

Figure 11. Comparison between the values of the % number of records having average errors less than 5, 10, 25 and 50% obtained using the equations developed in this study and other formulas for bed conditions i-ripples, ii-dunes and iii transition bed.

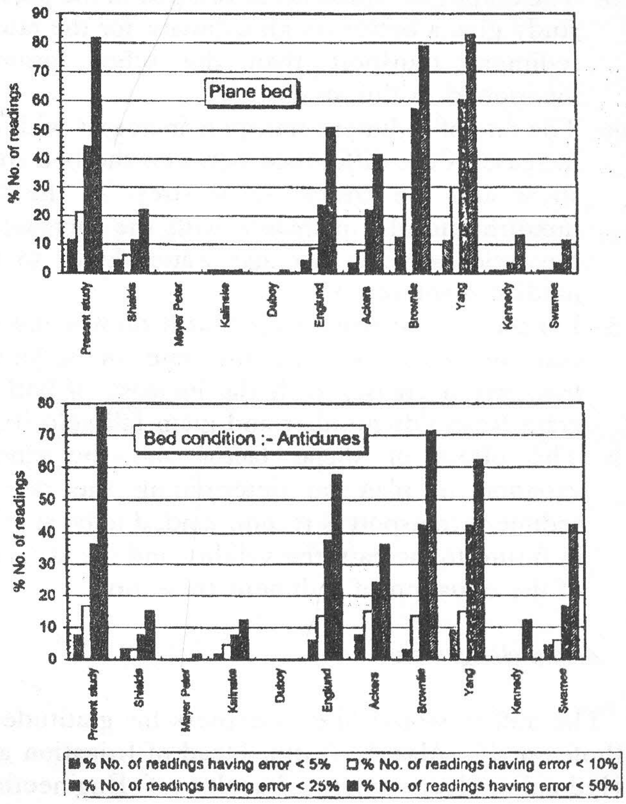

Figure 12. Comparison between the values of the % number of records having average errors less than 5, 10, 25 and 50% obtained using the equations developed in this study and other formulas for bed conditions iv-plane bed and v-antidunes.

CONCLUSIONS

The selection of the appropriate formula to determine the rate of sediment transport is very difficult because each sediment formula is based on a certain range of hydraulic condition and sediment properties. The analyses of the manner at which sediments are transported yields to the following conclusions:

1- The type of bed undulations plays an important roll for determining the rate of sediment transport.

2- For the range of collected data illustrated in table I, the total rate of sediment transport can be estimated using equations 5, 6, 7, 8 and 9 for rippled, dunned, transition, plane and antiduned bed, respectively.

3- The empirical equations developed in the present study give a better rough estimate for the rate of sediment transport than the other formulas mentioned in this study.
4- The rate of sediment transport increases with the increase of the difference between the bed shear stress and the critical shear stress of the bed mixture and also increases with the increase of the ratio between average water depth to the median diameter.
5- For the case of bed covered with ripples and the case of transition bed the rate of sediment transport decreases with the increase of both of grain Reynolds number and grain fall velocity.
6- The effect of shape factor and the stream variation in plan on determining the rate of sediment transport is recommended to be studied in future to increase the validity and the accuracy of the equation of sediment transport.

Acknowledgement

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REFERENCES

[1] P. Ackers and W.R. White, "Sediment Transport New Approach and Analysis", *Journal of Hydraulic Engineering, ASCE,* Vol. 99, No. 11, November 1973.
[2] W.R. Brownlie, "Compilation of Alluvial Channel Data: Laboratory and Field", *Report No. KH-R-43B,* W. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, California, 1981.
[3] W.R. Brownlie, "Prediction of Flow depth and Sediment Discharge", *Report No. KH-R-43A,* W. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, California, 1981.
[4] M.A. Carson and G.A. Griffths, "Influence of Channel Width on Bed Load Transport Capacity", *Journal of Hydraulic Engineering, ASCE,* Vol. 117, No. 12 December 1987.
[5] R.P. Ceballos, "Bed Load Coefficients", *Journal of Hydraulic Engineering, ASCE,* Vol. 118, No. 10 October 1992.
[6] P. Du-Boys, "Mechanic of Sediment Transport and Alluvial Stream Problems", *Willey Eastern Limited, ISBN 0 85226301* page No. 132 1977.
[7] F. Enguland and E. Hansen, "Monograph of sediment Transport in Alluvial Streams", *Teknish Forlag, Technical Press,* Copenhagen, Denmark, 62 pp, 1967.
[8] A.A. Kalinske, "Movement of Sediment as Bed Load in Rivers", *Trans. Geophy. Union, Vol. 28, No. 4,* August 1947
[9] F. Karim and J.F. Kennedy, "Menu of Coupled Velocity and Sediment Discharge Relations for Rivers", *Journal of Hydraulic Engineering, ASCE,* Vol. 116, No. 8 August 1989.
[10] Mayer-Peter and Muller, "Formulas for Bed Load Transport", *IAHR, 2nd Congress, Stockholm,* 1948.
[11] T. Nakato, "Tests of Selected Sediment Transport Formulas", *Journal of Hydraulic Engineering, ASCE,* Vol. 116, No. 3 March 1990.
[12] M.A. Osman and Thorne, "River bank stability analysis II: Application", *Journal of Hydraulic Engineering, ASCE,* Vol. 114, No. 2 February 1988.
[13] G. Parker, Colemon, "Simple Model of Sediment Laden flows", *Journal of Hydraulic Engineering, ASCE,* Vol. 112, No. 5 May 1986.
[14] W.W. Rubey, "Settling Velocities of Gravels, Sands and Silts particles", *Journal of Hydraulic Engineering, ASCE,* Vol. 25, No. 4 April 1933.
[15] A. Shields, "Anvendung der Aehnlichkeits-mechanik und der Turbulenzefuerschung auf die Geschiebebewegung", *Mitteilung der Preussichen Versuchsanstalt fuer Wasserbau und Schiffbau,* Heft 26, Berlin, Germany.
[16] G.M. Smart, "Sediment Transport Formula for Steep Channels", *Journal of Hydraulic Engineering, ASCE,* Vol. 110, No. 3 March 1984.
[17] P.K. Swamee and C.S. Ojha, "Bed Load and Susprnded Load Transport of Nonuniform Sediments", *Journal of Hydraulic Engineering, ASCE,* Vol. 117, No. 6 June 1991.
[18] T.C. Yang and A. Molinas, "Sediment Transport and Unit Stream Power Function", *Journal of Hydraulic Engineering, ASCE,* Vol. 108, No. 6 June 1982.

APPENDIX I

Du-Boy's formula [6]

$$q_b = C_d \tau(\tau - \tau_c) \tag{9}$$

Where $C_d = \dfrac{0.17}{d^{3/4}}$ (m^3/Kg/sec), q_b is the bed load in kg/sec/m and τ and τ_c are the average and the critical shear stress of the bed mixture in kg/m^2

Shields formula [15]

$$\frac{q_b(\gamma_s/\gamma - 1)}{\gamma_s q S} = 10\left[\frac{\tau - \tau_c}{(\gamma_s - \gamma)d}\right] \tag{10}$$

In which q is the unit discharge or the discharge per unit width of the channel.

Mayer - Peter - Muller formula [10]

$$\left[\frac{q_b(\gamma_s - \gamma)}{\gamma_s}\right]^{2/3}\left(\frac{\gamma}{g}\right)^{1/3}\frac{0.25}{(\gamma_s - \gamma)d} = \left(\frac{n_s}{n}\right)^{3/2}\frac{\gamma R S}{(\gamma_s - \gamma)d} - 0.047 \tag{11}$$

Where n is Manning's coefficient and $n_s = \dfrac{26}{d_{90}^{1/6}}$ where d_{90} in meters

Kalinske formula [8]

Kalinske used the experimental data to draw the relationship between the bed load and the ratio of the average shear stress to the critical shear stress of bed mixture. This relation is defined in the present study by applying the least square method on the points of his sediment load curve The relationship is expressed as follows:

$$\frac{q_b}{\gamma_s d u_*} = f_2\left(\frac{\tau}{\tau_c}\right) \tag{12}$$

$$= -0.075\left(\frac{\tau}{\tau_c}\right)^6 + 0.1878\left(\frac{\tau}{\tau_c}\right)^5 + 0.3236\left(\frac{\tau}{\tau_c}\right)^4 - 2.84\left(\frac{\tau}{\tau_c}\right)^3 + 5.848\left(\frac{\tau}{\tau_c}\right)^2 - 5.569\left(\frac{\tau}{\tau_c}\right) + 2.33$$

Engelund - Hansen formula [7]

$$C_s = 0.05\left(\frac{\gamma_s}{\gamma_s - 1}\right)\frac{VS}{[([\gamma_s - 1)gd]^{1/2}}\frac{RS}{(\gamma_s - 1)d} \tag{13}$$

Where C_s is the sediment concentration by weight ($C_s = q_s B/Q$)

Ackers - White formula [1]

$$C_s = c\frac{\gamma_s d}{R}\left(\frac{V}{u_*}\right)^n\left(\frac{F_g}{A} - 1\right)^m \tag{14}$$

Where F_g the mobility number which is defined as follows:

$$F_g = \frac{u_*}{gd(\gamma_s - 1)]^{1/2}}\left[\frac{V}{5.657\log\left(\frac{10R}{d}\right)}\right]^{1-n} \tag{15}$$

In which c, n A and m are coefficients depend on the values of the dimensionless grain diameter d_g which is equal to $d\left[\frac{g(\gamma_s - 1)}{\nu^3}\right]^{1/3}$

The values of the coefficients c, n, A and m are given in table III

Coefficient	$d_g > 60$	$60 \geq dg_{>1}$
c	0.025	$\log c = 2.86 \log d_g - (\log d_g)^2 - 3.53$
n	0	$1 - 0.56 \log d_g$
A	0.17	$0.23/(d_g)^{1/2} + 0.14$
m	1.5	$9.66/d_g + 1.34$

Parker et al. formula [13]

Parker developed a dimensionless parameter $\phi = \frac{\gamma Y S}{0.0875(\gamma_s - 1)d}$. For ϕ greater than 95 and less than 1.65 the rate of sediment transport is expressed as follows:

$$\frac{(\gamma_s - 1)q_b}{YSu_*} = 0.0025\exp\left[14.2(\phi - 1) - 9.28(\phi - 1)^2\right] \tag{16}$$

for ϕ greater than 1.65 the rate of sediment transport is expressed as follows:

$$\frac{(\gamma_s - 1)q_b}{YSu_*} = 11.2\left(1 - \frac{0.822}{\phi}\right)^{4.5} \tag{17}$$

Yang and Molinas formula [18]

$$\log C_s = 5.165 - 0.153\log\frac{\omega d}{\nu} - 0.297\log\frac{u_*}{\omega} + \left(1.78 - 0.36\log\frac{\omega d}{\nu} - 0.48\log\frac{u_*}{ome}ga\right)\log\left(\frac{VS}{\omega}\right) \tag{18}$$

Brownlie formula [3]

$$C_s = 9021.82\left[\frac{V}{\sqrt{\frac{\gamma_s - \gamma}{\gamma} g d}} - \frac{4.596 T_*^{0.5293}}{S^{0.1405}\sigma_g^{0.1606}}\right]^{1.978} S^{0.6601}\left(\frac{d_{50}}{R}\right)^{0.3301} \tag{19}$$

Karim and Kennedy formula [9]

$$\frac{q_s}{\sqrt{g(\gamma_s - 1)d^3}} = 0.00151\left[\frac{V}{\sqrt{g(\gamma_s - 1)d}}\right]^{3.369}\left[\frac{u_* - u_{*c}}{\sqrt{g(\gamma_s - 1)d}}\right]^{0.84} \tag{20}$$

Swamee and Ojha [17]

$$\frac{q_b}{d\sqrt{(\gamma_s - 1)d}} = \left[\left[\left(\frac{0.8}{M}\right)^{0.6} + M^{0.004}\right]^{1.75}\left(\frac{0.871}{T_*^{M^{0.25}}}\right)^{9} + \left[\left(\frac{0.01}{M}\right)^{0.6} + M^{3.25}\right]^{1.2}\left[\frac{0.339}{T_*^{9M/(8M^2+1)}}\right]^{1.6}\right]^{-1}$$

$$\frac{q_{ss}}{d\sqrt{(\gamma_s - 1)gd}} = \left[\left(\frac{0.073}{M}\right)^{4} + M^{3.8}\right]\left(\frac{0.567}{T_*^{M^{0.635}}}\right)^{6} + \left[\left(\frac{0.177}{M}\right)^{2} + M^{1.45}\right]^{2}\left[\frac{0.538}{T_*^{8M/(7M^{1.425}+1)}}\right]^{3}$$

Where M is Karmer uniformity coefficient and can be related to the geometric standard deviation as follows;

$$M = [0.1769 \ln \sigma_g + 1]^{-10}$$