SENSITIVITY ANALYSIS FOR 2D AND 3D MODELS FOR SALT WATER INTRUSION PROBLEMS

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ABSTRACT

The selection of either a two dimensional model or a three dimensional model to solve salt water intrusion problems will greatly depend upon their performance in terms of complexity and the accuracy required for the application. To obtain an insight into the problem of parameter identification for the models, it is desirable to perform a sensitivity analysis of the dependence of the model output on the accuracy of the model parameters. A bench mark problem is chosen to study the effects of changing the parameters on the potential and salt concentration values. The main aim of this research is to analyse and compare the simulation results of 2D and 3D models by performing a sensitivity analysis. This will include a comparison of the effect of perturbation of the parameters in the two models.

Keywords: Environmental aspects of salt water Intrusion, Pollution of ground water, Numerical modelling, Sensitivity analysis.

INTRODUCTION

The choice of parameters is among the most important outcomes of parameter identification (sensitivity analysis). The problem of parameter identification is not widely reported in salt water intrusion studies. A sensitivity analysis may illuminate the relative importance of parameters through the tendency of solutions to be affected by a certain change in a particular parameter. The speed of convergence of the calibration process depends greatly upon the sensitivity of the solution to changes following a change in the chosen system parameters. If the solution is not much affected by a change in the system parameters then the process of calibration becomes irrelevant. On the other hand, the solution may be too sensitive to the changes in a parameter. The effect of change of the system parameters on the calibration process in solving the problem using a two dimensional or a three dimensional model will be investigated in this research.

Most of the available research in literature concerns with flow parameters. Mandle and Kontis [1] studied the directions and rates of ground water movement in the vicinity of reservoirs. They concluded that combining low estimates of hydraulic conductivites with low values of leakance would define one end of a range of possible conditions while using high estimates of hydraulic conductivties and leakance would define the other end. Cooley et al [2] investigated non-linear- regression groundwater flow modelling of a deep regional aquifer system. They produced sensitivity maps to illustrate areas of the model most sensitive to change in a given parameter value. Soliman et al [3] investigated the western zone of the Nile Delta using areal two dimensional model. They concluded that the model dispersivity is many hundred times the theoretical and laboratory values. They also reported that the longitudinal dispersivity is about 100 km, while the lateral dispersivity is about 10 km. These values are in contradiction with values reported by Sherif [4] which are 100 m for longitudinal dispersivity and 10 m for lateral dispersivity. There is a three orders of magnitude difference between the two researches. Roberts et al [5] simulated different stages of the construction of a submerged tunnel in Kent area (UK) using both the two dimensional and three dimensional models. They found that there is a huge difference between the dispersivity values and leakage coefficient which are used in both models.

Objective Functions As Error Estimator:

The choice of objective function is among the most important factors for parameter identification. This function can be given in general form such as:

$$E = f\,(x) \tag{1}$$

in which E is the scalar quantity, f (x) is the function which is called the objective function and x is a vector which may have one or more variables. These variables may be the system parameters to be identified or dependent upon these parameters.

For salt water intrusion, with a fully mixed zone approach, different forms of mean objective functions may be written for instance[6] :

$$MOF^{\phi} = \frac{1}{N}\sqrt{\sum_{j=1}^{N}(\phi_o(j) - \phi_c(j))^2} \tag{2}$$

$$MOF^{c} = \frac{1}{N}\sqrt{\sum_{j=1}^{N}(c_o(j) - c_c(j))^2} \tag{3}$$

$$MOF = \frac{1}{N}\sqrt{\sum_{j=1}^{N}(\phi_o(j) - \phi_o(j))^2} + \frac{1}{N}\sqrt{\sum_{j=1}^{N}(c_o(j) - c_c(j))^2} \tag{4}$$

where N is the number of the samples, f is the potential head, c is the salt concentration and o, c are subscripts denoting the observed and calculated values.

The Selected Problem

A selected bench-mark problem is considered and solved for specific data. The values of leakage factor and dispersivity are changed by some known percentage and the effect of these changes on potential and salt concentration is studied.

The selected problem concerns three dimensional flow with the convection and dispersion of a solute species through a coastal confined aquifer with leakage through the river bottom. There is also a hydrological stress applied by a well in the aquifer. In practice, the situation may correspond to that involving the migration of a conservative contaminant from a sea boundary into a confined aquifer in which there is a steady uniform fresh water flux and an extensive pumping occurs. The aquifer region as well as the boundary conditions associated with the flow and transport equations are shown schematically in Figure (1). Used values of flow and transport parameters in the simulation are also given in Figure (1).

The simulated domain of interest extends from 2000 m inland from the shoreline, and 1500 m parallel to the sea, with a uniform depth of 100 m. The used finite element grid in the simulation of this aquifer consists of 2541 nodes (11 slices) and 2000 elements. Every slice represents a vertical cross-section through the aquifer. The aquifer is assumed to be homogeneous with steady flow.

The procedure of the Analysis

The procedure of the analysis is explained as follows:

1- The three dimensional finite element model (SWICHA)[7] has been run for the values of the parameters which are shown in Figure (1). The values of potential $\phi_i^0(x_i,y_i,z_i)$ and salt concentration $c_i^0(x_i,y_i,z_i)$ are obtained at each node.

2- The longitudinal dispersivity (α_L) and the leakage coefficient (K_z'/b') are perturbed by a known percentage to get new values of the potential $\phi_i^1(x_i,y_i,z_i)$ and the salt concentration $c_i^1(x_i,y_i,z_i)$ at each node. The mean objective function (average deviation summed over the nodes) can be obtained by applying the equations below:

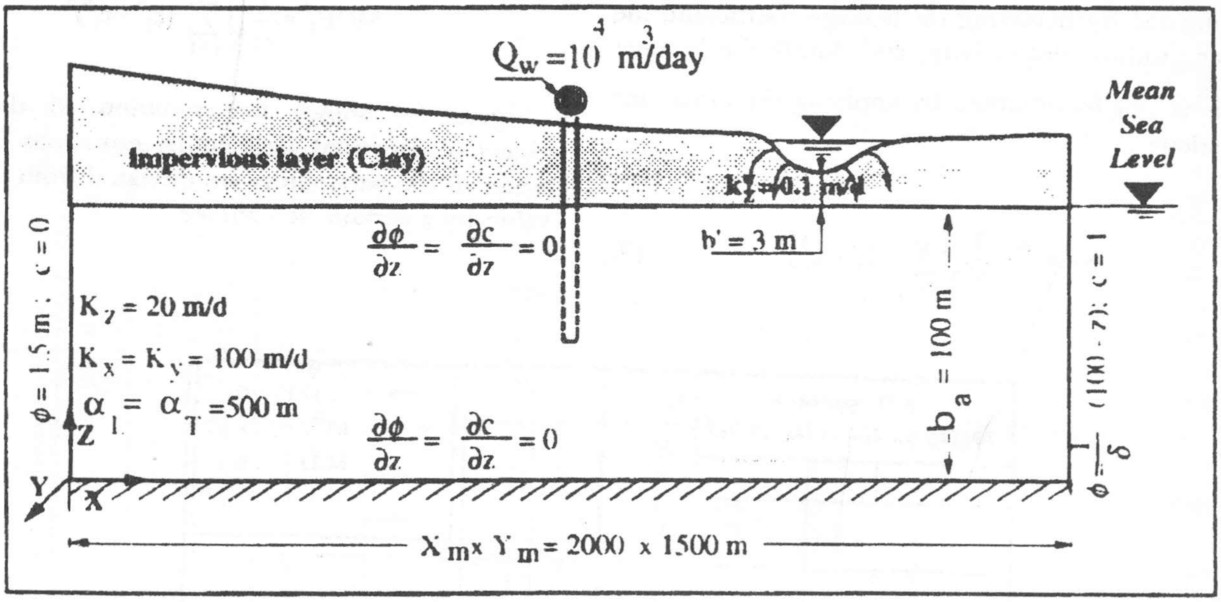

Figure 1. Physical model parameters used in the analysis.

$$MOF_1^{\phi} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(\phi_i^o - \phi_i^1)^2} \tag{5}$$

$$MOF_1^{c} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(c_i^o - c_i^1)^2} \tag{6}$$

where N is the number of samples which equals the number of the nodes in this case, superscript, o, denotes the original values while superscript, 1, denotes new calculated values after changing the parameters of concern. Figure (2) shows the variation of the mean objective functions defined by equations 5 and 6 when dispersivity and vertical leakage parameters are disturbed from their original values by a certain percentage, in the case of a three dimensional model which takes into account density effects.

3- The objective function is evaluated again by the consideration of a two dimensional areal model (SUTRA)[8], assuming that the original 3-D values (from step 1) averaged along the vertical z-direction $[(\phi_{avi}^o(x_i,y_i),(x_i,y_i)]$, are the correct values. The well discharge is distributed along the total depth so that the well is approximated by fully penetrating well. The mean objective functions are computed as before, while each parameter is disturbed from its original value by a certain percentage. The mean objective functions can be calculated in this case using these equations:

$$MOF_2^{\phi} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(\phi_{avi}^o - \phi_i^2)^2} \tag{7}$$

$$MOF_2^{c} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(c_{avi}^o - c_i^2)^2} \tag{8}$$

$\phi_i^2(x_i,y_i)$ and $c_i^2(x_i,y_i)$ obtained by running the programme in the case of the two dimensional model with the same used parameters in the case of the three dimensional model. The values of MOF_2^{ϕ} and MOF_2^{c} (equations 7 and 8) are plotted in Figure (3).

4- In a further study, trials have been done to minimise the mean objective functions for the potential and the salt concentration. These are calculated as the difference between solving the problem as a two dimensional approximation $\{\phi_i^3(x_i,y_i) \& c_i^3(x_i,y_i)\}$, assuming there is no variation in z-direction, and solving it as a three dimensional problem $\{\phi_i^o(x_i,y_i,z_i) \& c_i^o(x_i,y_i,z_i)\}$.

Trials are done to decrease the mean objective function by increasing the leakage coefficient, the longitudinal dispersivity, and thirdly the leakage $\phi^{3_1}(x_i, y_i)$ be obtained by applying the equations below:

$$MOF_3^{\phi} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(\phi_i^o - \phi_i^3)^2} \tag{9}$$

$$MOF_3^{c} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(c_i^o - c_i^3)^2} \tag{10}$$

Figure (4) shows the variation of the mean objective functions defined by equations 9 and 10 when all the parameters are deviated from their true value by a certain percentage.

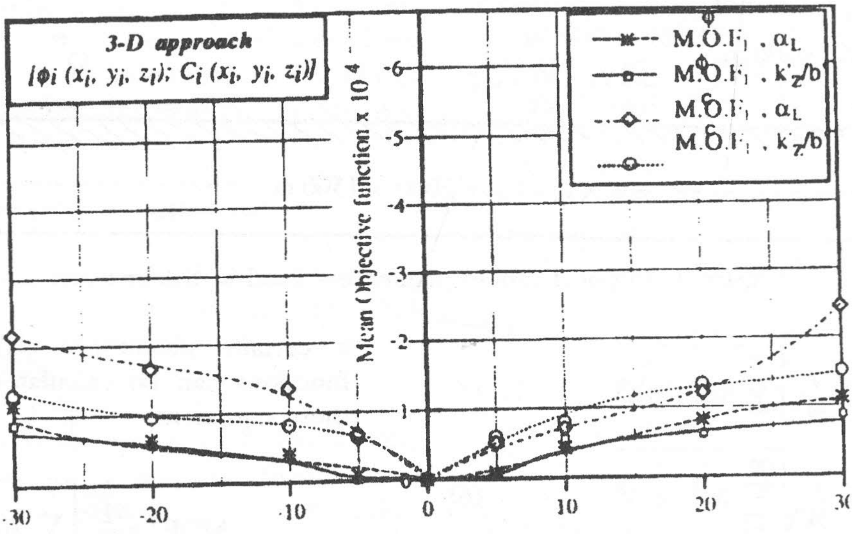

Figure 2. Effect of change parameters on the mean objective function in case of use 3-D model.

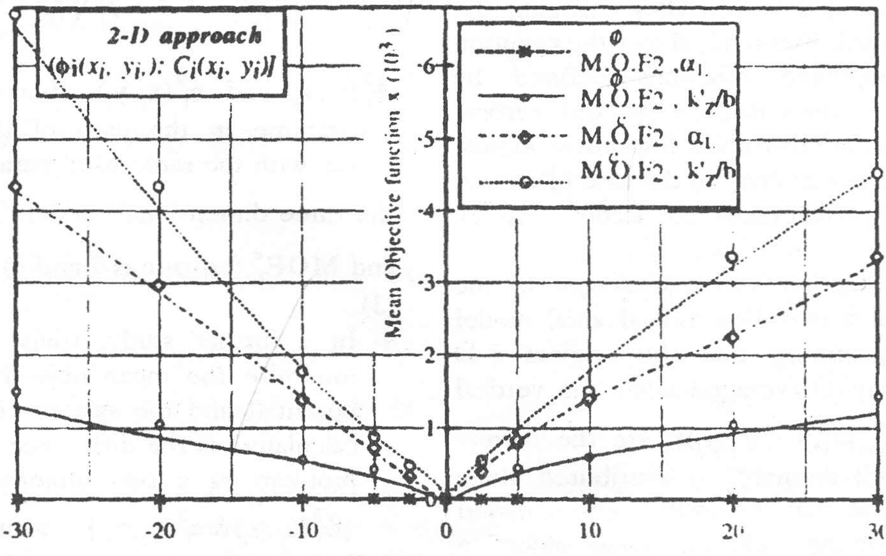

Figure 3. Effect of change parameters on the mean objective function in case of use 3-D model.

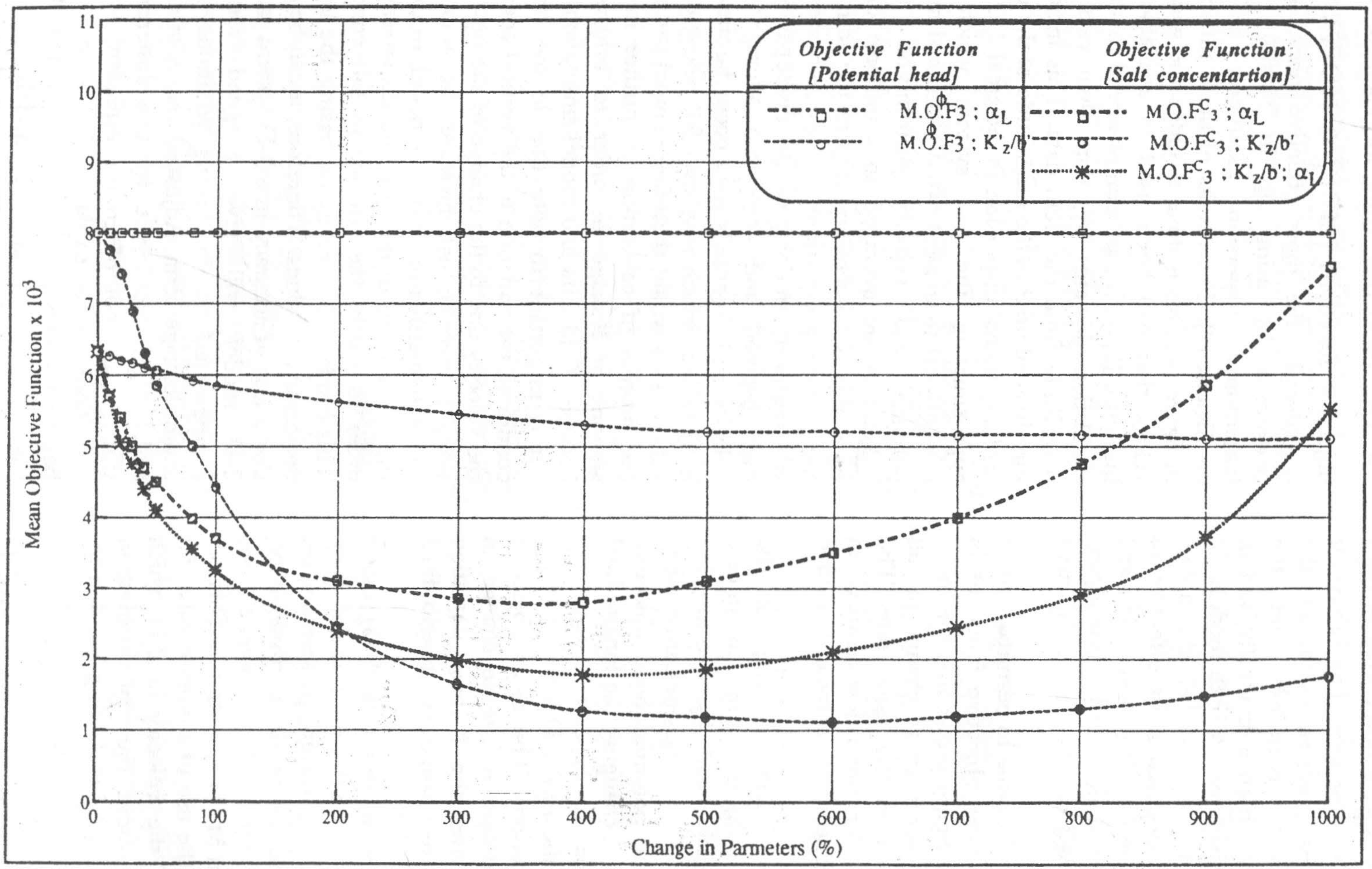

Fig. 4 Effect Of Increasing Leakage Coefficient Value and Longitudinal Dispersivity On Main Objective Function In Areal 2-D Model

Discussion of The Results

It is important to examine the performance of the 2-D and 3-D models through a comparison of their respective mean objective functions. The behaviour of the objective functions gives an insight into the effects of the leakage coefficient and the longitudinal dispersivity, both individually and in combination, on the sensitivity of both models. In carrying out the simulation work of the above selected problem, it is deemed most effective to determine the parameters for salt concentration and potential head distributions in order to investigate the separate and combined effects on the objective function.

Figure (2) shows the effect due to perturbations of the parameters on the mean objective functions in case of 3-D model. The figure shows that the mean objective functions are sensitive to change in all the parameters, but not in the same rate. The potential mean objective function is less sensitive, in general, than the salt concentration mean objective function.

Figure (3) shows the effect of change in the parameters on the mean objective functions in case of 2-D model. The potential head objective function is still less sensitive than the salt concentration mean objective function.. The potential head objective function is invariant with changing the longitudinal dispersivity (α_L). This is expected since the governing equations of the areal 2-D model does not account for α_L perturbations. The mean objective function for salt concentration due to change in leakage coefficient shows the highest deviation followed by that due to change in longitudinal disperstivity.

A comparison between figures (2)and (3) shows that the results in 2-D model appear to be more sensitive to perturbation in governing parameters, i.e. leakage coefficient and longitudinal dispersivity. The values of $M.O.F_1$, figure (2) are lower by one order of magnitude than $M.O.F_2$, figure(3). This can be partly attributed to the use of a unique value for vertical component of the velocity in 2-D which does not precisely represent the real distribution observed in the 3-D model. Neglecting the density effect on the potential head, and thereby on the distribution of the vertical velocity, which in turn is related to the advection of salt, is another possible source of the difference. Higher salt convectivity requires a further increase in longitudinal disperstivity to balance the concentration of salt in the region of study.

A few simulation runs for each of the mean objective functions (MOF_3^{ϕ} & MOF_3^{c}), which are considered the most effective ones, have been performed by using up to 1000% change in longitudinal dispersion parameter and leakage coefficient [9]. The results of these runs are shown in figure (4).This is done to judge the wide range of values that had been used in available previous literature [3,4] . It has been observed that the model simulation deviates by more than two orders of magnitude from the field data. This incidentally is mainly attributed to the magnitude of the used value for longitudinal dispersion in the areal 2-D model. In approximating the real system which is three dimensional to an areal two dimensional model, both potential and velocity distribution along the z-direction are averaged to a uniform distribution since this is a necessary step in the formulation of the governing equations (i.e. groundwater flow and salt transport). In doing so, the discrepancy between the observed and computed average values of velocity and potentials may occur because of some local values which may well fall below the average values. This would therefore compel the modeler in the process of calibration to further increase the amount of leakage in order to bridge the gap between field data and model simulation.

It is important to note that in the steady state condition, the variation of the vertical component of the velocity due to the change of the potential head along z-direction will increase the amount of the convective afflux in areal 2-D model. In this respect, there is a need to increase the dispersive afflux in order to satisfy the balance in salt concentration. This can be achieved by increasing the longitudinal dispersitivity values. Therefore, it can be concluded that a loss of accuracy in a 3-D system tackled with 2-D model will give a good result when compensated for by using additional dispersion and/or leakage term. Adjusted values of dispersivity and leakage coefficient, for two dimensional areal study, will not of course represent actual field conditions. This explains the wide discrepancy between values of the parameters that have been used with the 2-D and 3-D models.

Finally, it is interesting to note that in both 2-D and 3-D models, molecular diffusion has virtually no effect on the results. Therefore, it can be concluded that in the calibration process the molecular diffusion effect can be neglected for all practical purposes. The errors in moving from 3-D to 2-D is shown at the zero point on the x-axis. As the parameters change however it can be seen that the 2-D model may be forced to match the 3-D model more closely, particularly in the case of concentrations.

CONCLUSION

A parametric sensitivity analysis has been carried out for both SUTRA(2-D model) and SWICHA(3-D model) to examine the extent to which three dimensional effects may be accommodated in two dimensional effects by varying the model parameters. The following conclusions are derived:

1- The values of groundwater flow and salt transport parameters are strongly dependent upon the reliability of the model parameters. The three dimensional effects can be compensated for in two dimensional model by changing both model parameters, namely leakage coefficient (K_z'/b) and the longitudinal dispersivity (α_L). However, these fictitious parameters' values may be misleading in evaluating the field data.

2- The molecular diffusion does not have a significant effect on the objective function of the potential or the salt concentration.

3- In the three dimensional model, the longitudinal dispersivity is the main parameter of salt transport. It has an implicit effect on the potential objective function. This effect is due to the coupling of the flow and salt transport equations.

4- The rate of change of the objective function due to the change in the parameters in the two dimensional model is higher than that in the three dimensional model.

5- An objective function based on the sum of the squares of errors in the potential and the salt concentration is appropriate for the identification of salt water intrusion parameters in fully mixed zone models. This objective function is sensitive to the ground water flow parameters (K_x, K_y) more than salt transport parameters (α_L, α_T).

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