SEEPAGE SURFACE IN EARTH DAMS

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ABSTRACT

Because of its importance to the stability of earth dams, determination of the location of seepage surface developed on the downstream toe of the dam has received a great deal of attention. A two-dimensional finite element model has been developed in order to determine the height of seepage surface in homogeneous isotropic earth dams. Various values of stable side slopes of the dam section, bed width and downstream water depths have been investigated. The results are presented in dimensionless form. They agree well with the Pavlovsky equation, while noticeable discrepancies are observed with the application of Schaffernack and Iterson equation. The present results are useful to determine directly the height of seepage surface for any dam section under different boundary conditions.

Keywords: Finite elements, Earth dams, Seepage.

Notation

$A^{(e)}$	area of element e
B	dam bed width
D	horizontal distance between downstream toe and point at which water surface intersects the upstream face of the dam.
h	total head $= \frac{P}{\gamma} + z$
{h}	global head vector
h_1	upstream water depth
h_s	height of the exit point above the dam base
h_w	downstream water depth
h^e	nodal head vector
$\hat{h}^{(e)}$	approximate solution for hydraulic head within element e
H	dam height = maximum expected upstream water depth
$k^{(e)}$	element stiffness matrix
k	permeability coefficient
[K]	global stiffness matrix
m	height of seepage surface above the downstream water level
N_c	number of columns in the mesh.
N_R	number of rows in the mesh.
$N_i^{(e)}$	interpolation function for each node within element e
p	pressure head
q	quantity of seepage passed through the dam section
S	width of the middle section of dam
W	dam crest width
z	elevation head
α	downstream side slope
β	upstream side slope
γ	specific weight of water

INTRODUCTION

Earth dams for storage of water for irrigation have been built since earliest times. They are now being built to unprecedented heights. Development of soil mechanics, study of behaviour of earth dams, and the development of better construction techniques have all been helpful in creating confidence to build higher dams with improved designs and more ingenious details. Seepage of water through the embankment has been responsible for more than one third of earth dam failures [4]. Uncontrolled seepage may, however, cause erosion within embankment which may cause piping failure. The location of the free surface within an earth dam is necessary to assess the soil properties in order to test the stability of earth dam section. Referring to figure (1), the free surface meets the downstream face of a homogenous earth dam tangentially, if no filter arrangements are provided, and creats a seepage face of height m above the downstream water level. Accurate determination of seepage face will enable to draw accurately the free surface. As a result, it is easy to draw the flow net for the dam body to determine quantity of seepage passed through the embankment. Casagrand [8] proposed a graphical solution to determine the position of the central portion of the free surface. However, he left the exact position of the seepage surface to be estimated. Moreover, his solution was only considered for dry downstream water level (h_w=0.0). A number of attempts haven been made to determine the height of the seepage surface analytically. They were based on the subdivision of dam body into several sections and the flow through each section equals the flow through all other sections. Mikhailov [1] and Pavlovsky [1] introduced similar sets of equations to determine the quantity of seepage passed through the dam and the corresponding seepage surface height. These equations take the following form:

$$q = k \frac{h_1 - h}{\cot\beta} \ln \frac{H}{H-h} \quad (1)$$

$$q = \frac{k}{2S}[h^2 - (m + h_w)^2] \quad (2)$$

$$q = k \frac{m}{\cot\alpha}(1 + \ln \frac{m + h_w}{m}) \quad (3)$$

$$S = W + \cot\alpha \, [H^2 - (m+h_w)] \quad (4)$$

where

q = seepage rate passed through the dam section
k = permeability coefficient
h_1 = upstream water depth.
H = dam height
α, β= downstream and upstream side slopes.
h = height of free surface at the middle section of dam (see Figure (1).
S = width of middle section of dam
m = height of free surface above downstream water level
h_w = downstream water depth
W = crest width of dam.

These equations were derived based on Dupuit assumptions, where the flow is assumed to be horizontal everywhere in the dam body even in the region under the seepage surface. Although in the immediate vicinity of the downstream face of the dam, a strong curvature of the flow lines is noticeable. Schaffernak and Van Iterson [8] developed a similar solution, based on Dupuit assumptions, to determine the position of the seepage surface for only the case of dry downstream water level:

$$\frac{m}{\sin\alpha} = \frac{D}{\cos\alpha} - \sqrt{\frac{D^2}{\cos^2\alpha} - \frac{h_1}{\sin^2\alpha}} \quad (5)$$

in which D is the horizontal distance between the downstream toe and point b, where water surface intersects the upstream face of the dam and other symbols are defined earlier. It should be noted that this solution was based on an assumption that the portion of the dam to the left of $\overline{bg}$ is neglected in the analysis.

Several experimental approaches have been made to investigate the problem using electrical analogue [5], sand models [12] and viscous flow models [12]. However, the high cost of construction, the lack of accurate measurements due to capillary effects, local variation in permeability and the sensitivity to the temperature change put some restrictions to the use of such models in the study of unconfined flow problems. Many numerical attempts have been made to locate the position of the free surface in earth dams. Rushton [10] used the finite difference method and Liggett [7] used the boundary element method. Zienkiewicz [13], Toylor et al [11], France [3], Kazda [6], Neuman [9], and Finn [2] used the finite element method and they showed the flexibility and accuracy of the method to handle the unconfined seepage problems. However, these studies did not obtain expressions or relationships associated with the determination of the seepage surface height.

The main objective of this study is to determine numerically the height of the seepage surface in homogeneous earth dams, using the finite elements method. Various values of stable side slopes of the dam section, bed widths and downstream water depths are investigated.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Considering two-dimensional steady seepage flow through an earth dam (Figure (1)), the following assumptions are introduced in the study:

1. a homogenous and isotropic type of earth dam is assumed.
2. the compressibility of the water complex is neglected.
3. the flow through the unsaturated portion of the porous media is negligible and effects of capillarity is, however, neglected.

The associated governing differential equation for the flow in the steady state condition takes the form:

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\partial h}{\partial z}\right)=0 \qquad (6)$$

where

h total head $= \frac{p}{\gamma} + z$

p pressure head

γ specific weight of water

z elevation head

Equation (6) has an infinite number of solution and a unique solution can only be obtained when the boundary conditions of the flow domain are specified. The boundary conditions associated with seepage through the earth dam (see Figure (1)) are as follows:

$$\frac{\partial h}{\partial n}=0 \quad (\text{surface } \overline{ae}) \qquad (7)$$

$$h = h_1 \quad (\text{surface } \overline{ab}) \qquad (8)$$

$$h = h_w \quad (\text{surface } \overline{de}) \qquad (9)$$

$$\left.\begin{array}{l} h = z \\ \frac{\partial h}{\partial n}=0 \end{array}\right\} (\text{surface } \overline{bc}) \qquad (10)$$

$$h = z (\text{surface } \overline{dc}) \qquad (11)$$

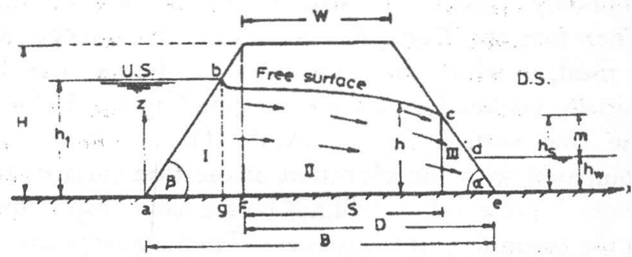

Figure 1. Seepage through a Homogenous eart dam.

Applying the finite elements technique to Eq. (6 and using the Galerkin's approach [13], the integra formulation for equation (6) takes the form:

$$\int\int_{A^{(e)}} N_i^{(e)}\left[k^e \frac{\partial^2 \hat{h}^{(e)}}{\partial x^2} + k^{(e)}\frac{\partial^2 \hat{h}^{(e)}}{\partial z^2}\right] dx\, dz = 0 \qquad (12$$

where:

$N_i^{(e)}$ interpolation function for each node withi element e

$\hat{h}^{(e)}$ approximate solution for hydraulic head withi element e

$A^{(e)}$ area of element e

Equation (12) can be written in a simple matri form;

$$[k^{(e)}]\,\{h^{(e)}\} = \{0\} \qquad (13$$

where

$k^{(e)}$ element stiffness matrix

$h^{(e)}$ nodal head vector

Equation (13) is written for each element in th mesh and combined to obtain a system of linea equations of the form

$$[K]\,\{h\} = \{0\} \qquad (14$$

where

[K] global stiffness matrix
{h} global head vector

Equation (14) can be solved by using the associated boundary conditions and then, values of unknown heads at nodes can be obtained. According to Eq. (10), the free surface is a boundary with two boundary conditions with an unknown location. Therefore, the Taylor Brown [11] iterative procedure is used, in which the location of the free surface is initially assumed and the corresponding head along the free surface is calculated. These values are compared with the elevation of the free surface. In general, these values will not be the same, especially at the beginning of the process. Then a new position of the free surface is assumed, for instance, by taking Z = h. This procedure is repeated until the assumed elevation of the free surface equals to the calculated head along the free surface.

Finite Elements Model

A finite element computer program has been developed, to perform the previous calculations, in order to determine the position of the free surface. To generate the initial finite element mesh, the input data to be given to the program are; the width of the dam at its toe (B), the left and right side slopes (β) and (α), water depths in the left right sides (h_1) and (h_w) and numbers of columns (NC) and rows (NR) of elements. Several attempts have been made to determine number of elements and modes in the mesh. Reasonable values of NC and NR were chosen equal to 20. This makes the total number of elements and nodes equal 441 and 400 respectively. On the basis of these data the computer program generates a mesh, by estimating a free surface, and subdividing the domain into NC sections, using equidistant points along the impervious base of the dam. Each section has a triangular shape, with its top angle in the intersection point of the upstream and downstream side slopes, and the section is divided into NR elements as shown in Figure (2). The origin of the cartesian coordinates are chosen at the upstream toe of the dam.

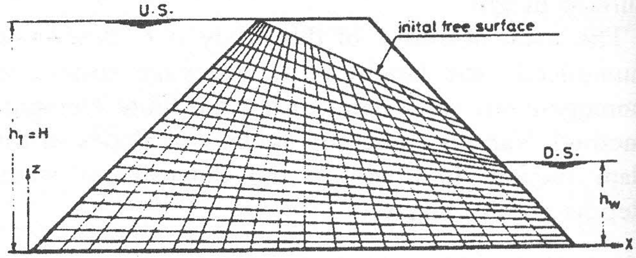

Figure 2. Finite element mesh.

Model Dimensions

(a) *Crest Width;* there are several considerations controlling the width of the crest such as nature of embankment material, height of dam and roadway requirements, ... etc. Gloze [4] recommended the following formula to determine the crest width:

$$W = \frac{5}{3}\sqrt{H} \tag{11}$$

where W = crest width
H = height of dam

Equation (11) will be used in the present study to design the dam cross section.

(b) *Side slopes;* for fixing tentative section of the earth dam the upstream and downstream side slopes may be taken from Table (I) given by Terzaghi [4], Moreover two more cases are also investigated. They are α = 45°, B = 33.7° (h:v = 1.1, 1.5:1) and α=33.7°, β = 2.656° (h:v = 1.5:1, 2:1) respectively.
(c) *Upstream and downstream water depths;* for more safety of the dam design the upstream water depth h_1 is assumed equal the dam height H. Four values of H are investigated. They are H = 10, 20, 30, and 40 m. For each value of H, five cases of downstream water depth h_w are considered (h_w/H = 0.0, 0.2, 0.4, 0.6 and 0.8).

RESULTS AND ANALYSIS

Using the developed finite element program, the steady position of the free surface is determined. Accordingly, height of exit point c above the dam base h_s can be determined for any dam section.

Table I. Recommended side slopes of earth dams (after Terzaghi [4]).

Type of material	U.S. slope h : v	D.S. slope h : v
-homogeneous well graded material and homogeneous silt clay or clay (height less than 15 m)	2.5:1 (β = 21.8°)	2:1 (α = 26.6°)
-homogeneous coarse silt and homogeneous silt clay or clay (height more than 15 m)	3:1 (β = 18.43°)	2.5:1 (α = 21.8°)

Table II: A Summary of the finite element model calculations.

H (m)	W (m)	h_w (m)	Case I (α=45°,β=33.7°)			Case II (α=33.7°, β=26.56°)			Case III (α=26.6, β=21.8°)			Case IV (α=21.8°, β=18.43)		
			B (m)	h_s (m)	m (m)	B (m)	h_s (m)	m (m)	B (m)	h_s (m)	m (m)	B (m)	h_s (m)	m (m)
		0		2.04	2.04		1.99	1.99		1.972	1.972		1.971	1.971
		2		3.4	1.40		3.277	1.277		3.18	1.18		3.14	1.14
		4	31	4.70	0.70	41	4.65	0.65	51	4.6	0.60	61	4.57	0.57
10	6	6		6.415	0.415		6.397	0.397		6.375	0.375		6.37	0.37
		8		8.342	0.342		8.39	0.33		8.31	0.31		8.305	0.305
		0		4.19	4.19		4.14	4.14		4.08	4.08		4.05	4.05
		4		6.94	2.94		6.78	2.78		6.74	2.14		6.64	2.64
20	8	8	58	9.6	1.60	78	9.58	1.56	98	9.44	1.44	118	9.36	1.36
		12		12.964	0.964		12.9	0.9		12.82	0.82		12.78	0.78
		16		16.85	0.85		16.804	0.804		16.74	0.74		16.69	0.69
		0		6.35	6.35		6.30	6.30		6.191	6.191		6.15	6.15
		6		10.59	4.59		10.0	4.26		10.14	4.14		10.0	4.0
30	10	12	85	14.58	2.58	115	14.46	246	145	14.34	2.34	175	14.25	2.25
		18		19.55	1.55		19.47	1.47		19.38	1.38		19.32	1.32
		24		25.32	1.32		25.317	1.317		25.3	1.30		25.29	1.29
		0		8.54	8.54		8.48	8.48		8.34	8.34		8.26	8.26
		8		14.35	6.35		13.9	5.9		13.6	5.68		13.48	5.48
40	11	16	111	19.6	3.60	151	19.4	3.40	191	19.24	3.24	231	19.12	3.12
		24		26.2	2.20		26.08	2.08		25.92	1.92		25.8	1.80
		32		33.92	1.92		33.88	1.88		33.88	1.88		33.81	1.81

Height of seepage surface m above the downstream water level is, then, determined by subtracting the downstream water depth h_w from h_s. A summary of these calculations is listed in Table (II). Referring to this table and at specific values of H, W and h_w, it can be noticed that the decreases of upstream and downstream side sloped (β and α) cause an increase in the dam bed width B and a decrease in the seepage surface height. This is attributed to the fact that the decrease of β and α cause an increase in the dam section if the dam height and dam crest width are kept constant. Accordingly, the path of the free surface will be increased and more resistance to the seepage is occurred. As a result, less height of seepage surface is happened. Figures (3), (4), (5) and (6) show dimensionless relationship between dam bed width B, seepage surface height m, upstream and downstream water depths H and h_w for four cases of α and β. These figures are useful to determine directly the height of the seepage surface for any dam section under different boundary conditions. It can be noticed from these figures that, for certain values of B/H, the seepage surface height m increases rapidly as the decrease of h_w/H and reaches a maximum at h_w = 0.0 (case of dry downstream).

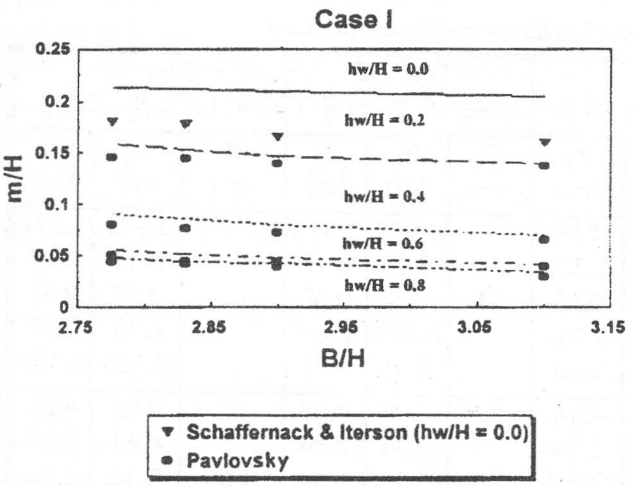

Figure 3. Height of the seepage surface (m) for α = 45° and β = 33.7°.

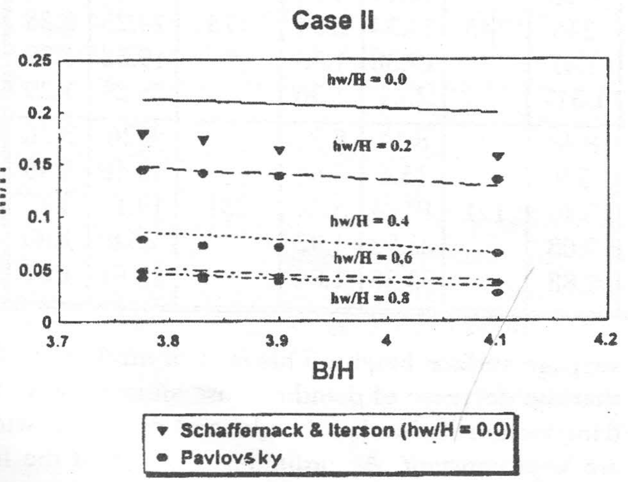

Figure 4. Height of the seepage surface (m) for α = 33.7° and β=26.56°.

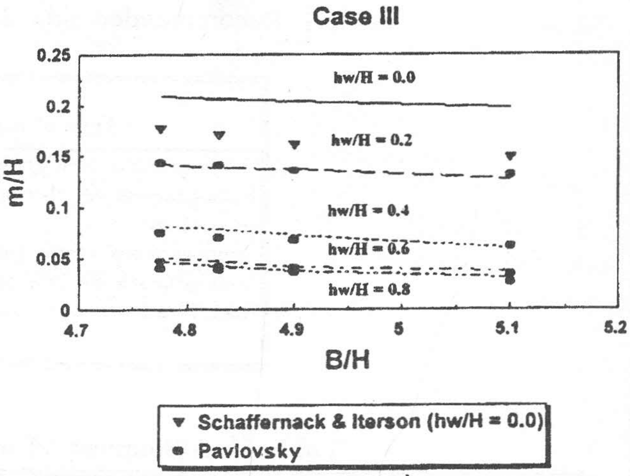

Figure 5. Height of the seepage surface (m) for α 26.6° and β =21.8°.

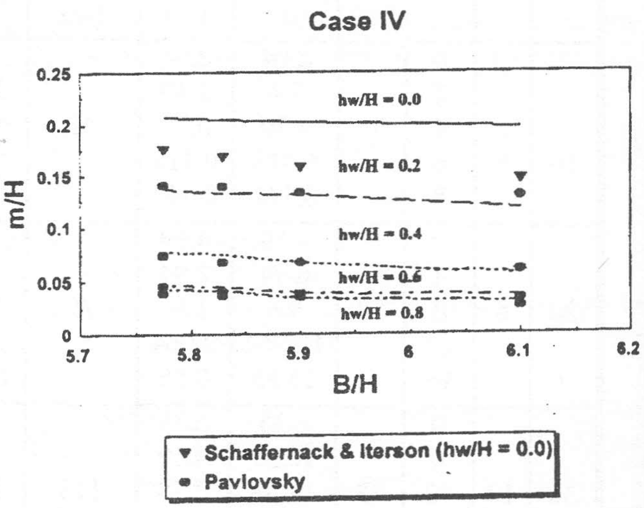

Figure 6. Height of the seepage surface (m) for α = 21.8° and β = 18.43°.

Comparisons between model results and the corresponding values of m, using the Schaffernach and Iterson equation (Eq. 5), are shown in Figures (3) to (6) for the case of h_w = 0.0. Noticeable differences are observed between both results and varied form 16% to 23%. This may be attributed to the fact that Eq. (5) was based on the Dupuit assumption. Moreover, the upstream portion of the dam body was neglected in derivation of Eq. (5) as introduced earlier.

Comparisons are also made between the finite element results and those obtained with the application of the Pavlovsky equations (Eqs. 1 to 4) and are shown in figures (3) to (6) for $h_w/H = 0.2$, 0.4, 0.6 and 0.8. Good agreement are noticed between both studies at high values of h_w/H and some differences are observed at $h_w/H = 0.2$ and 0.4. However, these differences do not exceed 5%

CONCLUSIONS

A finite element program has been developed to determine the height of the seepage surface, developed at the downstream side of the homogeneous earth dam. Various upstream and downstream side sloped and water depth are investigated. The results are presented in dimension-less forms and they agree very well with the Pavlovsky equation. Comparisons with the Schaffernack and Iterson equation showed noticeable discrepancies . The study showed that for specific dam height and crest width, the seepage surface height decreases as the decrease of the upstream and downstream side slopes and as the increase of both the downstream water depth and the dam bed width. The results will be useful for the designer of earth dams to creat more accurate design to the dam section.

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