

A GENERALIZED ALGORITHM FOR GENERATION OF ORTHOGONAL POLYNOMIALS FOR EQUAL AND UNEQUAL INTERVALS AND ITS USE FOR ANALYTICAL SPECTROMETRIC METHODS

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ABSTRACT

The use of orthogonal polynomials is popular in spectrometric data fitting and analysis. The use of these polynomials at equal and unequal intervals is considered to be very efficient in correcting background interference in different spectrometric techniques. Examples of these uses are; UV-Visible spectrophotometry, spectrofluorometry and spectropolarimetry. In this paper a generalized algorithm that generates orthogonal polynomials of any degree, any segment length, and for equal and unequal intervals is presented and applied for spectrometric data analysis. This algorithm could be a very useful tool for computerized analysis of spectrometric data taken for single component, two-component, and multicomponent systems.

Keywords: Orthogonal polynomial, Nystatin, Methylparaben, Spectrometric analysis.

INTRODUCTION

For the last three decades, orthogonal polynomials for equal intervals were used extensively in correcting background interference in spectrophotometric analyses [1,2]. In [3] a method reported for smoothing and differentiation of data by simplified least squares procedures. The smoothing procedure was based on convoluting orthogonal polynomials with the spectral data. The orthogonal polynomial method has been used for a wide range of applications in spectrophotometry. For example, in the analysis of some weakly absorbing compounds [4], two-component analysis [5], multicomponent analysis [6], and also for the analysis of compounds in the presence of their degradation products [7], and in the presence of other interfering constituents [8]. In [9] a linear least square approach has been programmed to analyze mixtures of drugs in the presence or absence of known background. Other modifications for the applications of orthogonal polynomials for equally spaced intervals have often been reported in literature. These include the delta p_j method [10], the combined polynomial method [11]. The orthogonal polynomial method has been

also extended to correct interferences in spectropolarimetric [12] and spectrofluorimetric analyses [13].

In 1986 Korany et al [14]. Introduced the use of orthogonal polynomials for unequal intervals to eliminate interferences in spectrophotometric analysis. This method is considered more versatile when absorption spectra have considerable overlap. Also, this method was applied for the analysis of single component in presence of other interfering component [15], and for two components having spectra with noticeable serious overlap [14].

Recently, an interactive computer program (CNVSP1) has been developed and presented in [16]. The main function of the CNVSP1 was to analyze spectrophotometric data for multicomponent system using orthogonal polynomials for equally spaced intervals.

The CNVSP1 enables the analyst to store, edit, display, and to process absorbance data. Examples of processing, are; convolution with orthogonal polynomials, solving linear simultaneous equations and applying the least squares principle on data. One

major application of the CNVSP1 was the analysis of complex multicomponent mixtures. However, one of its limitation was that the program user should enter the polynomial coefficients in its integer format manually from the tabulated figures of Fisher and Yates [17]. On the other hand orthogonal polynomials for unequal intervals were not supported by the CNVSP1 program.

The first attempt to use the orthogonal polynomials for unequal interval was presented in [18]. The orthogonal polynomials were calculated using desk calculator using the method of Grandage [18]. In this connection Whabi [19] reported a curve fitting program that facilitated the calculation of orthogonal function coefficients for equally and non-equally spaced points during spectrophotometric analysis. The program is a modified version of the BASIC program designed and developed for curve fitting process using least squares [20].

In this work a generalized algorithm has been developed and implemented to generate the orthogonal polynomials for equal and unequal intervals, any segment length, and for any polynomial degree. The generalized algorithm has been integrated with the CNVSP1 resulting in a generalized, versatile, user friendly program G_CNVSPC. The functional block diagram of the G_CNVSPC is illustrated in Figure (1).

Mathematical Background

The orthogonal polynomial of degree K will be

$$\phi_{kT}(t) = t^k + C_{k-1}(k,T) t^{k-1} + \dots + C_1(k,T)t + C_0(k,t)$$

$$k = 1,2,\dots, T-1,$$

and $\phi_{0T}(t) = 1$. For $\phi_{kT}(t)$ to be orthogonal to

$\phi_{0T}(t), \phi_{1T}(t), \dots, \phi_{k-1,T}$ or equivalently [21] to $1, t, \dots, t^{k-1}$ We must have

$$\sum_{t=1}^T \phi_{kT}(t) t^i = 0, i = 0,1,\dots,k-1;$$

that is,

$$C_0(k,t) \sum_{t=1}^T t^i + C_1(k,T) \sum_{t=1}^T t^{i+1} + \dots + C_{k-1}(k,T) \sum_{t=1}^T t^{i+k-1} = - \sum_{t=1}^T t^{i+k}$$

$$i = 0,1,\dots,k-1$$

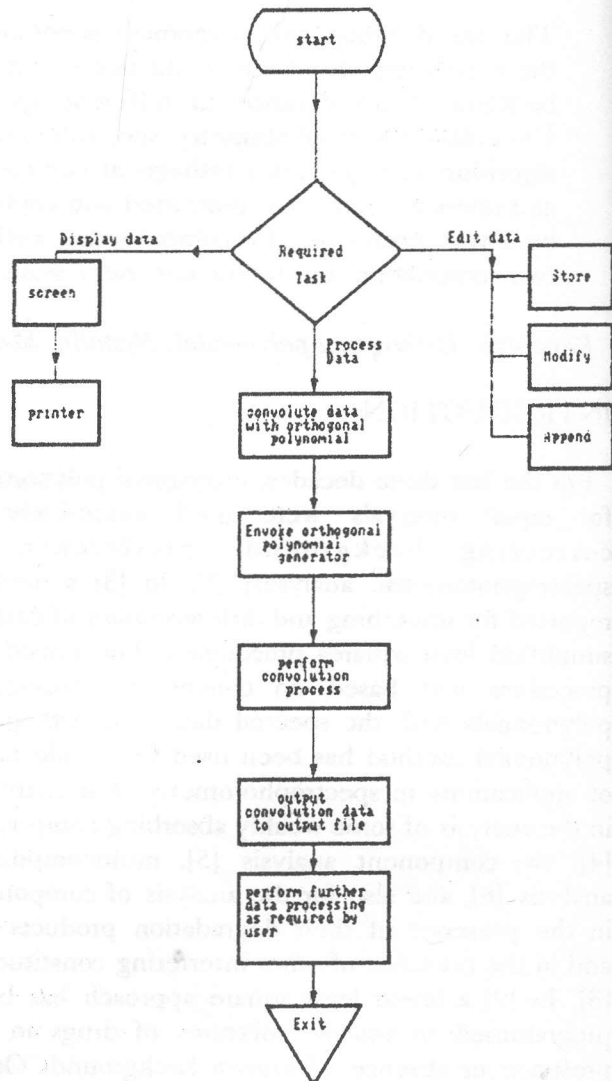


Figure 1. The function block diagram of G_CNVSPC.

Then the system of the following equations can be formed;

$$\begin{aligned}
 &C_0(k,t) \sum_{t=1}^T 1 + C_1(k,T) \sum_{t=1}^T t + \dots + C_{k-1}(k,T) \sum_{t=1}^T t^{k-1} = - \sum_{t=1}^T t^k \\
 &C_0(k,t) \sum_{t=1}^T t + C_1(k,T) \sum_{t=1}^T t^2 + \dots + C_{k-1}(k,T) \sum_{t=1}^T t^k = - \sum_{t=1}^T t^{k+1} \\
 &\vdots \\
 &C_0(k,t) \sum_{t=1}^T t^i + C_1(k,T) \sum_{t=1}^T t^{i+1} + \dots + C_{k-1}(k,T) \sum_{t=1}^T t^{i+k-1} = - \sum_{t=1}^T t^{k+i} \\
 &\vdots \\
 &C_0(k,t) \sum_{t=1}^T t^{k-1} + C_1(k,T) \sum_{t=1}^T t^k + \dots + C_{k-1}(k,T) \sum_{t=1}^T t^{2k-2} = - \sum_{t=1}^T t^{2k-2}
 \end{aligned}$$

Solving the above system of equations yields the k coefficients;

$C_0(k,T), C_1(k,T), \dots, C_{k-1}(k,T)$. It should be noted that in the case of generating the polynomial for unequal intervals the summation $\sum_{t=1}^T t^i$ is computed as $\sum_{v \in V} t^i$ where v is the set of points (wavelengths) of the data segment at unequal intervals.

The orthogonal polynomial generator flow chart is illustrated in Figure (2). The system of equations is solved by the Gauss elimination method. The greatest common divisor (GCD) program provides the facility to determine the GCD for the orthogonal polynomial coefficients to be displayed in the integer format, Thus providing compatibility with the standard tables of orthogonal polynomials of equal intervals. The polynomial coefficients are not always needed to be in the integer format. That is the polynomial coefficients are directly written to a disk file for further computer processing. For example for convolving the absorption data with an orthogonal polynomial, the orthogonal polynomial generator is invoked by the main program. The output of the orthogonal polynomial generator is directed to a disk file in the real data format. the convolution program then reads this data from that file and performs the convolution process as required.

The machine used to process absorption data was an IBM AT compatible based on the 80286 microprocessor with 16 MHz clock speed. The memory size of the machine was 1 megabytes and the hard disk of 40 Megabytes. The implemented algorithm was tested to generate orthogonal polynomials for unequal intervals. The data type used in the algorithm were of type double precision. Double precision data are stored with 16 digits of precision and printed with up to 16 digits. The reason for using double precision data type is to get the maximum accuracy from the AT machine.

Practical Examples

All materials and solvent were analytical grade, Nystatin and Methylparaben are pharmaceutical grade substances. Spectrophotometric measurements were made on Perkin-Elmer 550S. Reference standard solution of Nystatin (10 mg/dl) in methanol and Methylparaben (2.7 mg/dl) were prepared in methanol. Nystatin with characteristic multippeak absorption spectrum (Figure 3.A) is formulated [22] in solution with Methylparaben as preservative. The absorption data showed that Nystatin and Methylparaben have a nearly overlapping spectrum.

In the present example, the orthogonal polynomial for unequal interval has been used for the determination of nystatin directly without

interference of methylparaben, on the other hand Methylparaben was determined by applying two component method of orthogonal polynomials for unequal intervals [12].

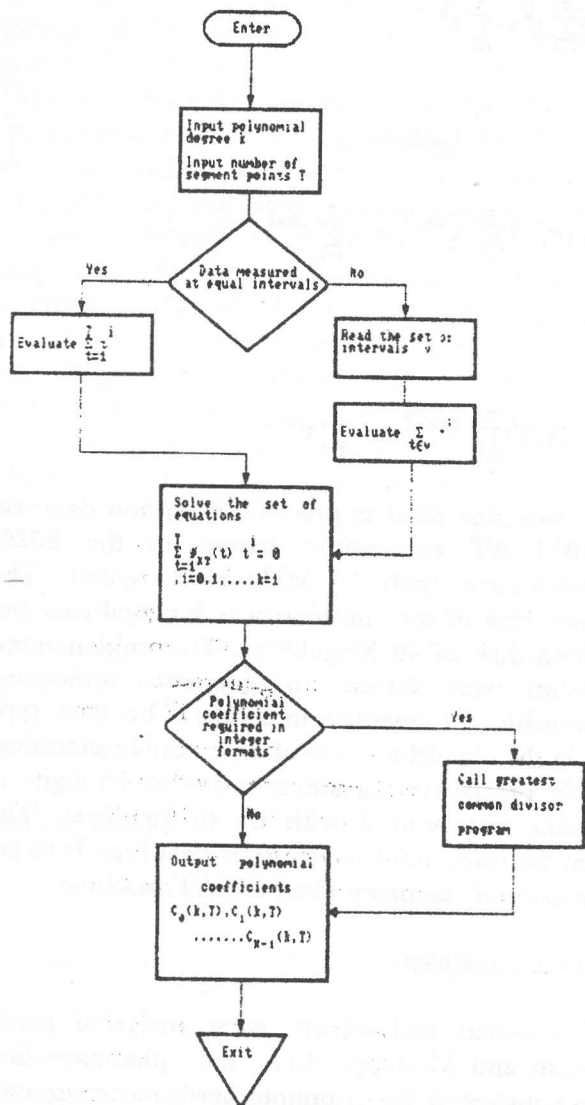


Figure 2. Flow chart of the orthogonal polynomial generator.

The first parameter to be selected is the working segment and its intervals. this selection is based on the shape of the specified segment on the absorption curve. For nystatin, the segment (abcdefg) was selected as shown in Figure 3.A. This segment suggests that a K6 (sixth degree) polynomial can fit

the absorption curve. The segment (abcdefg) corresponds to the wavelengths 280_{min}; 290_{max}; 296_{min}; 304_{max}; 312_{min}; 318_{max}; 330_{min} nm. The polynomials generated by the generator algorithm are tabulated in Table 1. Their graphs are shown in Figure (3.B).

For Methylparaben, the segment (hijklmn) which corresponds to wavelengths 230, 236, 244, 256, 260, 270 and 280 nm was selected. This segment is shown in Figure (3.A). This segment suggests that a K2 polynomial would fit the absorption curve. Therefore, a K2 polynomial is generated by the generator algorithm. The orthogonal polynomials for degree 1 to degree 6 for this segment are tabulated in Table 2. The graphs for these orthogonal polynomials are shown in Figure (3.C).

Synthetic mixtures of nystatin and methylparaben were prepared in concentrations presented in Table 3. and assayed for both component using orthogonal polynomials for unequal intervals (proposed) and the results obtained are compared with A_{max} method for nystatin and Vierordt's method for Methylparaben (Table 3). The slightly higher results using A_{max} was due to the slight contribution of methylparaben at the 303 nm (λ of nystatin). On the other hand the higher results Table (3) for methylparaben using Vierordt's method were due to the overlapping spectra in the vicinity of 240-280 nm. the results obtained using the proposed methods are 100.0 + 0.558 % and 101.3 + 1.634 % (mean % recovery + coefficient of variation) for nystatin and methylparaben, respectively.

In conclusion the potential of the orthogonal polynomials for unequal interval could be summarized in the following points;

- i) the choice of the polynomial degree is the most critical factor that affects the fitting of the fine structure of the absorption curve. For example in nystatin absorption spectra, choice of higher order such as K4, K5, and K6 is preferable). This means that if we select K6 polynomials the constant, linear, quadratic,..etc, are covered by the orthogonal polynomial and hence components of irrelevant absorption are corrected.
- ii) In two component analysis, errors due to considerable overlapping spectra could be minimized by extracting the best fit polynomial which reflects the characteristic features of the absorption curve. This means that the resolution between spectra can be increased by using an appropriate mathematical technique.

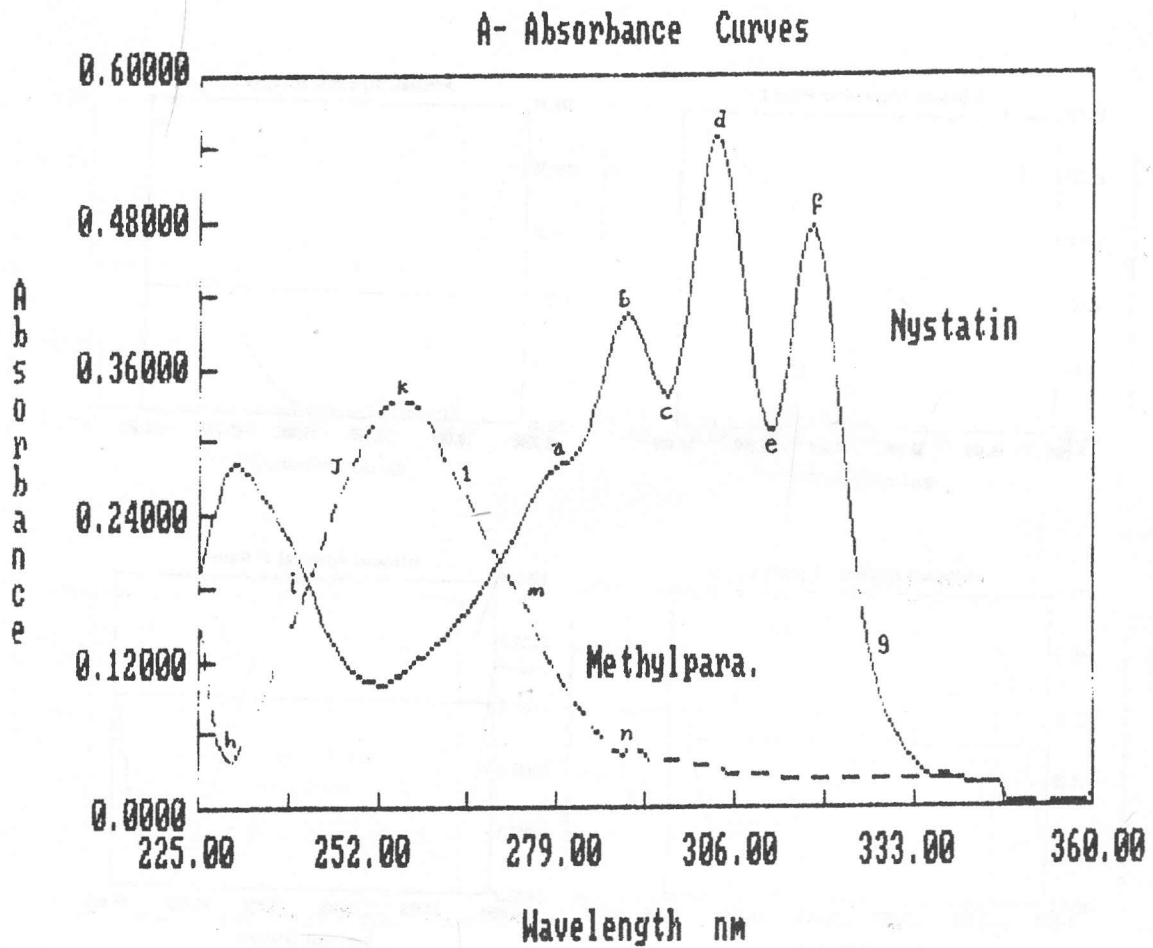


Figure 3a. Absorption spectra of 10 mg/dl Nystatin (solid line) and 2.7 mg/dl Methylparaben (dashed line) in Methanol.

Table 1. Generated polynomials K_j for the determination of nystatin in two component mixtures with methylparaben.

i	l	intv	k1	k2	k3	k4	k5	k6
0	280	0	-24.28572	367.3548	-3595.85	15598.62	-45097.16	205611.3
1	290	10	-14.28571	-30.96774	3859.123	-46751.14	291981.1	-1736194
2	296	6	-8.285715	-173.9613	3602.425	5364.767	-366384.4	3247323
3	304	8	-0.2857143	-252.6194	430.1981	47863.4	-24218.48	-2891160
4	312	8	7.714286	-203.2774	-3160.272	11934.35	384317.3	1666362
5	318	8	13.71429	-82.27096	-4279.596	-45962.83	-271289.6	-503162.2
6	330	12	25.71429	375.742	13143.97	11952.84	30691.29	11219.49

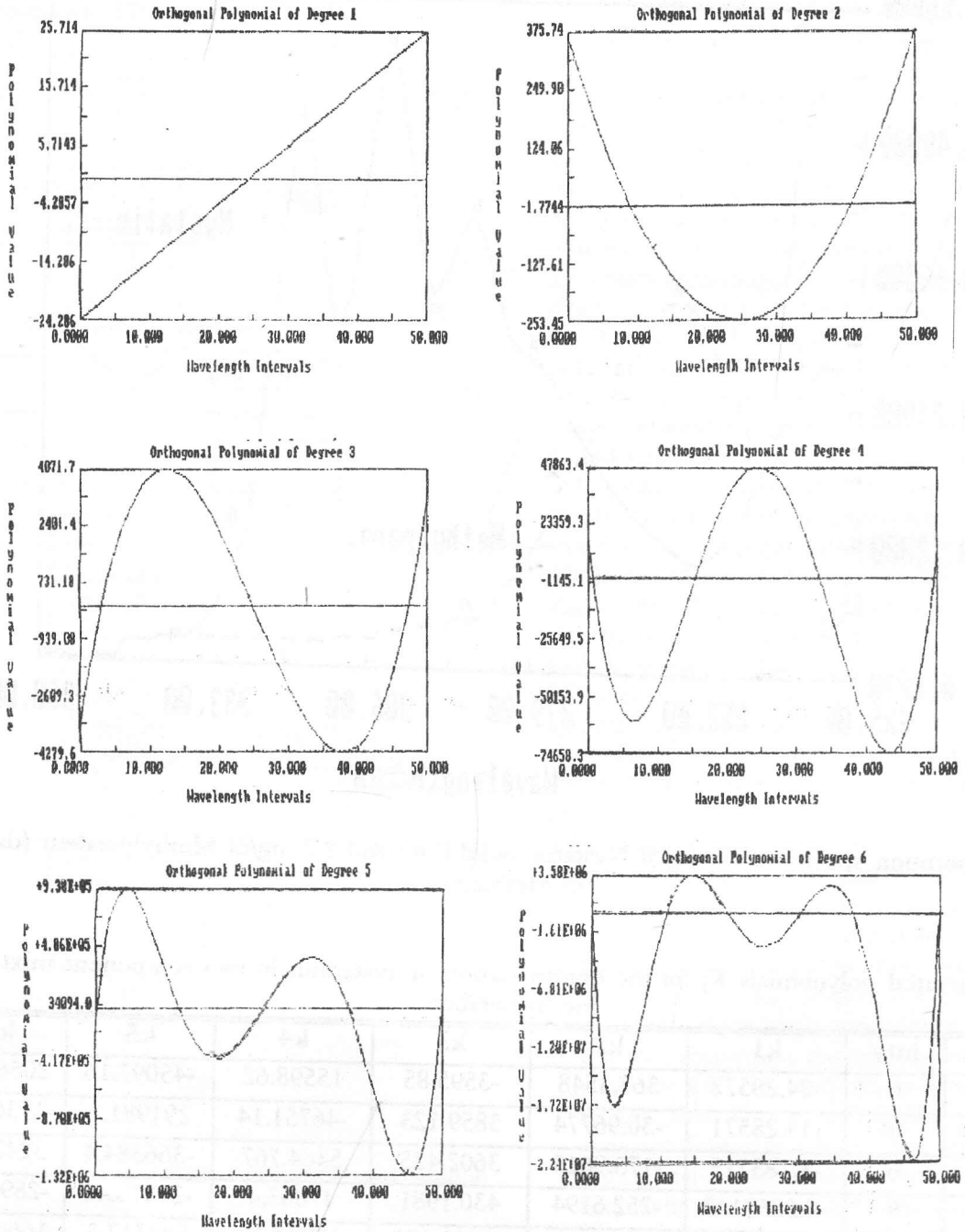


Figure 3.B Graphs of the generated orthogonal polynomials for the unequal interval segment corresponding to table 1.

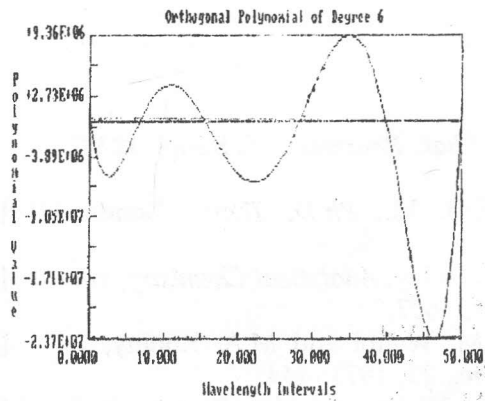
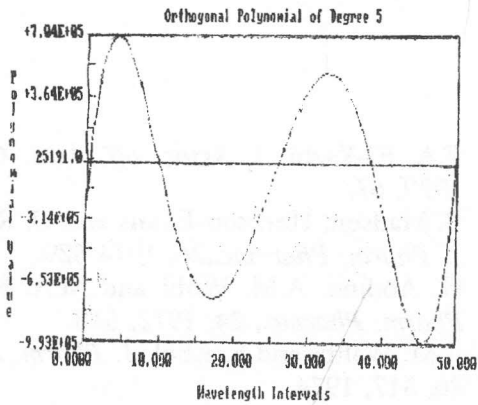
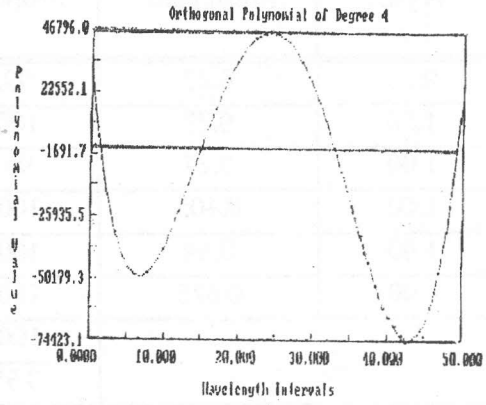
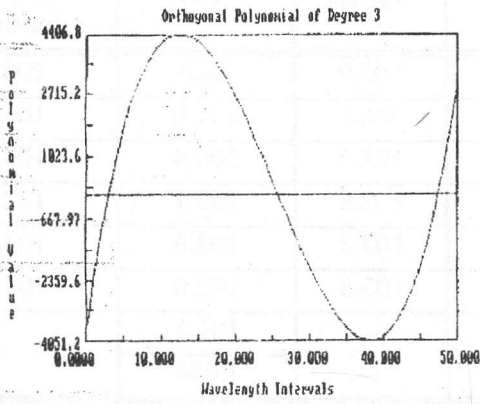
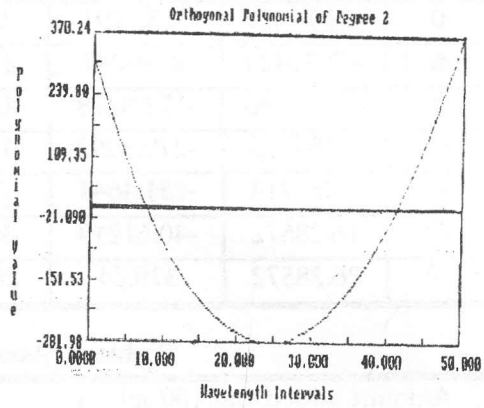
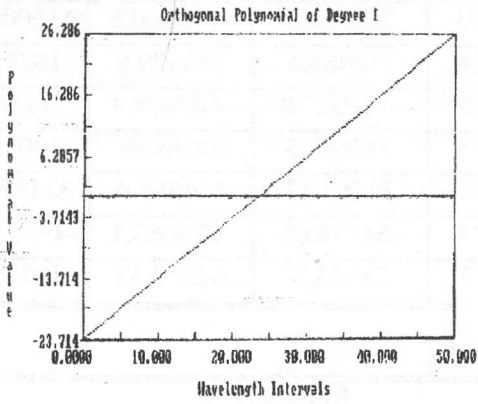


Figure 3.C Graphs of the generated orthogonal polynomials for the unequal interval segment corresponding to table 2.

Table 2. Generated polynomials K_j for the determination of methylparaben in two component mixtures with nystatin.

i	l	intv	k1	k2	k3	k4	k5	k6
0	230	0	-23.71429	315.9791	-3657.993	30036.12	-199100.6	413000.1
1	236	6	-17.71429	58.49044	2467.824	-48868.4	583329.8	-1421349
2	244	8	-9.714286	-172.8278	4337.975	-7149.778	-602698.4	2113629
3	256	12	2.285715	-279.8052	-111.6684	44268.05	20046.46	-4005589
4	260	4	6.285714	-251.4644	-1993.515	24265.47	414565.4	3344265
5	270	10	16.28572	-40.61218	-3930.138	-64194.65	-278683.1	-493732.7
6	280	10	26.28572	370.24	2887.515	21643.2	62540.33	49776.4

Table 3. Assay Results.

Experiment No.	Amount added mg/100 ml		Recovery %			
			Nystation		Methyparaben	
	Nyst.	Meth.parb.	Proposed	A_{303nm}	Proposed	Vierordt's method
1	0.75	0.27	99.6	103.9	98.6	103.3
2	1.50	0.27	100.5	99.2	101.0	105.6
3	1.00	0.27	99.1	102.2	100.4	104.8
4	1.00	0.405	100.5	102.8	103.1	102.0
5	1.00	0.54	100.0	103.2	102.6	103.5
6	1.00	0.675	100.3	103.5	102.0	102.2
mean			100.0		101.3	
C.V.%			0.558		1.634	

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