

# THE ELASTIC ELECTRON SCATTERING FROM $^{15}\text{N}$

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## ABSTRACT

The Coulomb transition (CO) of elastic electron scattering form factor of  $^{15}\text{N}$  has been measured over a momentum transfer range  $q = 0.4\text{--}3.2 \text{ fm}^{-1}$ . From these form factor data the ground state charge density and its RMS radius ( $2.612 \pm 0.009 \text{ fm}$ ) were determined. This charge density as well as its difference with that of  $^{16}\text{O}$  were compared to recent large-basis shell model calculations. Although these calculations describe the individual charge densities reasonably, the difference between  $^{16}\text{O}$  and  $^{15}\text{N}$  cannot be reproduced satisfactorily.

*Keywords: Electron scattering, Polarisation effects, Charge density.*

## INTRODUCTION

Charge density differences between neighbouring nuclides provide a tool for studying nuclear structure details like the validity of a pure single particle description and the influence of core polarisation effects. For the pair  $^{16}\text{O}\text{--}^{15}\text{N}$ , Gerace and Hamilton [1] showed that the charge density difference, determined from the data of Dally et al. [2] and Sick and McCarthy [3], could be explained satisfactorily by a single-particle calculations. Core polarisation effects were taken into account by adjusting the harmonic oscillator parameter to fit the experimental RMS charge radii. The experimental charge density of  $^{15}\text{N}$ , however, is not very well established. Dally et al. [2] performed an electron scattering experiment at energies of 250 and 400 MeV, using a liquid  $^{15}\text{NH}_3$  target. Their measurements cover a momentum transfer range  $q = 0.9\text{--}2.6 \text{ fm}^{-1}$ . The statistical errors in the data range from 4 to 7 %, but systematic errors due to variations in the thickness of their liquid  $^{15}\text{NH}_3$  target might be significant. Schutz [4] measured this form factor using a gas target at energies between 30 and 65 MeV, spanning a momentum transfer range from 0.3 to  $0.5 \text{ fm}^{-1}$ . The statistical errors in the data amounted to 0.3%. The RMS charge radius given by Schutz is  $\langle r^2 \rangle^{1/2} = 2.580 \pm 0.026 \text{ fm}$ . Dally et al. give  $\langle r^2 \rangle^{1/2} = 2.65 \text{ fm}$  for the 250 MeV data. The uncertainty of these values are not given.

This paper describes a study of the  $^{15}\text{N}$  ground

state by longitudinal electron scattering. The transverse data have been measured by Singhal et al. [5]. The data will be used as test of large-configuration-space shell model calculations, for which  $^{15}\text{N}$  is considered a sensitive testing-ground, carried out by Utrecht theory group [6].

The experimental was performed at the high-energy electron scattering facility of NIKHEF-K with the QDD spectrometer [7]. A room temperature gas target was used. It consists of a thin-walled cylinder with its symmetry axis perpendicular to the scattering plane, and is made out of an Al-alloy with a high tensile strength, especially at elevated temperatures. The height of the gas cell is 45 mm, its diameter is 40 mm and its wall thickness  $40 \mu\text{m}$ . After the cell has been filled with 4-5 atm gas it is sealed permanently. Calculations [8] showed that the QDD spectrometer accepts electron scattered from a 2 cm wide volume around the axis of the cylinder. Electron scattered from the walls of the gas target are not detected for scattering for angles between  $40^\circ$  and  $140^\circ$ , which prevents contamination of the spectra with Al-peaks. The effective thickness of such a  $^{15}\text{N}$  gas target at a scattering angle  $\theta = 90^\circ$  is  $10 \text{ mg/cm}^2$ . This depends on the scattering angle and on the solid-angle defining slit of the spectrometer. Local variations of the density of the gas due to heating in the beam were investigated by varying the beam current. For currents up to  $50 \mu\text{A}$

this effect is less than 2%. The temperature of the gas target was monitored during the experiment and did not rise above 70°C. Background from the walls of the target was measured by observing the scattering from an empty gas cell.

The extension of the beam spot in the beam direction hampers the reconstruction of the particle trajectories through the spectrometer. This reconstruction is needed to correct for kinematic broadening i.e. the spread in recoil shift caused by the horizontal acceptance of the spectrometer. Furthermore, the angular resolution need to perform the kinematic correction deteriorates because of multiple scattering in the walls of the target. For electron energies above 200 MeV, where the kinematic broadening becomes sizeable, the resolution obtained with this gas target is therefore worse than in other experiments with solid targets; e.g. at 300 MeV the resolution is 90 keV, whereas 30 KeV would be easily attainable with solid targets. For a study of the ground state of  $^{15}\text{N}$  this 90 keV resolution is, however, sufficient as the first excited state in this nucleus is located at an excitation energy of 5.3 MeV.

Elastic electron scattering cross sections were measured at energies between 70 and 430 MeV at scattering angles from 40° to 98°. They cover the range  $q=0.43.2 \text{ fm}^{-1}$ . The spectra of elastic electron scattering at different angles are presented in Figure (1). The error varies from 4-5% for  $q \leq 2.3 \text{ fm}^{-1}$  to 17% for  $q=3.2 \text{ fm}^{-1}$ . The form factor was calculated from the measured cross sections according to De Forest and Walecka [9]. The absolute normalisation was deduced from the overlap of the data with those of Schutz [4]. The stability of this normalisation was checked with QDQ spectrometer as a monitor.

As the ground state of  $^{15}\text{N}$  has spin and parity  $J^\pi=1/2^-$  the form factor consists of CO and M1 multiples only. The M1 form factor has been measured by Singhal et al. [5] in an electron scattering experiment at 180°. Their data were used to subtract in PWBA the M1 component from our data. The shift of the form factor due to DWBA effects is accounted for by applying the  $q_{\text{eff}}$  formalism. The DWBA effects on the absolute values of the form factor at a fixed  $q$  are at most 2.5% for  $q < 1 \text{ fm}^{-1}$ , where the longitudinal component in the form factor is dominant. At higher

$q$ -values, these effects are even smaller and could be neglected with respect to the experimental uncertainty.

To determine a model-independent charge density, a phase shift code, in which Coulomb distortions are fully accounted for, was used to fit a Fourier-Bessel expansion to the CO data together with those obtained by Schutz [4]. The resulting FB coefficients are listed in Table (1).

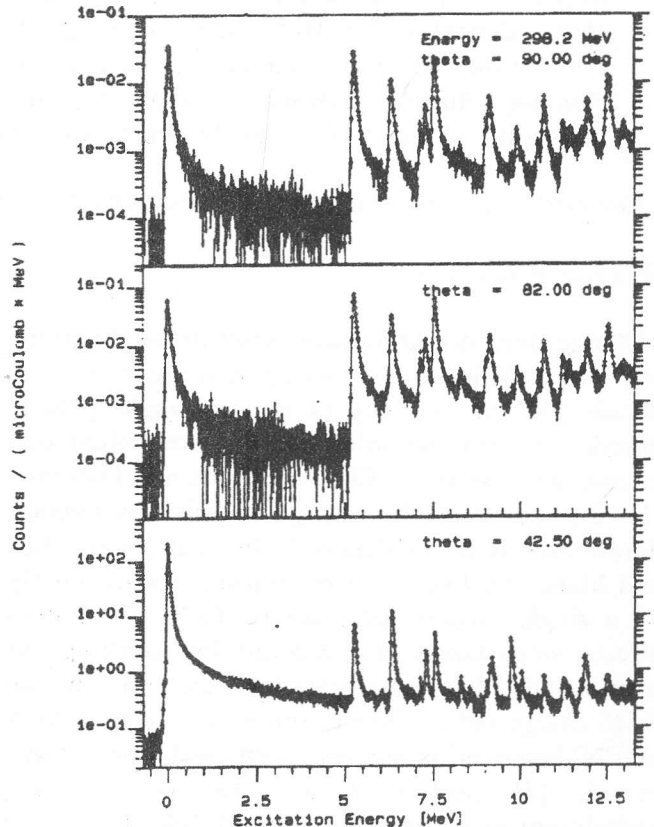


Figure 1. Spectra of elastic electron scattering at different angles.

The RMS charge radius resulting from this fit is  $\langle r^2 \rangle^{1/2} = 2.612 \pm 0.009 \text{ fm}$ , in agreement with the value  $\langle r^2 \rangle^{1/2} = 2.5802 \pm 0.026 \text{ fm}$  reported by Schutz. Figure (2) shows the data together with the Fourier-Bessel fit. The difference between the charge density calculated from the data of Sick and McCarthy [3] and Schutz [4], is shown in Figure (3). The RMS charge radius of  $^{16}\text{O}$  is  $1.125 \pm 0.012 \text{ fm}$  larger than that of  $^{15}\text{N}$ .

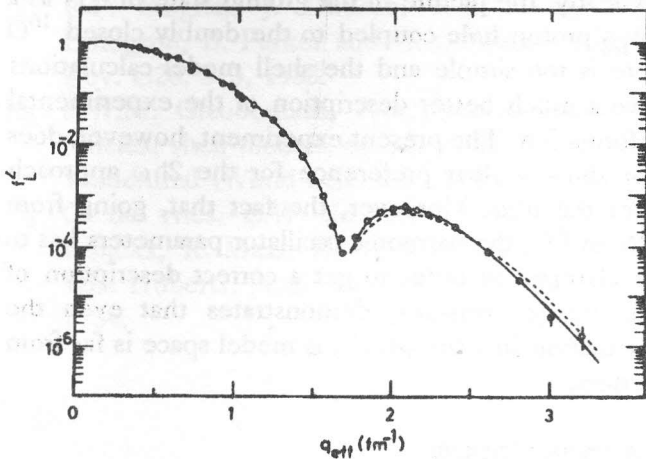


Figure 2. Longitudinal elastic form factor of  $^{15}\text{N}$ . The data from the present experiment (circles) and the data measured by Schutz [4] (plusses) are shown. The solid curve is the best fit to the data. The dashed curve is the result of the  $2\hbar\omega$  shell-model calculation.

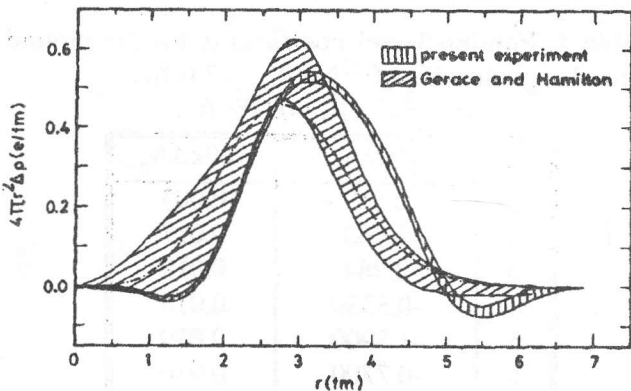


Figure 3. The charge density difference from Gerace and Hamilton [1] compared to that from the present experiment. The band includes both statistical and systematic errors. The dot-dashed curve represents the single-particle calculation by Gerace and Hamilton [1].

Shell-model calculations to describe this CO form factor were performed by the Utrecht theory group [6]. To describe the structure of  $A=4-16$  nuclei an effective interaction is used that can be expressed in terms of 13 Talmi integrals. These integrals as well as the value of  $\hbar\omega$  have been fitted to 76 energy levels of 1p-shell nuclei. In the calculations no

closed  $^4\text{He}$  core is assumed. For  $^{15}\text{N}$  all possible excitations up to  $2\hbar\omega$  were included. The resulting ground-state wave function can be expressed as:

$$0.65.(1s)^4 (1p)^{11} + 0.19. (1s)^4 (1p)^{10} (1f2p)^1 \\ + 0.14.(1s)^4 (1p)^9 (2s1d)^2 + 0.02 (1s)^3 (1p)^{11} (2s1d)^1.$$

The form factor was calculated with harmonic oscillator wave functions with a harmonic oscillator parameter  $b=1.77$  fm, that reproduces the form factor minimum. The calculation is shown in Figure (2) and describes the experimental data well, except for the height of the second form factor maximum. The influence of using Wood-Saxon wave functions in the calculations of the form factor was also investigated. The only effect of that is somewhat steeper decrease for  $q>2.3$  fm $^{-1}$ .

A shell-model in a  $0\hbar\omega$  model space is able to describe the form factor equally well. However,  $b=1.67$  fm has to be used to reproduce the position of minimum. This can be explained by the fact that the major  $2\hbar\omega$  contributions come from  $2p_{3/2}$  densities. Interference of these with the  $1p_{1/2}$  terms yields a charge density that has a shape very similar to that originating from only  $1p_{1/2}$  terms. Near the edge of the nucleus the  $1p_{1/2}$  and the  $2p_{3/2}$  wave functions interfere destructively and hence a smaller radial size parameter is sufficient in the  $0\hbar\omega$  calculation.  $A^{16}\text{O} (e,e'p)^{15}\text{N}$  experiment [10] indicates that the  $^{15}\text{N}$  ground state is only of 60% single-hole character. Although this would seem to favour the  $2\hbar\omega$  shell-model calculation, a quenching by 40-60% of the spectroscopic factor for valence orbits however observed as a general trend in the results from the  $\{e,e'p\}$  reaction over the system [11]. At present it is not clear whether this should be ascribed to configurations of higher-lying shells or that there is a fundamental lack understanding of the reaction mechanism.

In a single particle description the difference between the charge densities of  $^{16}\text{O}$  and  $^{15}\text{N}$  should reflect the  $1p_{1/2}$  proton orbit. Gerace and Hamilton [1] showed that such a description was sufficient to describe the data available at that time, when core polarisation effects are taken into account in a phenomenological way. The density difference

determined from the present experiment is, however inconsistent with that used by Gerace and Hamilton and can no longer explained by a pure  $1p_{1/2}$  calculation, as is shown in Figure (3). Furthermore, a simultaneous analysis undertaken by us for the data of Dally et al. [2] and that of Schutz [4] showed that renormalisation the data of Dally by 20% reduces the chi-square from 3.8 to 2.4 per point, while the charge density obtained by this way does not significantly differ from the results of the present experiment. In Figure (4) the difference between the  $^{16}\text{O}$  and  $^{15}\text{N}$  densities, determined in the present experiment is compared to a single-particle calculation, in which the  $b$  parameter was adjusted to reproduce the maximum of the experimental curve, and the Utrecht shell-model calculation. For comparison, the  $^{16}\text{O}$  charge density was calculated from the  $2\hbar\omega$  shell model matrix elements with  $b=1.83$  fm, which reproduces the position of the first minimum in the form factor. If the same  $b$ -value is used as in the calculation for  $^{15}\text{N}$ . The difference between the RMS charge radii becomes only 0038 fm, in disagreement with the experimental value. The calculation in a full  $0\hbar\omega$  model space gives similar results if smaller  $b$ -values are chosen for both nuclei. Although the density difference at 1.3 fm and 5.5 fm is not well reproduced by the shell-model calculation, its agreement with the experimental data is much better than that of the pure  $1p_{1/2}$  calculation.

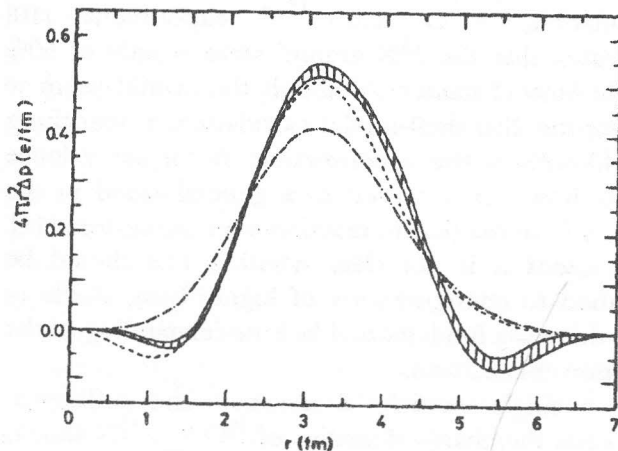


Figure 4. The band gives the charge density difference between  $^{16}\text{O}$  and  $^{15}\text{N}$ . The dot-dashed curve is a single-particle calculation in which the  $b$ -value was adjusted to reproduce the maximum of the experimental curve. The dashed curve gives the difference calculated with the  $2\hbar\omega$  shell-model.

Clearly, the picture of the ground state of  $^{15}\text{N}$  as a  $1p_{1/2}$  proton hole coupled to the doubly closed  $^{16}\text{O}$  core is too simple and the shell model calculations give a much better description of the experimental information. The present experiment, however, does not show a clear preference for the  $2\hbar\omega$  approach over the  $0\hbar\omega$ . Moreover, the fact that, going from  $^{15}\text{N}$  to  $^{16}\text{O}$ , the harmonic oscillator parameters has to be changed in order to get a correct description of the charge densities, demonstrates that even the calculation in a full  $(0+2)\hbar\omega$  model space is far from perfect.

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Table 1. Fourier-Bessel coefficients for the ground state charge density of  $^{15}\text{N}$ .  $R_{\text{max}}=7.0$  fm,  $\langle r^2 \rangle^{1/2}=2.612 \pm 0.009$  fm.

$n$	$100xA_n$	$100x\Delta A_n$
1	2.549	0.003
2	5.063	0.017
3	2.984	0.019
4	-0.5530	0.015
5	-1.5900	0.030
6	-0.7700	0.030
7	-0.2300	0.050
8	-0.0400	0.070

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