A VISCOUS FLOW ANALOGY FOR STUDYING PUMPING FROM MULTIPLE ARTESIAN WELL SYSTEMS

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ABSTRACT

This paper addresses a new design technique for a horizontal Hele-Show model to simulate pumping from an artesian multiple well system forming a square with a well in the center. Full details of the construction techniques are presented. Observed discharges of wells and piezometric heads agree very well with the Muskat and Dupuit formulae as well as the superposition principles. The designed model has the flexibility to simulate different complicated geometries and boundary condition of artesian multiple well systems.

Keywords: Viscous flow, Wells, Pumping, Hele-Shaw model.

Notation

B	side of the square,
D	thickness of the confined aquifer,
g	gravity acceleration,
h	piezometric head at any point,
h_w	water depth in well,
H	depth of the original piezometric surface,
k	permeability coefficient,
n	number of wells in group,
Q	well discharge,
r	distance from well to point at which drawdown is computed,
r_w	well radius,
R	well radius of influence,
R_o	radius of region at which a group of wells is pumping, and
υ	kinematic viscosity.

INTRODUCTION

Closely spaced wells are used where the demand of water is larger or where a number of wells have to feed a common water supply system. Multiple well systems are also used for lowering the groundwater level in a given area in order to facilitate excavation for foundation works. In such cases a group of wells are arranged in different configurations and the lowering of groundwater is more effective when wells are spaced at distances smaller than their radii of influences. Under these conditions, they affect each other's drawdown and rate of discharge. According to the superposition principles [4], the drawdown at any point in multiple well systems is equal to the sum of drawdown due to each individual well. For confined aquifer, the total drawdown (H-h) at any point equals (Todd [10]):

$$H - h = \frac{1}{2\pi kD}\sum_{i=1}^{i=n} Q_{wi}\ln\frac{R_i}{r_i} \tag{1}$$

where:

H	depth of the original piezometric surface
h	piezometric head at any point
k	permeability coefficient
D	thickness of the confined aquifer
Q_{wi}	the discharge from ith well
R_i	radius of influence for ith well
r_i	distance from ith well to point at which drawdown is computed
n	number of wells in a group

Similarly, the head h_{wj} at any well, for example, well j in a system consists of n wells can be determined as follows:

$$H\text{-}h_{wj} = \frac{1}{2\pi kD}\left(Q_{wj}\ln\frac{R_i}{r_{wj}} + \sum_{i=n}^{i=n-1} Q_{wi}\ln\frac{R_i}{r_{i,j}}\right) \quad (2)$$

where

Q_{wj}	discharge from well j
R_j	radius of influence of well j
r_{wj}	effective well radius of well j
$r_{i,j}$	distance from each well to well j

For group of wells pumping in island with radius R_o, the radius of influence for all wells can be assumed constant, where R_i and R_j are assumed equal to R_o (Powers [6]).

Muskat [2] developed solutions for well discharges for various well patterns localized near the centre of a region of radius R_o. He assumed that all the wells fully penetrate a confined aquifer and have the same drawdowns, and discharges over the same period of time. The discharge of each of four wells forming a square of side B (see Figure (1-a)) is:

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{2\pi kD(H-h_w)}{\ln(R_o^4/\sqrt{2}r_wB^3)} \quad (3)$$

For four wells on the corners of a square with a fifth well in the center (see Figure (1-b)), the corner wells yield:

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{2\pi kD(H-h_w)\ln(B/\sqrt{2}r_w)}{4\ln(\sqrt{2}R_o/B)\ln(B/\sqrt{2}r_w) + \ln(R_o/r_w)\ln(B/4\sqrt{2}r_w)} \quad (4)$$

while the center well discharges only:

$$Q_5 = \frac{2\pi kD(H-h_w)\ln(B/4\sqrt{2}r_w)}{4\ln(\sqrt{2}R_o/B)\ln(B/\sqrt{2}r_w) + \ln(R_o/r_w)\ln(B/4\sqrt{2}r_w)} \quad (5)$$

Accordingly, it can be stated that Muskat formulae can not be used if the drawdowns in the center well and the corner wells are not equal.

Hansen [3], Babbitt and Caldwell [1] studied the interference effects of small group of wells using sand models. They compared their results with the theoretical equations developed by Muskat (Eqs. 3 to 5). A gratifying agreement was noted between the observed and the computed discharges. Mansur and Kaufman [9] and Powers [6] developed a simple solution to determine the total yield and drawdown at any point for a group of equally spaced wells. They considered that the wells act as a single large well of radius r_s (see Figure (1-c)):

$$r_s = \frac{\sqrt{ab}}{\pi} \quad (6)$$

and the discharge from this well equals:

$$Q = \frac{2\pi kD(H-h_w)}{\ln\left(\frac{R_o}{r_s}\right)} \quad (7)$$

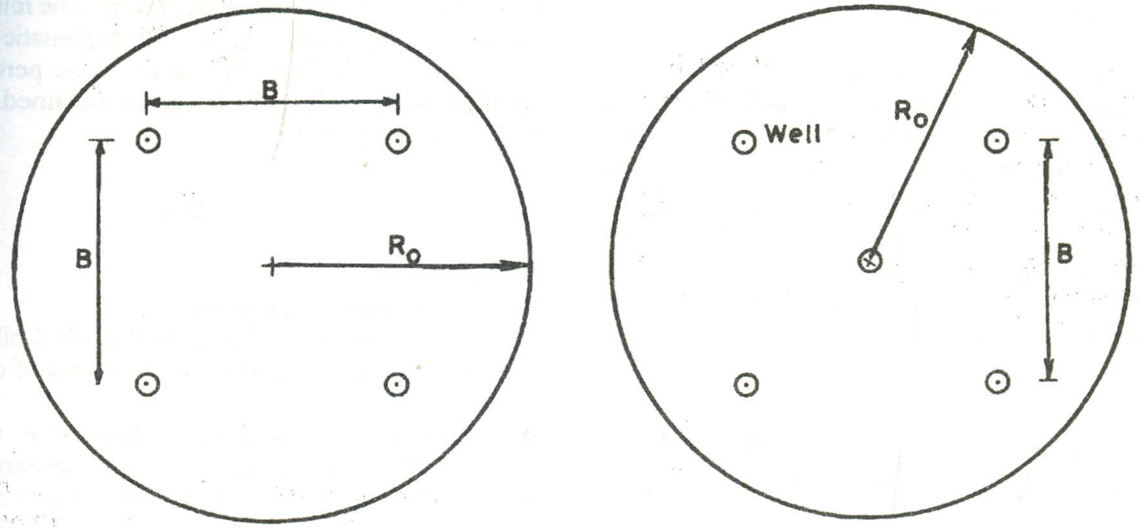
Eq. (7) can be used to determine the piezometric head h at any point and takes the form:

$$Q = \frac{2\pi kD(H-h)}{\ln\left(\frac{R_o}{r}\right)} \quad (8)$$

in which r is the radius distance from the well.

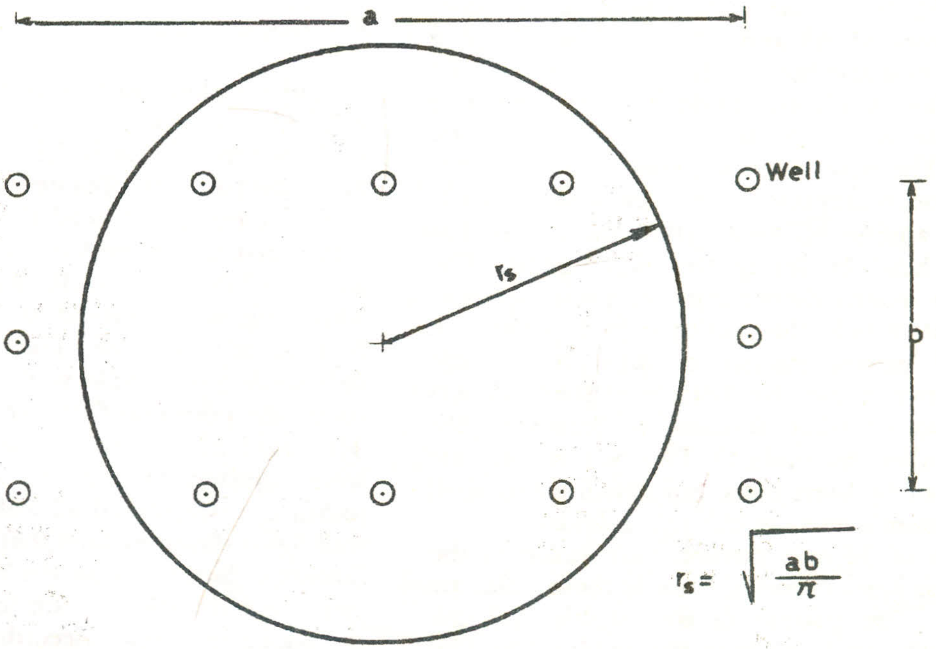
Hele-Shaw model proved to be a versatile tool for investigating groundwater problems. Santing [8] and Marino [5] have used horizontal Hele-Shaw model to study the seepage through confined aquifer. Rezk and Moghazi [7] showed the high accuracy of the model to simulate seepage towards a single artesian well. According to the Authors knowledge, horizontal Hele-Shaw model has not be used before to simulate the pumping from a group of artesian wells arranged in different configurations.

The main objective of this research is to design and investigate the accuracy of the horizontal Hele-Shaw model to simulate the pumping from artesian multiple well systems under different boundary conditions.



a : Four interfering wells

b: Five interfering wells.



c : Approximation of equivalent radius r_s.

Figure 1. Multiple well systems in different configurations.

Description of the Model

The following assumptions are considered in the design of the model; (1)the flow of groundwater is purely horizontal in the confined aquifer; (2) the aquifer is homogeneous and isotropic, and (3) the multiple well system is completely penetrating the confined aquifer and have the same diameter. The designed well system consists of four wells on the corners of a square with a fifth well in the center to simulate the case shown in Figure (2).

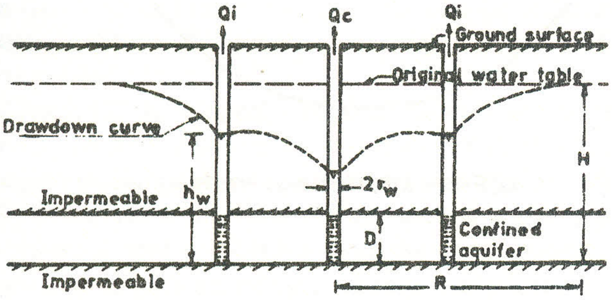


Figure 2. Steady radial flow to a multiple well system penetrating a confined aquifer.

A schematic diagram (not into scale) of the designed horizontal Hele-Show model is illustrated in Figure (3). It consists of two prespex cylinders with diameters of 60 and 70 cms. They have a height of 20 cms and bottom thickness of 1 cm. The small cylinder is placed inside the big one in a vertical position, and their vertical axes are coincident. The outer cylinder represents a constant head reservoir to simulate the original piezometric head of the aquifer H. An overflow tube is used to control the head H during the experiments. The radius of the inner cylinder represents the radius of island R_o at which the multiple well system is pumping. A narrow interspace (0.15 cm) is kept constant between the two bottom of the cylinders with the aid of fiber washers to represent the confined aquifer thickness D. The two bottoms are fixed together by brass bolts, equally distributed all over the bottom's areas of the two cylinders.

Five holes of 10 mm diameter are made in the bottom of the inner cylinder. Four of these holes are forming a square pattern of side width 21.2 cm, while the fifth one is made in the center. The four holes are symmetrical and located at 15 cm from the centric well. Five glass tubes (10 mm diameter) are glued into these holes to represent the multiple well system. A viscous oil (supper 7500-20 w/50) is used to simulate groundwater movement. The relationship between oil temperature and the kinematic viscosity of the oil is plotted in Figure (4). The permeability coefficient of the aquifer k can be obtained from the formula (Todd [10]):

$$K = \frac{D^2 g}{12 \nu} \tag{9}$$

where

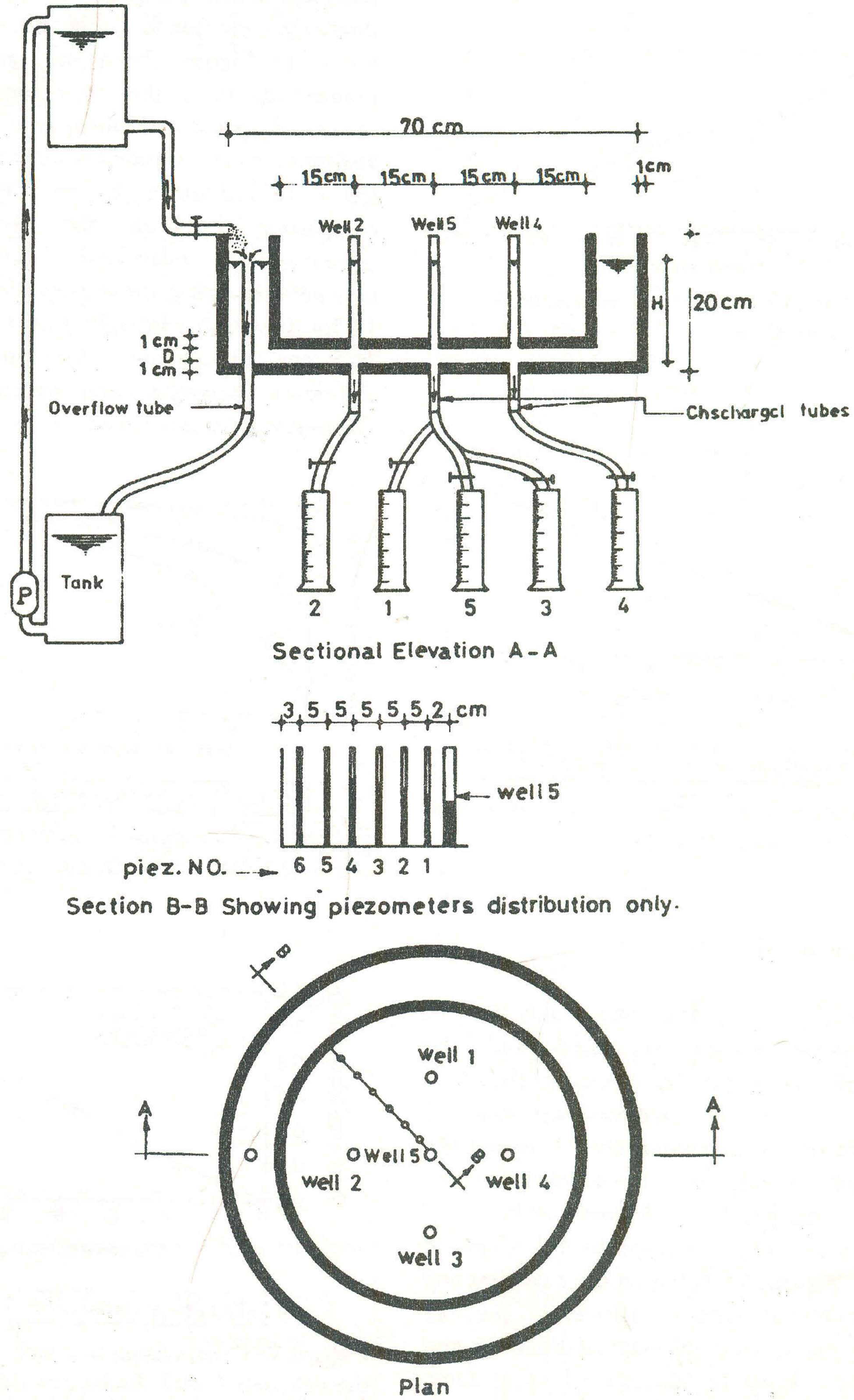
g the gravity acceleration
ν the kinematic viscosity of the used oil, and
D spacing between the two bottoms of cylinders

Withdrawals of groundwater from the wells are initiated by withdrawing certain amount of oil through five discharge tubes connected to the center of the outer cylinder's bottom and opposite to the positions of wells. The rate of wells' discharges Q_i are regulated by control valves and calibrated using graduated tubes and stop watches. In order to observe piezometric heads during experiments, Six piezometer tubes are fixed into the inner cylinder's bottom, in one line with the centric well and passing in the mid-distance between wells 1 and 2, as shown in Figure (3). They are fixed at distance 2,7,12,17,22 and 27 cms from the centric well axis.

Experimental Tests

Four complete tests were made. The centric well referred to hereafter as well 5 while the other four wells referred as wells 1,2,3 and 4 respectively. All experiments were operated at a fixed value of the original piezometric head H and it was equal 16 cms.

The first test considered only the pumping from well 5 while other wells were completely closed. This case is considered to simulate the pumping from an artesian well. The second test considered only wells 1 to 4 while well 5 was closed in order to study the pumping from wells in a square pattern. The third test considered all wells (1 to 5). They were adjusted to give the same drawdowns. The fourth test considered all well, but the drawdown in well 5 was different than that in the other four wells. Since the four wells on the corners are symmetrical and their drawdowns are equal, their discharges should be the same. Accordingly, discharge of well 1 was only measured to represent discharges of well 2,3 and 4. During all the above tests, various drawdown in the wells were tried, and the corresponding well discharges, piezometric heads and oil temperature were measured.



Sectional Elevation A - A

Section B-B Showing piezometers distribution only.

Plan

Figure 3. Diagram of the horizontal Hele-Shaw model (Not into scale).

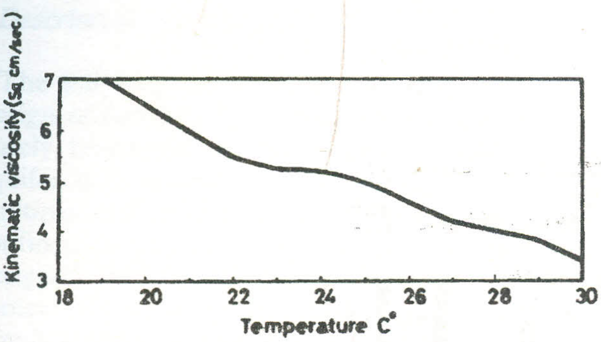


Figure 4. Relationship between temperature and viscosity for the used oil.

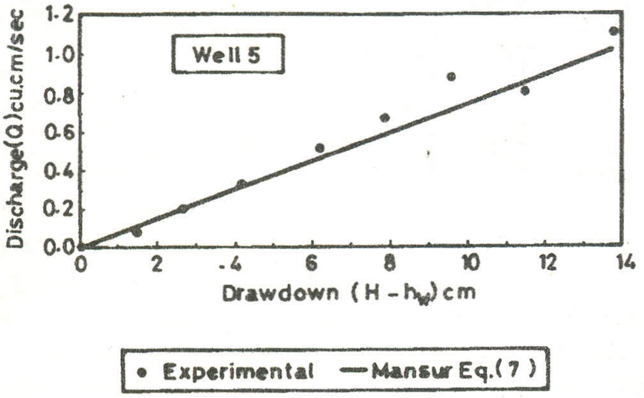


Figure 5. Comparison between measured and calculated well discharge (test 1).

ANALYSIS OF RESULTS

For the case of pumping from well 5 only (test 1), comparison between the measured and the calculated well discharges Q, based on Eq. 7, is shown in Figure (5) for various drawdowns in well 5 $(H-h_w)$. Figure (6) shows comparisons between the observed and calculated drawdowns in the piezometers, using Eq. 8, for drawdowns in well 5 equal 1.5, 6.2, and 11.5 cm respectively. It can be noticed close agreements between the experimental and the theoretical results. However, possible variations in measurement the rates of discharge and time increments leads to little variations at some points. By operating only wells 1,2,3 and 4 (test 2), with the same drawdowns $(H-h_w)$, comparisons between the measured and the calculated well discharges are made using Eq. 3 and 6 and are shown in Figure (7). A very good agreement is noticed between the experimental and Muskat results compared to Mansur results (Eq. 6). This is attributed to the approximation made in developing Eq. 6 as introduced earlier. Figure (8) shows a comparison between the observed and the corresponding calculated drawdowns in the piezometers, using the superposition principle (Eq. 1), for drawdowns in wells 1 to 4 equal 3.5, 8.5 and 14.5 cm respectively. Although there are small difference between both values, the maximum difference does not exceed 12%.

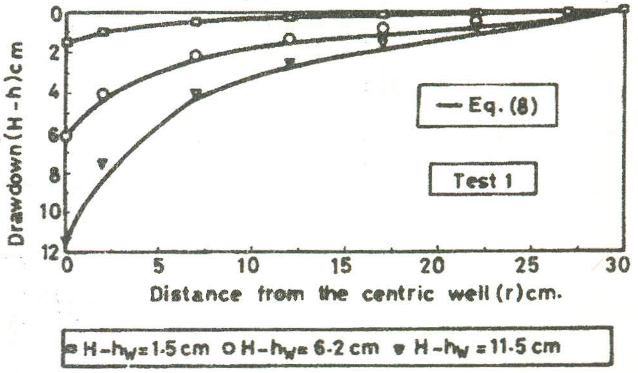


Figure 6. Comparison between observed and calculated drawdowns in the piezometers.

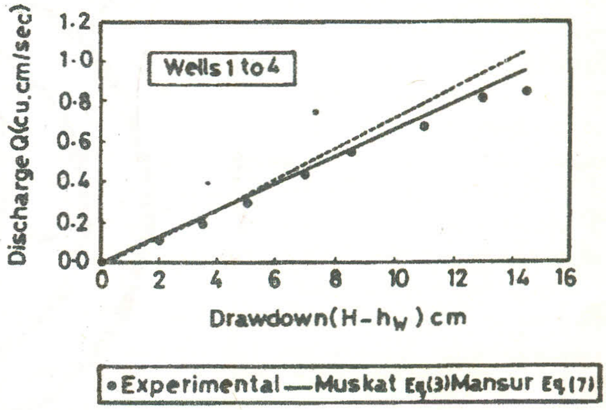


Figure 7. Comparison between the measured and the calculated well discharges (test 2).

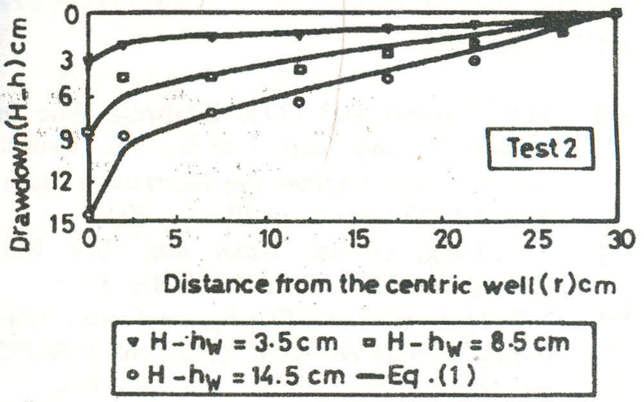


Figure 8. Comparison between observed and calculated drawdowns in the piezometers.

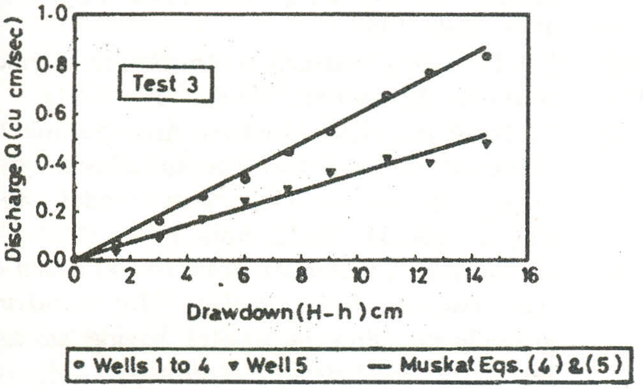


Figure 9. Comparison between observed and calculated well discharges.

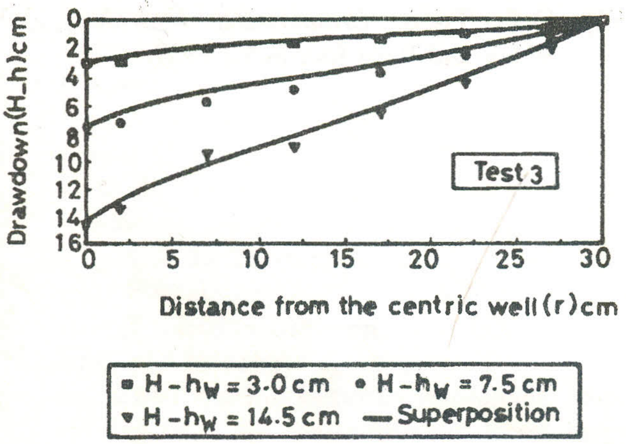


Figure 10. Comparison between the observed and the calculated drawdowns in the piezometers.

For the case of operating all wells under the same drawdowns in the wells (test 3), comparisons between the observed and calculated well discharge using the Muskat formulae (Eqs. 4 and 5) are made for four wells on the corners of a square and a fifth well in the center as shown in Figure (9). It can be noticed that the Muskat equations are the best fit curves for the experimental results. Figure (10) shows a comparison between the observed and calculated piezometric heads, using the superposition principles, for drawdowns in all wells equal 3,7.5 and 14.5 cm respectively. Negligible deviations are noticed between both results.

For the case of operating wells 1 to 4 with the same drawdown $(H-h_w)$, while the centric well operating under a different drawdown $(H-h_w)$, a comparison between discharges of wells 1 and 5 is listed in Table (1). According to the Authors knowledge, no relative references or forms are found to be compared with the experimental results of test 4. This indicates the superiority of the horizontal Hele-Show model to simulate different geometries of artesian multiple well systems, under different boundary conditions. A comparison between the observed and the calculated drawdowns, using the superposition principles, is shown in Figure (11) for drawdowns in the centric well $(H-h_w)$ equal 3,0, 9.0 and 14.5 cm and drawdowns in the corners $(H-h_w)$ equal 1.5, 3.0 and 4.5 cm respectively. A good agreement is noticed between the experimental and the theoretical results.

CONCLUSIONS

1. An innovated horizontal Hele-Shaw model is designed to simulate the pumping from a group of artesian wells forming a square with a well in the center.
2. The results of the model are in good agreement with those based on the widely used Muskat and Dupuit equations as well as the superpositions principles.
3. The designed Hele-Shaw model has the flexibility and ability to simulate the pumping from artesian multiple well systems with different geometries and operating modes under various boundary conditions when numerical solutions are difficult.

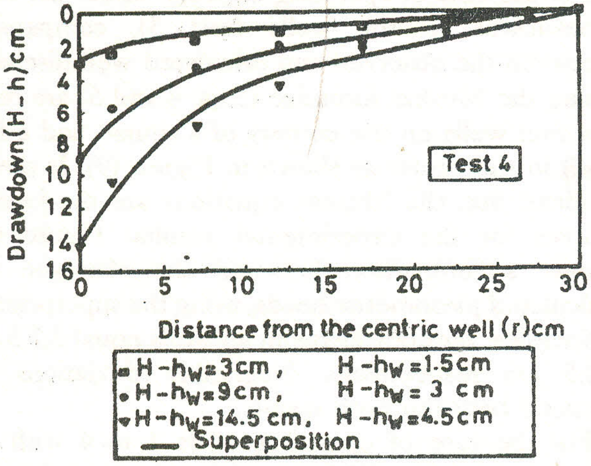


Figure 11. Comparison between the observed and the calculated drawdowns in the piezometers.

Table (1). Comparisons between discharges of wells 1 and 5.

Group	$H-h_w$ 5 (cm)	$H-h_w$ 1 (cm)	Q_5 (cm^3/sec)	Q_1 (cm^3/sec)	Temp °C
A (h_w=14 cm)	2	1.2	0.0833	0.0417	23.0°
	3	1.5	0.208	0.0417	
	5	2	0.383	0.033	
	7	2.5	0.5833	"	
	9	3	0.7667	"	
	11	3.5	1.0	"	
	13	4	1.15	"	
	14.5	4.5	1.34	"	
B (h_w=12 cm)	4	3.5	0.1413	0.152	23.5
	6	4	0.233	0.244	
	8	4.5	0.544	0.144	
	10	5	0.711	0.133	
	12	5.8	0.944	0.133	
	14.5	6.5	1.156	0.144	
C (h_w=11 cm)	5	5.2	0.15	0.217	23.5°
	7	5.5	0.325	0.217	
	9	6	0.55	0.2	
	11	6.5	0.717	0.2	
	13	7	0.917	0.2	
	14.5	7.5	1.067	0.2	
D (h_w=9.5 cm)	6.5	7.3	0.1417	0.317	23.5°
	8.5	7.7	0.3167	0.317	
	10	8	0.483	0.3	
	11.5	8.5	0.617	0.283	
	13	8.8	0.75	0.275	
	14.5	9.2	0.883	0.267	

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