PREDICTION OF BED FORMS IN ALLUVIAL CHANNELS

Khaled Hassan Baghdadi
Irrigation and Hydraulics Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

The present study investigates the formation of bed forms in Alluvial channels. Criterions for formation of ripples, dunes, plane bed and antidunes are introduced. The effects of grain Reynolds number, flow Froude number, particle fall velocity on the formation of bed forms are studied. Regime criterion charts are constructed from the collected data. Equations expressing the height and length of bed forms are presented in terms of Froude number, grain Reynolds number, the dimensionless bed shear stress and the ratio of shear velocity to fall velocity.

Keywords: Bed forms, Ripples, Dunes, Transition regime, Antidunes.

NOTATIONS

- c bed forms celerity
- d grain diameter
- d_g geometric diameter
- d_{50} median diameter of bed mixture
- Fr Froude number of alluvial stream
- g gravitational acceleration
- H height of bed forms
- L Length of bed form
- R hydraulic radius
- R' hydraulic radius due to effective bed roughness
- R_b hydraulic radius due to bed resistance
- Re_* grain Reynolds number
- S longitudinal channel slope
- T_* dimensionless shear stress
- V average velocity
- u local velocity
- u_c critical local velocity at which sediment start to move
- u_* shear velocity
- u'_* effective friction shear velocity
- Y depth of flow
- δ thickness of laminar sublayer
- γ specific weight of fluid
- γ_s specific weight of sediment
- ω particle fall velocity
- ν kinematic viscosity
- τ average shear stress
- τ_c critical shear stress

INTRODUCTION

Any sediment movement in alluvial streams is usually associated with bed deformation. The Task Committee [12] has classified the bed forms into ripples, dunes, transition regime, antidunes and chutes pools
In river engineering, it is very important to predict the type of bed forms because it influences the velocity distribution, resistance of flow and rate of sediment transport.

Liu [8] studied the mechanism of ripples formation. He found that ripples are formed due to instability of the interface between sand and water. The beginning of ripples formation are governed by grain Reynolds number and u_*/ω. Where u_* is bed shear stress and ω is the fall velocity of the mean diameter of bed mixture. He developed a criterion for beginning of ripples formation shown by Figure (1). Liu stated that for coarse bed material ($Re_* > 100$) the ripples will form as soon as material starts moving. While for fine bed material ($Re_* < 100$), ripples will start to form at shear stress greater than the critical shear stress. However, Vanoni and Brooks [13] stated that the beginning of ripples formation curve should coincide with the curve for incipient motion.

Albertson, Simons and Richardson [1] stated that the type of bed forms depends on the ratio of sediment diameter "d" to the thickness of laminar sublayer δ. They found that ripples form when d/δ is less than 10, while dunes form when $10 < d/\delta < 40$. The transition regime occurs when $0.6 < Fr < 1.2$. They constructed a diagram (Figure 2) showing a criterion for ripples formation, dunes formation, transition regime and Antidunes formation. Their criterion for bed formation is a function of bed roughness "f", Reynolds number "Re" and bed grain diameter "d".

Engelund and Hassan [4] studied theoretically and experimentally the effect of the characteristics of the deformed bed on the hydraulic conditions. Their results were given in a stability criterion shown by figure (3) in which the abscissa designates the Froude number of the alluvial channel, while the ordinate is the ratio of average velocity "V" to the effective friction shear velocity u'_*. They defined the effective friction shear velocity as follows:

$$\frac{V}{u'_*} = \frac{V}{\sqrt{gR'S}} = 6 + 2.5\ln\left(\frac{R'}{2d_{65}}\right) \qquad (1)$$

In which S is the longitudinal channel slope and d_{65} is the grain diameter for which 65% of bed material is finer by weight.

According to figure (3), the increase of Froude number changes the feature of bed from ripples and dunes to plane bed which can be considered transition regime to antidunes.

Ranga Raaju and Soni [9] did experimental studies on the formation of bed undulation. They used the experimental data and field data from Luzince river and Japanese canal to obtain empirical equations describing the height "H" and the length "L" of bed undulation (as shown by figure (4)). Their relation for "H" and "L" is valid for grain diameter varying from 0.1 mm to 0.93 mm and were expressed as follows:

$$\frac{H}{d}\left(\frac{V}{\sqrt{gR}}\right)^3\left(\frac{V}{\sqrt{\left(\frac{\gamma_s}{\gamma}-1\right)d}}\right) = 6500\,T'^{\frac{8}{3}}_*$$

$$\frac{L}{d}\left(\frac{V}{\sqrt{gR}}\right)^3\left(\frac{V}{\sqrt{\left(\frac{\gamma_s}{\gamma}-1\right)d}}\right)\frac{R}{d} = 3\times10^8\,T'^{\frac{10}{3}}_* \qquad (2)$$

Where $T'_* = \dfrac{\gamma R'S}{(\gamma_s-\gamma)d}$, and R' are given by the following equation:

$$V = \frac{43.657}{d^{1/6}} R^{\frac{2}{3}} S^{\frac{1}{2}} \quad \text{fps}$$

Equation (2) shows that the size of bed wave increases with the increase of the dimensionless grain shear stress T_* and the size of the grains forming the bed material. Sumer and Bakioglu [11] used the results of a linear stability analysis for both smooth and transitional flows over erodible bed to develop a theory describing the formation of ripples. The theory was based on the assumption that the formation of ripples depends on the bed load transport and on the gravity force presented by the local inclination of the wavy bed surface. They found that the ripples will not occur when the grain Reynolds number is greater than 26 and the ripple length increases with the grain size.

Raudkivi and White [10] built a mathematical model to explain the development of bed features. Their model is based on the assumption that the initial bed consists of large number of arbitrary bed disturbance and the velocity of bed features is inversely proportion to their height. Since bed features have different heights and different velocities, the bed features will grow up because small features will join bigger ones till they reach their maximum height. Using the well known Exner equation for erosion and Fuehrboeter formula for rate of sediment transport the following equation was obtained to determine the maximum height of a dune:

$$\frac{H}{Y} = \left[\left(\frac{2.3V-1.6}{2.2}\right)^2 + \frac{V-1}{2.2}\right]^{1/2} - \frac{2.3V-1.6}{2.2} \qquad (3)$$

Where "H" is the maximum height of the bed feature. Equation (3) is valid only for Froude numbers less than 0.3 and specific weight of grains equals to 2.65 gm/cm^3. Their model does not differentiate between ripples and dunes.

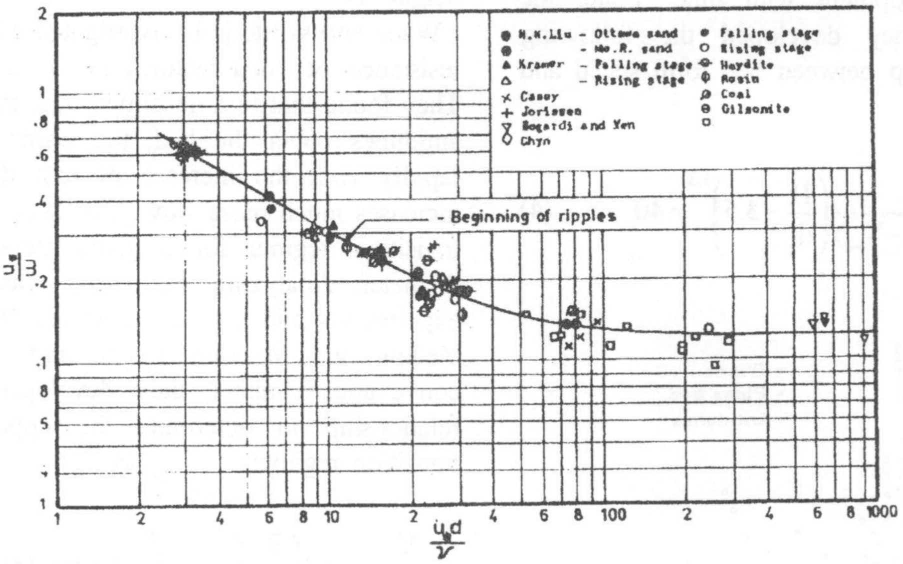

Figure 1. Criterion of ripples formation after Liu [8].

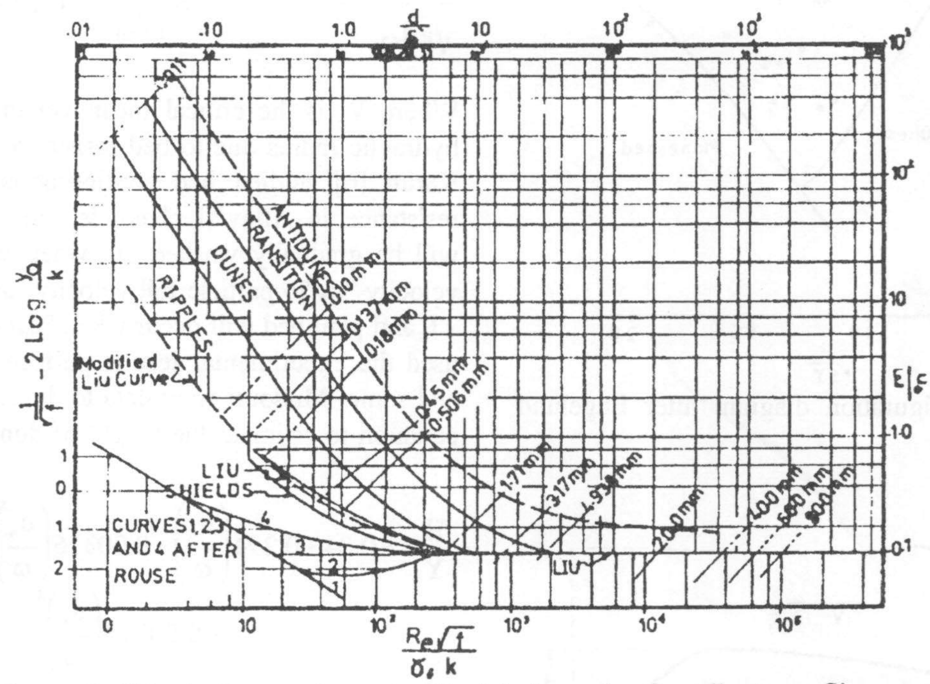

Figure 2. Criteria for roughness in alluvial channels after albertson, Simons and Richardson [1].

Coleman and Melville [3] studied experimentally the evolution of bed features subsequent to their initial generation from flat bed conditions. They examined experimentally the mathematical model developed by Raudkivi and White and found that the convergence of two bed forms of different sizes did not result in coalescence of these two features at every instance.

They concluded that bed form coalescence was clearly dictated by local bed form spacing and the potential flow theory could provide a comprehensive description of bed feature development with time. Using the experimental data, they developed the following quantitative relationship between bed form speed and bed form height:

$$\frac{c}{(u_* - u_{*c})(T_* - T_{*c})}\left(\frac{H}{d_g} - 3.5\right)^{1.3} = 40 \qquad (4)$$

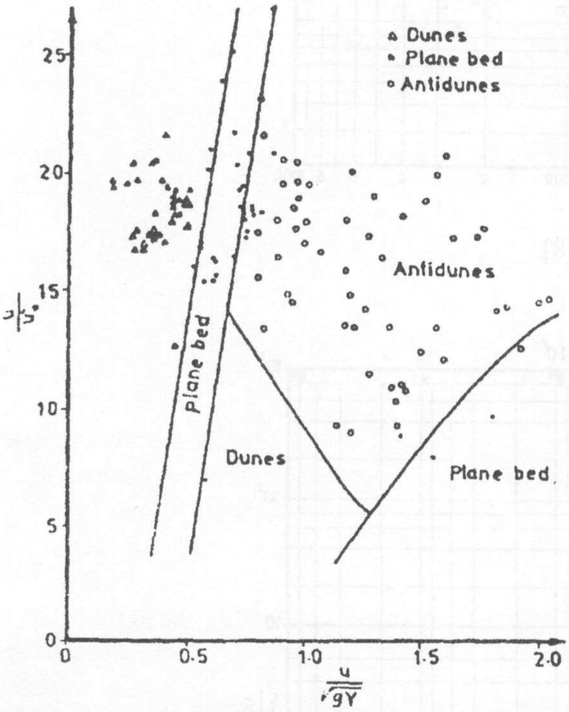

Figure 3. Bed configuration diagram after Engelund [4].

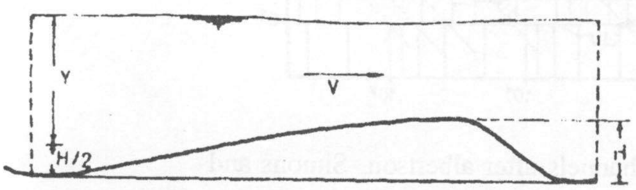

Figure 4. Definition sketch.

Where c is bed form celerity, d_g geometric mean size diameter which is defined as $d_g = (d_{16} d_{84})^{0.5}$. Equation (4) complies with Raudkivi assumption in which bed form velocity decreases with the increase of bed form height "H".

Wang and White [14] investigated experimentally the resistance of bed features in the transition regime. They found that in a deformed bed; in which dunes or antidunes cover the bed, the water depth increases rapidly with the increase of unit discharge, but it increases much more slowly, or even decreases in the transition regime for a given slope and sediment diameter. Analyzing the results obtained from their experiments and the data collected from the Yangtze Yellow and Missouri rivers, and the Rio Grande conveyance channel, they developed the following relationship to determine the upper limit of the transition regime:

$$\frac{V_c}{\sqrt{g d_{50}}} = 2.8\left(\frac{R_b}{d_{50}}\right)^{0.3} \quad ;\text{for}\left(\frac{R_b}{d_{50}} \le 10{,}000\right)$$
$$\frac{V_c}{\sqrt{g d_{50}}} = 44.4 \quad ;\text{for}\left(\frac{R_b}{d_{50}} > 10{,}000\right) \qquad (5)$$

Where V_c is the critical mean velocity and R_b is the hydraulic radius due to bed resistance.

Karim [6] studied bed configurations and hydraulic resistance in Alluvial channels. He found that dunes will be gradually washed out when the ratio of shear velocity "u_*" to particle fall velocity "ω" is greater than 2.0 and plan bed will occur when "u_*/ω" equals 3.6. He used the experimental results performed by Guy et al and some Missouri river data to develop the following equation to estimate the height of dunes:

$$\frac{H}{Y} = -0.04 + 0.294\left(\frac{u_*}{\omega}\right) + 0.00316\left(\frac{u_*}{\omega}\right)^2 - 0.0319\left(\frac{u_*}{\omega}\right)^3 + 0.00272\left(\frac{u_*}{\omega}\right)^4 \qquad (6)$$

Equation (6) is valid for $0.15 < u_*/\omega < 3.64$.

The present study investigates only the prediction of bed forms in dynamic equilibrium state, where the flow is steady and the size of bed forms is constant and is transported either to downstream in case of bed covered with ripples and dunes or to the upstream direction in case of antidunes.

DEVELOPMENT OF BED FORMS

According to Vanononi and Brooks [13] study, grain particles will be entrained at incipient motion conditions causing disturbance at bed surface and the bed roughness will increase. When the bed shear stress is greater than the critical shear stress, the entrained particles will be transported. The perturbed bed will hinder some of the transported particles forming a small undulation. This undulation will grow up to form ripples or dunes. If the flow at incipient motion is smooth turbulent flow ($Re_* < 11.6$), only very fine particles will be entrained. In this case, the surface deformation is too small to be observed. Therefore, the initiation of ripples at Reynolds number less than 11.6 are recognized at boundary shear stress greater than the critical shear stress.

According to equations (2) and (3), ripples will grow up with the increase of the boundary shear stress till they reach a certain height. For a further increase of boundary shear stress, sand waves will be washed out and plane bed can be formed. This condition is called lower limit of transition regime. For this condition dunes or ripples are removed because all grains forming the sand wave are subjected to local velocity equal or greater than the critical velocity.

To determine the shear stress at which bed features will be washed out, it is assumed that this condition will happen when the local velocity at distance 0.1 of the height of bed undulation equals the local critical velocity.

Using the well known Prandtl-Karman velocity distribution equation for rough turbulent flow, and assuming the equivalent bed roughness of the boundary equals the maximum height of bed features "H", the velocity distribution equation can be written as follows:

$$\frac{u}{u_*} = 5.75 \log\left(30.1 \frac{y}{H}\right) \tag{7}$$

Where u is the local velocity at distance "y" from the bed.

Garde [5] expressed the critical velocity for rough turbulent flow as follows:

$$u_c = 1.51 \sqrt{(\gamma_s - \gamma)\frac{d}{\rho}} \tag{8}$$

Substituting equation (8) into equation (7) at y = 0.1 H, the dimensionless shear stress at which bed undulation starts washing is:-

$$T_* = \frac{\tau}{(\gamma_s - \gamma) d} = 0.60 \tag{9}$$

Where τ is the average shear stress required to wash out dunes having grains of diameter "d". Equation (9) means that the coarser the grains the greater is the shear stress required to wash out bed undulation. Consequently, the maximum height of bed undulation increases with the increase of grain diameter because the height of bed undulation increases with the increase of shear stress (equations 2 and 6). Since the maximum height of ripples is about 3 cm (Task Committee [12]), ripples constructed of very fine sand may be washed out before they grow to dunes.

According to Kennedy [7] antidunes are formed when the free water surface of the stream is disturbed. The free surface of the stream will be disturbed when Froude number is equal or greater than 0.8. This case is called the upper limit of transition regime. If the bed material consists of fine sand only, the dunes can be washed out at Froude number less than unity. Sand dunes constructed of coarse sand needs higher boundary shear stress to remove it. The value of Froude number may reach unity before the dunes are washed away. In this case dunes are directly transformed to antidunes. Therefore, the range between the upper and lower limit of transition regime decreases with the increase of grain diameter.

PREDICTION OF THE TYPE OF BED FORMS

According to the above analysis, ripples and dunes will form only when Froude Number is less than unity and the boundary shear stress is greater than the critical shear stress of the grains forming the bed. Therefore, Shields criteria for incipient motion can be used as a criterion for initiation of bed form which is expressed as follows:

$$\frac{\tau_c}{(\gamma_s - \gamma)d} = f(Re_*) \quad (10)$$

Using the least square method and the readings obtained from Shields diagram, the function of equation (10) can be expressed in a polynomial form as follows:-

$$\log\left(\frac{\tau_c}{(\gamma_s - \gamma)d}\right) = -0.0442 \times \log^6(Re_*) + 0.4218 \times \log^5(Re_*) - 1.5085 \times \log^4(Re_*) + 2.349 \times \log^3(Re_*) - 1.0479 \times \log^2(Re_*) - 0.7965 \times \log(Re_*) - 0.8841 \quad (11)$$

The correlation coefficient of equation (11) is 0.98. Equation (11) is valid for Re_* less than 500. For Re_* equals or greater than 500, $T_* = \tau/(\gamma_s - \gamma)\ d = 0.06$ According to the foregoing review, the height of bed form "H" depends on the depth of flow "Y", the average velocity "V", the shear velocity "u_*", the average bed shear stress "τ", the median diameter of bed mixture "d_{50}", the submerged specific weight of bed sediments "$(\gamma_s - \gamma)$", the particle fall velocity "ω", the kinematic viscosity of fluid "ν", and gravitational acceleration "g". The height of bed form can be expressed as follows:

$$H = f(Y, V, u_*, d_{50}, (\gamma_s - \gamma), \omega, \nu, g) \quad (12)$$

Applying Buckingham theorem (π- theorem), equation (12) can be written as follows:

$$\frac{H}{Y} = f_1\left(Fr, Re_*, \frac{\tau}{(\gamma_s - \gamma)d_{50}}, \frac{u_*}{\omega}, \frac{u_*}{V}\right) \quad (13)$$

In which Fr is the Froude number of alluvial stream and $Re_* = u_* d_{50}/\nu$. The function of equation (13) can be determined using the available data.

BED FORMS DATA

The source of data used in this study was collected by Brownlie [2]. His report contains 5263 laboratory records and 1764 field records collected from 77 references. Each record consists of observation of 10 basic parameters. These parameters are discharge, channel width, average depth of flow, energy slope, median particle size of bed material, geometric standard deviation of bed mixture, specific gravity, concentration of sediment load, temperature, and the condition of bed form. Not all observation parameters were available to Brownlie and he was not certain of some of them.

In the present study only records for water depths greater than 20 cm and records containing complete data of undulated bed were used. The sum of the utilized records is 677.

Also results of the experiments conducted by Wang and White [14] were used in the analysis in the present study. These experiments were performed on two sediment mixtures having specific weight 2.65 t/m^3 with a "d_{50}" of size 0.076 mm and 0.76 mm and geometric standard deviation of 1.2 and 1.12, respectively. In each run, they measured the rate of flow, water depth, longitudinal slope of the alluvial bed, height, length, and the celerity of bed forms.

ANALYSIS OF THE COLLECTED DATA

For each collected record, the values of Froude number, the average shear stress, the shear velocity, and grain Reynolds number were calculated. Also the critical shear stress of the median diameter was determined using Shields criteria for incipient motion. The fall velocity was also calculated using Rubey's [5] formula which is expressed as follows:

$$\omega = \sqrt{\frac{36\nu^2}{d^2} + \frac{2}{3}\frac{(\gamma_s - \gamma)d}{\rho}} - \frac{6\nu}{d} \quad (14)$$

Where ν is the kinematic viscosity of water and ω is the particle fall velocity.

Based on the collected data, the relationship between grain Reynolds number "Re_*" and the dimensionless shear stress "T_*" is shown in Figure (5). According to Figure (5), ripples are formed at Re_* less than 16 and T_* less than 1.0, while dunes are formed at Re_* less than 200 and greater than 20. The transition regime is found at dimensionless shear stress equals 0.6. The standing waves and antidunes are formed at T_* greater than 0.8.

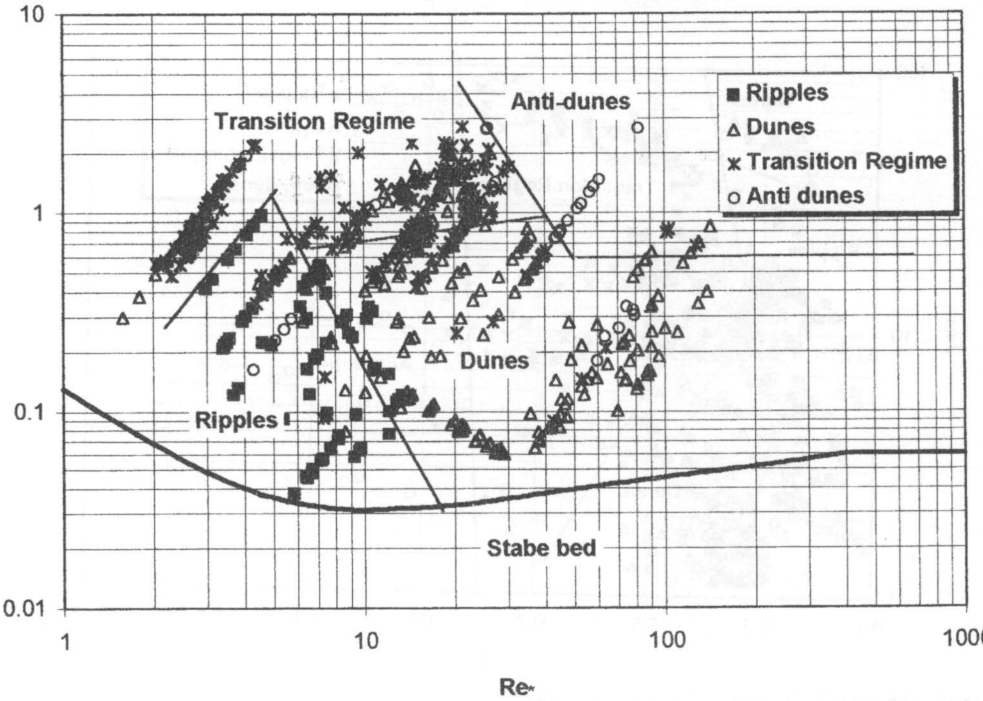

Figure 5. Criterion for bed formation as a function of grain Reynolds number and the dimensionless average shear stress (Brownlie [2] data).

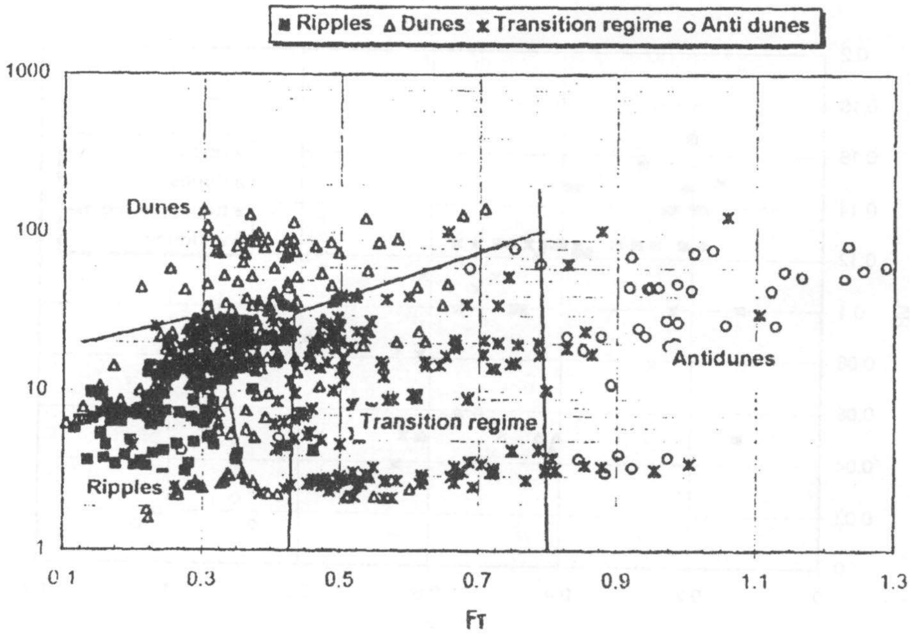

Figure 6. Criterion for bed form s a function of Froude number and grain Reynolds number (Broenlie [2] data).

The relationship between Froude number and grain Reynolds number is shown in Figure (6). It can be concluded from Figure (6) that ripples are formed at Froude number less than 0.3 and Re_* less than 16 and dunes can be formed at Froude number less than 0.42. At Re_* less than 16, the bed forms are washed out to plane bed and the so called transition regime occurred at Froude number less than 0.8 and greater than 0.45.

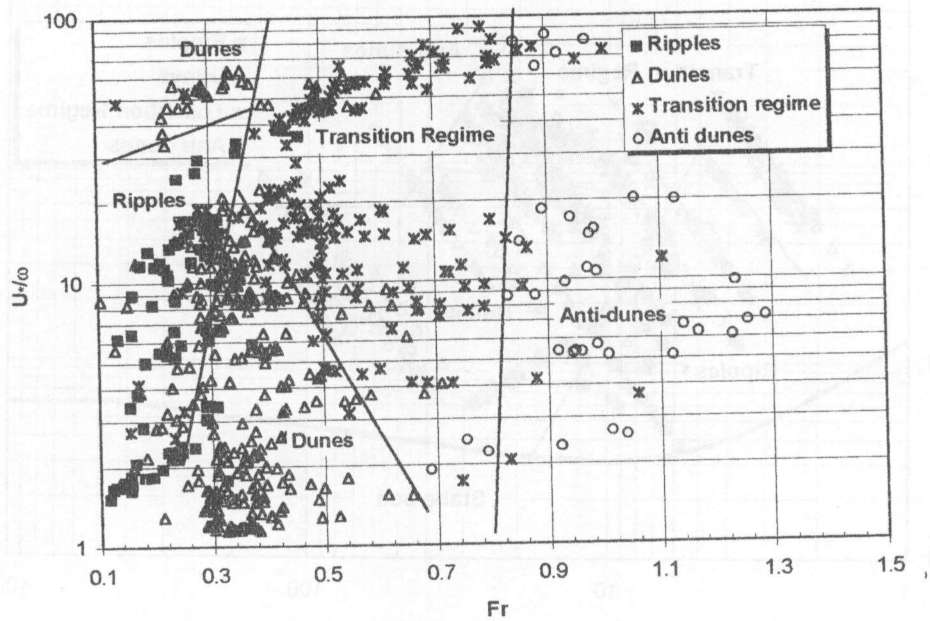

Figure 7. Criterion for bed forms as a function of Froude number and the ratio of shear velocity to particle fall velocity (Brownlie [2] data).

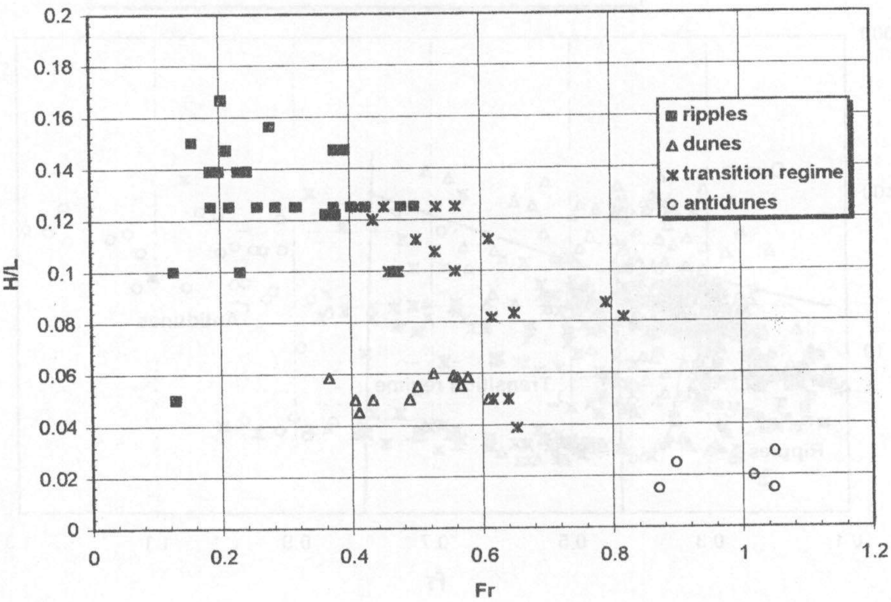

Figure 8. Variation of Froude number with the ratio of height to length of bed form (Wang & White [14] data).

At Froude number equals and greater than 0.8 anti dunes start to form.

The variation of the ratio of shear velocity to particle fall velocity "u_*/ω" and Froude number of alluvial stream is shown in Figure (7). In which ripples are formed for Froude numbers less than 0.45 and u_*/ω less than 20. Dunes are formed at u_*/ω greater than 20 and Froude number less than 0.45. Also dunes are formed at u_*/ω less than 20 and Froude number greater than 0.25 and less than 0.7. The transition regime is found to form at "u_*/ω" greater than 4.0 for Froude numbers greater than 0.45 and less than 0.8. Antidunes are created at Froude number greater than 0.8 independently of the ratio of "u_*/ω".

Using the data collected by Wang and White [14], the relationship between the ratio of bed form height to its length "H/L" and Froude number is given in Figure (8). It can be concluded from Figure (8) that the geometric configurations of bed forms can be characterized according to the ratio of bed form height to its length as follows: ripples have H/L greater than 0.12 and dunes have H/L greater than 0.04 and less than 0.06 while anti dunes have H/L greater than 0.01 and less than 0.03.

Based on the data collected by Wang and White [14], a comprehensive multiple regression analysis was carried out to obtain the following best fit equation expressing the geometry of bed forms:

$$\frac{H}{L} = \frac{0.057931}{Re_*^{0.1251}\ Fr^{0.71}} \qquad (15)$$

The correlation coefficient of equation (15) is 0.91 and the average percentage standard error of estimate is 13.2%. The plot of the actual and the estimated ratio of H/L is shown in Figure (9).

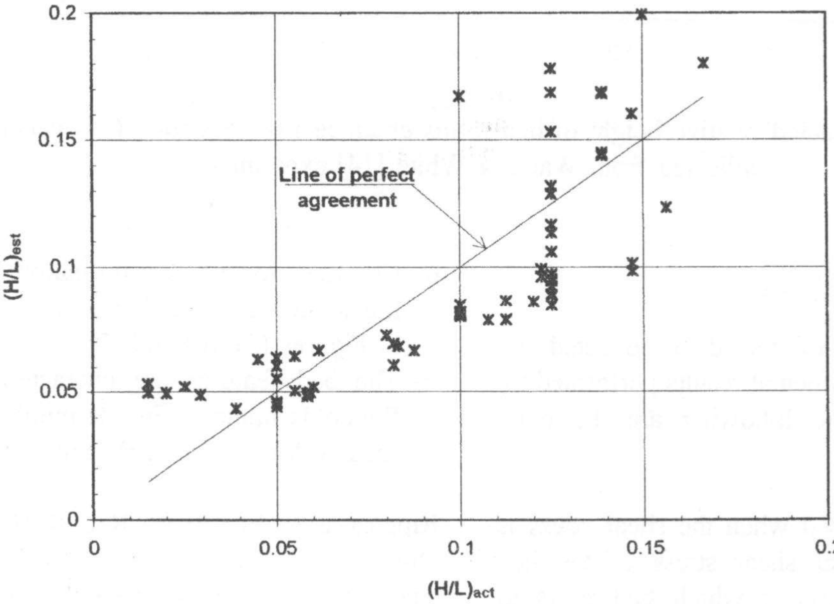

Figure 9. Plot of estimated ratio of height to length of bed forms obtained from equation (15) versus actual ratio collected by Wang & White [14].

Also a multiple regression analysis was carried out to obtain the following best fit equation to determine the height of ripples or dunes:

$$\frac{H}{Y} = 0.000632\,\frac{Fr^{0.74}}{T_*^{1.917}} \times Re_*^{1.451} \times \left(\frac{u_*}{\omega}\right)^{2.431} \qquad (16)$$

The correlation coefficient of equation (16) is 0.963 and the percentage average standard error of estimate is 33.8%. The plot of the actual and the estimated values of the ratio "H/Y" is given in Figure (10). Equation (15) and (16) is valid for depth of flow Y, 0.10-0.365 m and slope S, 0.011×10^{-3}-3.054×10^{-3}, and for the range of experimental runs performed by White and Wang [14]. According to equation (16), the height of bed form increases with the increase of Froude number, grain Reynolds number and decreases with the increase of particle fall velocity.

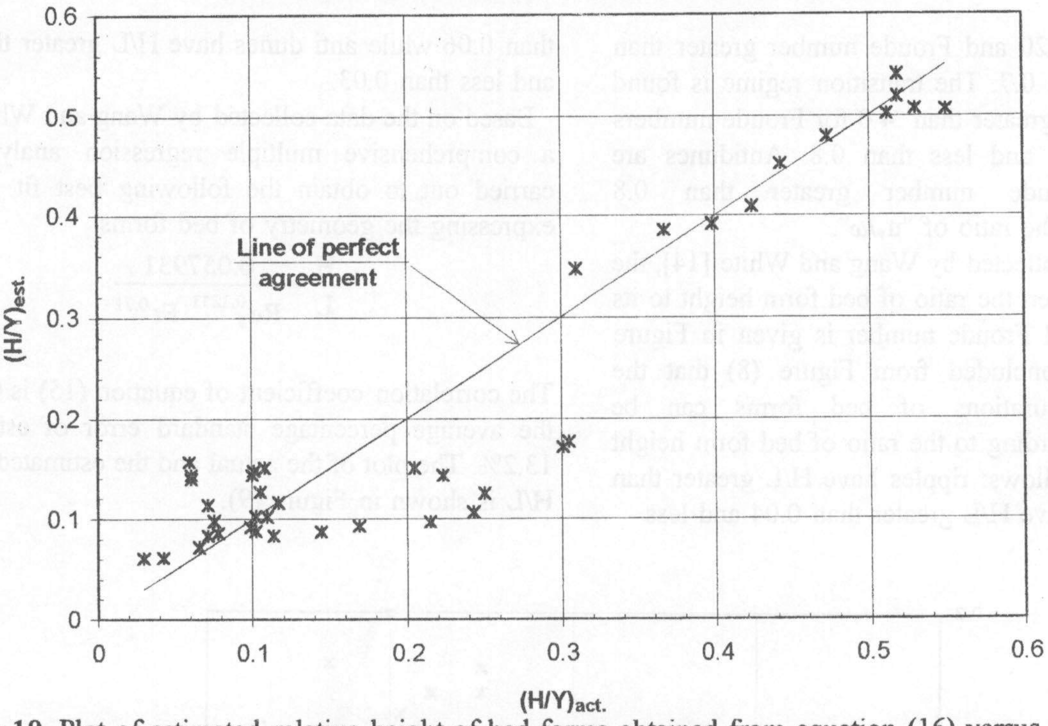

Figure 10. Plot of estimated relative height of bed forms obtained from equation (16) versus actual values collected from Wang & White [14] experiments.

CONCLUSIONS

Based on the analysis of the data collected by Brownlie [2] and the experimental results performed by Wang and white [14], the following are the main conclusions:

1- Bed Forms are developed when the shear stress is greater than the critical shear stress of the bed mixture. The shear stress at which bed starts to deform can be determined from equation (11) for Re_* less than 500.
2- The height of ripples and dunes increases with the increase of grain Reynolds number and flow Froude Number.
3- Ripples and dunes are washed out when the dimensionless bed shear stress is equal or greater than 0.6 and Froude number is less than unity.
4- The coarser the grains the greater is the maximum height of bed forms.
5- For the range experimental runs performed by Wang and White [14], the height and length of bed undulations can be determined using equations (15) and (16).
6- Criterion for ripples formation, dunes formation, transition regime, and antidunes formation are given in Figures (5), (6) and (7).
7- The bed features are characterized by the grain Reynolds number, Froude number, and the ratio of shear velocity to particle fall velocity as follows:

Ripples at	$Fr < 0.30$, $Re_* \leq 16.0$, $u_*/\omega < 3.0$
Dunes at	$0.30 \leq Fr < 0.45, 16 < Re_* \leq 200, u_*/\omega > 20.0$
Transition at	$0.45 \leq Fr < 0.80$, $Re_* \leq 16.0, u_*/\omega > 4.0$
Antidunes at	$Fr \geq 0.8$

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REFERENCES

[1] M. Albertson, D. Simons and E. Richardson" Discussion of Mechanics of sediment ripples

formation", *Journal of Hydraulic Engineering, ASCE*, HY1, February 1958.

[2] W. Brownlie, "Compilation of Alluvial Channel Data: Laboratory and Field", *California institute of Technology, Pasadena*, report No. KH-R-43B, November 1981.

[3] S. Coleman and B. Melville, "Bed-Form Development", *Journal of Hydraulic Engineering, ASCE*, vol. 120, No. 4, April 1994.

[4] F. Engelund, "Hydraulic Resistance of Alluvial Streams", *Journal of Hydraulic Engineering, ASCE*, vol. 92, No. HY2, March 1966.

[5] R.J. Garde and K. Ranga Raju, "Mechanics of sediment transport and Alluvial stream problems", *Published by WILY EASTERN LIMITED*, 1977.

[6] F. Karim, "Bed Configuration and Hydraulic Resistance in Alluvial-Channel Flows", *Journal of Hydraulic Engineering, ASCE*, vol. 121, No. 1, January 1995.

[7] J.F. Kennedy, "The Mechanics of Dunes and Antidunes in Open Channels.", *Journal of Hydraulic Engineering, ASCE*, vol. 92, No. 2, March 1966.

[8] H.K. Liu, "Mechanics of Sediment-Ripple Formation", *Journal of Hydraulic Engineering, ASCE*, vol. 83, No. 2, April 1957.

[9] K. Ranga Raju and J. Soni, "Geometry of Ripples and Dunes in Alluvial Channels", *Journal of Hydraulic Engineering, ASCE*, vol. 14, No. 3, March 1976.

[10] A. Raudkivi and H. White, "Development of Bed Features", *Journal of Hydraulic Engineering, ASCE*, vol. 116, No. 9, September 1990.

[11] B. Summer and Bakioglu, "On The Formation of Ripples on an Erodible Bed", *Journal of Fluid Mech.*, vol. 144 pp. 177-190, January 1984.

[12] The Task Force on Bed Forms In Alluvial Channels of the Committee on Sedimentation, "Nomenclature For Bed Forms in Alluvial Channels", *Journal of Hydraulic Engineering, ASCE*, vol. 92, No. 3, May 1966.

[13] V. Vanoni and N. Brooks, "Discussion on Mechanics of Sediment-ripple Formation", *Journal of Hydraulic Engineering, ASCE*, No. HY-1, February 1958.

[14] Wang S. and White R.W., "Alluvial Resistance in Transition Regime", *Journal of Hydraulic Engineering, ASCE*, vol. 119, No. 6, June 1993.

formation", *Journal of Hydraulic Engineering*, ASCE, HY1, February 1958.

[2] W. Brownlie, "Compilation of Alluvial Channel Data, Laboratory and Field", *California Institute of Technology*, Pasadena, report No. KH-R-43B, November 1981.

[3] S. Coleman and B. Melville, "Bed-Form Development", *Journal of Hydraulic Engineering*, ASCE, vol. 120, No. 4, April 1994.

[4] F. Engelund, "Hydraulic Resistance of Alluvial Streams", *Journal of Hydraulic Engineering*, ASCE, vol. 92, No. HY2, March 1966.

[5] R.J. Garde and K. Ranga Raju, "Mechanics of sediment transport and Alluvial stream problems", *Published by WILEY EASTERN LIMITED*, 1977.

[6] F. Karim, "Bed Configuration and Hydraulic Resistance in Alluvial-Channel Flows", *Journal of Hydraulic Engineering*, ASCE, vol. 121, No. 1, January 1995.

[7] J.F. Kennedy, "The Mechanics of Dunes and Antidunes in Open Channels", *Journal of Hydraulic Engineering*, ASCE, vol. 92, No. 2, March 1966.

[8] H.K. Liu, "Mechanics of Sediment-Ripple Formation", *Journal of Hydraulic Engineering*, ASCE, vol. 83, No. 2, April 1957.

[9] K. Ranga Raju and J. Soni, "Geometry of Ripples and Dunes in Alluvial Channels", *Journal of Hydraulic Engineering*, ASCE, vol. 14, No. 2, March 1976.

[10] A. Raudkivi and H. White, "Development of Bed Features", *Journal of Hydraulic Engineering*, ASCE, vol. 116, No. 9, September 1990.

[11] B. Summer and Bakioglu, "On The Formation of Ripples on an Erodible Bed", *Journal of Fluid Mech.*, vol. 144 pp. 177-190, January 1984.

[12] The Task Force on Bed Forms in Alluvial Channels of the Committee on Sedimentation, "Nomenclature For Bed Forms in Alluvial Channels", *Journal of Hydraulic Engineering*, ASCE, vol. 92, No. 3, May 1966.

[13] V. Vanoni and N. Brooks, "Discussion on Mechanics of Sediment-ripple Formation", *Journal of Hydraulic Engineering*, ASCE, No. HY-1, February 1958.

[14] Wang S. and White R.W., "Alluvial Resistance in Transition Regime", *Journal of Hydraulic Engineering*, ASCE, vol. 119, No. 6, June 1993.