

# OBLIQUE SURFACE WAVES IN THE PRESENCE OF POROUS PLATES

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## ABSTRACT

A linearised surface wave motion is considered for a fluid of infinite extent and of infinite or finite depth in the presence of an impermeable plate and a porous wall immersed in the fluid parallel to each other. The motion is generated by small horizontal oscillations of the impermeable plate or the porous wall oblique to its plane. The scattering problem of a time-harmonic wave incident obliquely on the porous plate which is kept fixed is considered also. The effect of porosity and the angle of incident on the reflection and transmission coefficients are discussed.

*Keywords: Surface waves, Porous wall, Oblique waves.*

## 1. INTRODUCTION

The scattering of surface waves obliquely incident on a partially immersed or completely submerged vertical barrier and plate in deep water was studied by Faulkner [1,2], Jarvis and Taylor [3], Evans and Morris [4], Rhodes-Robinson [5] and Mandel and Goswami [6], employing different methods. (In these works the vertical barrier was represented by a vertical impermeable plate).

Chwang [7] considered a porous wave maker oscillating normally to its surface with constant amplitude. In this linearised analysis, the wave maker is located in the middle of an infinitely long channel with constant depth.

Chwang and Li [8] applied the linearised porous wave maker method developed in [7] to investigate the small amplitude surface waves produced by piston-type porous wave maker near the end of semi-infinitely long channel of constant depth. Gorgui and Faltas [9] considered the wave motion for a fluid of infinite extent and of infinite or finite constant depth in the presence of an impermeable plate and a porous wall immersed in the fluid parallel to each other. The motion is generated by small horizontal oscillations of the impermeable plate or porous wall normally to the surface of the impermeable plate or porous wall.

In the present paper we propose to investigate the two-dimensional gravity waves. The waves are generated by arbitrary prescribed horizontal oscillations oblique to the plane of a porous wall or an impermeable plate. The boundary condition on the

surface of the porous plate is derived on the basis of Taylors assumption [10] that the relative velocity normal to the porous wall is linearly proportional to the difference in pressure on the two sides of the wall.

## 2. OBLIQUE WAVES GENERATED IN SEMI-INFINITELY LONG-CHANNEL

Assume irrotational motion of fluid with a free surface. This fluid also is assumed to be an incompressible and inviscid flow under the action of gravity; surface tension is neglected. Cartesian axes are chosen so that  $y$  is directed vertically downwards and  $x, z$  are in the plane of the free surface. The wave motion is induced by an impermeable plate oscillating horizontally about its mean position at the end of a semi-infinitely long channel. We assume that the oscillations of the plate are periodic in time and in  $z$ -direction, let its velocity at time  $t$  be  $V U(y) \exp(-i\omega t + i\nu z)$  where  $\nu = K \sin \beta$  in which  $K = \omega^2/g$ ,  $\beta$  is the angle that the produced train of waves makes with  $x$ -axis,  $g$  is the gravitational constant,  $U(y)$  is complex valued and suitably limited. At a distance  $d$  from the plate a porous wall is fixed in the fluid parallel to the plate see Figure (1). The resulting fluid motion is therefore time and  $z$  harmonics with the same  $\omega$  and  $\nu$  as that of the plate, respectively. The motion is assumed small so that linearisation is permissible. We consider first the case when the fluid is of infinite depth. When the initial transients disappear, the

subsequent motion can be conveniently described by velocity potentials

$$\bar{\Phi}_j(x,y,z;t) = \text{Re} \{ \phi_j(x,y) \exp(-i\omega t + i v z) \}, j=1,2$$

refer to the regions  $0 < x < d, x < 0$ , respectively, and the functions  $\phi_j$  satisfy

$$\phi_{jxx} + \phi_{jyy} - v^2 \phi_j = 0 \text{ in } y > 0 \quad (2.1)$$

subject to the free surface condition

$$K \phi_j + \phi_{jy} = 0 \text{ on } y = 0 \quad (2.2)$$

On the impermeable plate we have

$$\phi_{1x} = V U(y) \text{ on } x = d \quad (2.3)$$

and on the porous wall

$$\phi_{1x} = \phi_{2x} \text{ on } x = 0 \quad (2.4)$$

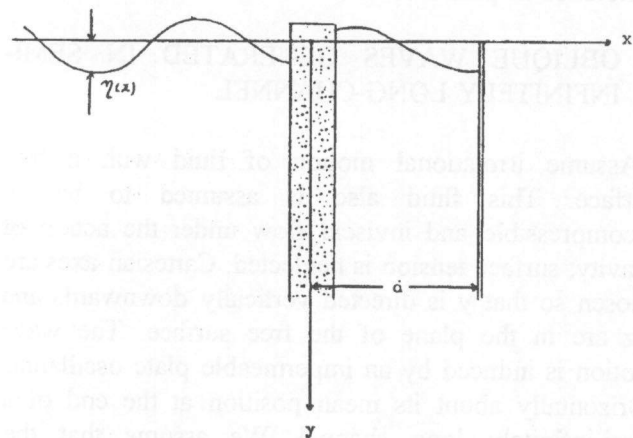


Figure 1. Schematic diagram of a porous wall parallel to an impermeable plate.

We shall assume that the porous wall is made of material with very fine pores. Thus according to Taylor's assumption [10], we have

$$\phi_{jx} = i G (\phi_1 - \phi_1) \text{ on } x = 0 \quad (2.5)$$

Where  $G = \rho \omega b / \mu^*$ ,  $\mu^*$  is the dynamic viscosity,  $\rho$  is the constant density of the fluid and  $b$  is coefficient which has the dimension of length. It should be noted that if

the porous flow through the plate is significant, Taylor's assumption and condition (2.5) may be not accurate enough. Hence we should confine our investigation to porous walls with fine pores.

Finally we have the conditions for no motion at infinite depth

$$\nabla \phi_j \rightarrow 0 \text{ as } y \rightarrow \infty \quad (2.6)$$

and the radiation condition for the outgoing waves

$$\phi_2 \rightarrow C e^{-i\mu x - Ky} \text{ as } x \rightarrow -\infty, y > 0 \quad (2.7)$$

Where  $C$  is a constant and  $\mu = K \cos \beta = \sqrt{K^2 - v^2}$ .

The relevant physical parameters of the problem are the distance  $d$ , the wave number  $K$ , the angle of incident waves  $\beta$  and the porous effect parameter  $G$ .

### 3. SOLUTION OF THE PROBLEM

Using the method of separation of variables and superposing basic solutions of Laplace's equation (2.1) appropriate to the present case, let

$$\phi_1 = \int_0^\infty [ A(k) e^{\lambda(k)x} + B(k) e^{-\lambda(k)x} ] f(k,y) dk + (\alpha e^{i\mu x} + \beta^* e^{-i\mu x}) e^{-Ky}, \quad (3.1)$$

$$\phi_2 = \int_0^\infty C(k) e^{\lambda(k)x} f(k,y) dk + \gamma e^{-i\mu x - Ky}, \quad (3.2)$$

where  $f(k,y) = k \cos ky - K \sin ky$ ,

$$\lambda(k) = \sqrt{k^2 + v^2}.$$

Equations (3.1), (3.2) satisfy all the conditions of the boundary value problem (2.1)-(2.7) except (2.3), (2.4), (2.5). Conditions (2.4), (2.5) are satisfied if

$$A - B = C, \quad \alpha - \beta = -\gamma$$

$$iG(C-A-B) = \lambda(k) C, \quad G(\gamma - \alpha - \beta^*) = -\mu \gamma$$

Solving the above equations to find A and B in terms of C and  $\alpha, \beta^*$  in terms of  $\gamma$  and then substituting in (2.3) we get

$$V U(y) = \int_0^\infty C \frac{\lambda(k)}{iG} f(k,y) \Delta(i\lambda(k)) dk - \frac{\gamma \mu}{G} \Delta(\mu) e^{-Ky} \quad (3.3)$$

Where

$$\Delta(\mu) = iGe^{-i\mu d} + \mu \sin \mu d. \quad (3.4)$$

Multiplying both sides of (3.3) by  $e^{-Ky}$  and integrating with respect to y from 0 to  $\infty$ , we get

$$\gamma = \frac{2\pi K V G A}{\mu \Delta(\mu)},$$

where  $A = -\frac{1}{\pi} \int_0^\infty U(y) e^{-Ky} dy.$

But U(y) has the unique expansion

$$U(y) = 2 \int_0^\infty \frac{ka(k)}{k^2 + K^2} f(k,y) dk - 2\pi K A e^{-Ky}, \quad (3.5)$$

where  $a(k) = -\frac{1}{\pi k} \int_0^\infty U(y) f(k,y) dy$

Which can be easily proved by straight forward application of the Fourier sine of U(y). Comparing (3.3) with (3.5) we get

$$C(k) = \frac{-2ikVGa(k)}{(k^2 + K^2) \Delta(i\lambda(k)) \lambda(k)}$$

Hence

$$\begin{aligned} \phi_1 = & -2V \int_0^\infty \frac{ka(k)f(k,y)}{\lambda(k)(k^2 + K^2) \Delta(i\lambda(k)) - \lambda(k) \cosh(\lambda(k)x)} [ie^{\lambda(k)x} \\ & + \frac{2\pi VAK}{\mu \Delta(\mu)} [Ge^{-i\mu x} + \mu \cos \mu x] e^{-Ky}, \quad (3.5.1) \end{aligned}$$

$$\begin{aligned} \phi_2 = & -2iVG \int_0^\infty \frac{ka(k)f(k,y)}{\lambda(k)(k^2 + K^2) \Delta(i\lambda(k))} e^{\lambda(k)x} dk \\ & + \frac{2\pi V GK}{\mu \Delta(\mu)} e^{-i\mu x - Ky}. \quad (3.5.2) \end{aligned}$$

The free surface elevation given by  $\eta_j = \frac{-i\omega}{g} \phi_j(x,0)$  is

$$\begin{aligned} \frac{\omega}{V} \eta_1 = & 2iK \int_0^\infty \frac{k^2 a(k)}{\lambda(k)(k^2 + K^2) \Delta(i\lambda(k)) - \lambda(k) \cosh(\lambda(k)x)} [iGe^{\lambda(k)x} \\ & - \frac{2\pi iK^2 A}{\mu \Delta(\mu)} [Ge^{-i\mu x} + \mu \cos \mu x], \quad (3.6.1) \end{aligned}$$

$$\begin{aligned} \frac{\omega}{V} \eta_2 = & 2GK \int_0^\infty \frac{k^2 a(k)}{\lambda(k)(k^2 + K^2) \Delta(i\lambda(k))} e^{\lambda(k)x} dk \\ & - \frac{2\pi iAGK^2}{\mu \Delta(\mu)} e^{-i\mu x} \quad (3.6.2) \end{aligned}$$

These results represent the incident and reflected waves in the finite region  $0 < x < d$  and the transmitted waves propagating in the unbounded region  $x < 0$ .

The amplitude of the free surface gravity waves given by (3.6.2) varies with distance from the porous wall because of the contribution from the non-propagating wave represented by the integral term. As  $|x| \rightarrow \infty$  this contribution tends to zero as  $\frac{1}{x}$  and the wave amplitude approaches to the constant quantity  $\frac{2\pi GK^2}{\mu} \left| \frac{AV}{\Delta(\mu)} \right|$ . It is evident that for any given values of G this amplitude decreases as K increases. On the other hand for any given value of K this amplitude increases monotonically from 0 to  $2\pi |AV| K^2 \mu$  as G increases from 0 to  $\infty$ .

When the porous wall is completely permeable ( $G \rightarrow \infty$ ), the velocity potential in the region  $x < d$  is

$$\phi = -2V \int_0^\infty \frac{ka(k)f(k,y)}{\lambda(k)(k^2 + K^2)} e^{\lambda(k)(x-d)} dk$$

$$-\frac{2\pi i V A K e^{-Ky}}{\mu} e^{-i\mu(x-d)} \quad (3.7)$$

This results represent the waves generated by the oblique oscillations of a wave maker in the unbounded region  $x < d$ .

For an impermeable wall ( $G=0$ ), the results (3.5.1), (3.5.2) reduce to

$$\begin{aligned} \phi_1 = & -2V \int_0^\infty \frac{ka(k)f(k,y)}{\lambda(k)(k^2+K^2)} \frac{\cosh(\lambda(k)x)}{\sinh(\lambda(k)d)} dk \\ & + \frac{2\pi VAK}{\mu} \frac{\cos \mu x}{\sin \mu d} e^{-Ky}, \end{aligned} \quad (3.8.1)$$

$$\phi_2 = 0 \quad (3.8.2)$$

This solution is valid only when  $\mu d \neq s\pi$ , where  $s$  is an integer. However it indicates that when  $\mu d = s\pi$ , resonance occurs and the linearised theory for small motion cannot be applied.

In the particular case when  $U(y) = e^{-Ky}$  we have

$$\phi_1 = \frac{-V}{\mu \Delta(\mu)} (G e^{-i\mu x} + \mu \cos \mu x) e^{-Ky}, \quad (3.9.1)$$

$$\phi_2 = \frac{-VG}{\mu \Delta(\mu)} e^{-i\mu x - Ky}, \quad (3.9.2)$$

#### 4. THE FINITE DEPTH CASE

Now we consider the case of finite depth  $h$ . Using the same notation and coordinates, the complex potentials  $\phi_{j,j} = 1,2$  for  $h$  motion in the fluid regions  $0 < x < d$  and  $x < 0$  are the solutions of the boundary value problem stated in section 2 with conditions (2.6), (2.7) replaced by

$$\phi_{j,y} = 0 \text{ on } y = h, \quad (4.1)$$

$$\phi_2 \rightarrow G \cosh k_0 (h-y) e^{-\mu x} \text{ as } x \rightarrow -\infty, \quad (4.2)$$

where  $\mu = \sqrt{k_0^2 - v^2}$  and  $k_0$  is the real positive root of  $k \sinh kh - K \cosh h = 0$ .

The method of separation of variables can be also used to get solutions for equations (2.1) that satisfy (2.2) (4.1) and (2.2), (4.1), (4.2). Let

$$\begin{aligned} \phi_1 = & \sum_{n=1}^{\infty} (A_n e^{\mu_n x} + B_n e^{-\mu_n x}) \cos k_n (h-y) \\ & + (\alpha e^{i\mu x} + \beta^* e^{-i\mu x}) \cosh k_0 (h-y) \end{aligned} \quad (4.4)$$

$$\begin{aligned} \phi_2 = & \sum_{n=1}^{\infty} (C_n e^{\mu_n x} \cos k_n (h-y) \\ & + \gamma e^{i\mu x} \cosh k_0 (h-y) \end{aligned} \quad (4.5)$$

where  $\mu_n = \sqrt{k_n^2 + v^2}$  and  $k_n$  are the real positive roots of  $k \sinh kh + K \cos kh = 0$ .

The remaining conditions are satisfied if

$$VU(y) = -\frac{i}{G} \sum_{n=1}^{\infty} \mu_n C_n \Delta(i\mu_n) \cos k_n (h-y) - \frac{\mu \gamma}{G} \Delta(\mu_0).$$

Since the eigenfunctions  $\cosh k_0 (h-y)$  and  $\cos k_n (h-y)$  are orthogonal over the interval  $[0, h]$ , we obtain the constants as

$$C_n = \frac{-4\pi V G a_n k_n \cos k_n h}{\delta_n \mu_n \Delta(i\mu_n)}, \quad \gamma = \frac{4\pi G k_0 a_0 \cosh k_0 h}{\mu \delta_0 \Delta(\mu)}$$

where  $\delta = 2 k_0 h + \sinh 2 k_0 h$ ,  $\delta_n = 2 k_n h + \sin 2 k_n h$ ,

$$a_0 = -\frac{1}{\pi \cosh k_0 h} \int_0^h U(y) \cosh k_0 (h-y) dy,$$

$$a_n = -\frac{1}{\pi \cos k_n h} \int_0^h U(y) \cosh k_n (h-y) dy.,$$

therefore

$$\begin{aligned} \phi_1 = & -4\pi V \sum_{n=1}^{\infty} \frac{k_n a_n \cos k_n h}{\mu_n \delta_n \Delta(i\mu_n)} [i G e^{\mu_n x} \\ & - \mu_n \cosh \mu_n x] \cos k_n (h-y) \\ & + \frac{4\pi V a_0 k_0 \cosh k_0 h}{\mu \delta_0 \Delta(\mu)} [G e^{-i\mu x} + \mu \cos \mu x] \cosh \mu (h-y), \end{aligned} \quad (4.6.1)$$

$$\phi_2 = -4\pi i V G \sum_{n=1}^{\infty} \frac{k_n a_n \cos k_n h}{\mu_n \delta_n \Delta(i\mu_n)} e^{\mu_n x} \cosh k_n (h-y) + 4\pi V G \frac{a_0 k_0 \cosh k_0 h}{\delta_0 \mu \Delta(\mu)} e^{-i\mu x} \cosh \mu (h-y), \quad (4.6.2)$$

For  $G = 0$

$$\phi_1 = -4\pi V \sum_{n=1}^{\infty} \frac{k_n a_n \cos k_n h \cosh \mu_n x}{\mu_n \delta_n \sinh \mu_n d} \cos k_n (h-y) \quad (4.7.1)$$

$$+ 4\pi V \frac{a_n k_0 \cosh k_0 h \cos \mu x}{\mu_n \delta_0 \sin \mu d} \cos k_0 (h-y) \quad (4.7.2)$$

$$\phi_2 = 0,$$

provided that  $\mu d \neq s\pi$ ,  $s$  is an integer.

As a particular case, when  $U(y) = \cosh k_0 (h-y)$

$$\phi_1 = -\frac{V G}{\mu \Delta(\mu)} (G e^{-i\mu x} + \mu \cos \mu x) \cosh k_0 (h-y),$$

$$\phi_2 = -\frac{V G}{\mu_0 \Delta(\mu)} e^{-i\mu x} \cosh k_0 (h-y).$$

### 5. OBLIQUE WAVES GENERATED BY THE POROUS WALL

If we let the impermeable at  $x=d$  be kept fixed while, we let the porous wall oscillate horizontal about its mean position with velocity  $V U(y) \exp(-i\omega t + i\nu z)$ , then the new boundary value problem is the same as stated in section 2 for the infinite depth case or as a stated in section 4 for the finite depth case, except that the boundary conditions (2.3), (2.5) are replaced by

$$\phi_{1x} = 0 \text{ on } x = d, \quad (5.1)$$

$$\phi_{jx} - V U(y) = i G (\phi_2 - \phi_1) \text{ on } x = 0. \quad (5.2)$$

Thus

$$\phi_1 = -2V \int_0^{\infty} \frac{ka(k)f(k,y)}{(k^2 + K^2) \Delta(i\lambda)} \cosh \lambda (d-x) dk$$

$$- \frac{2\pi V K A}{\Delta \mu} \cos \mu (d-y) e^{-Ky}, \quad (3.5.1)$$

$$\phi_2 = -2V \int_0^{\infty} \frac{ka(k)f(k,y)e^{\lambda x}}{(k^2 + K^2) \Delta(i\lambda)} \sin \lambda d dk$$

$$- \frac{2\pi V K A}{\Delta \mu} \sin \mu d e^{i\mu x - Ky} \quad (3.5.2)$$

for the infinite depth case; and

$$\phi_1 = 4\pi V \sum_{n=1}^{\infty} \frac{k_n a_n \cos k_n h}{\delta_n \Delta(i\mu_n)} \cos \mu_n (d-x) \cos k_n (h-y)$$

$$- \frac{4\pi V k_0 a_0 \cosh k_0 h}{\delta_0 \Delta(\mu)} \cos \mu (d-x) \cosh k_0 (h-y), \quad (5.4.1)$$

$$\phi_2 = -4\pi V \sum_{n=1}^{\infty} \frac{k_n a_n \sin h \mu_n d \cos k_n d}{\delta_n \Delta(i\mu_n)} e^{\mu_n x} \cos k_n (h-y)$$

$$- \frac{4\pi i k_0 a_0 \sin \mu d \cosh k_0 h}{\delta_0 \Delta(\mu)} e^{i\mu x} \cosh k_0 (h-y) \quad (5.4.2)$$

for the finite depth case.

### 6- OBLIQUE WAVES GENERATED IN INFINITELY LONG CHANNEL

Her we consider a wave motion induced by a porous plate situated at the middle of an infinitely long channel. With the same notation as before, the complex potentials  $\phi_j, j=1,2$ , for the motion in the fluid regions  $x > 0$  and  $x < 0$ , are the solutions of the boundary value problem with conditions (2.1), (2.2), (2.4) (2.6) and

$$\phi_{jx} - V U(y) = i G (\phi_2 - \phi_1) \text{ on } x = 0, \quad (6.1)$$

$$\phi_j = \rightarrow C_j e^{\pm i\mu x - Ky} \text{ as } x \rightarrow \pm \infty. \quad (6.2)$$

Applying the same technique as before we get

$$\phi_j = \pm 2V \int_0^{\infty} \frac{ka(k)e^{-\lambda(k)x} f(k,y)}{(k^2 + K^2) [\lambda - 2iG]} dk \pm \frac{2\pi i V A K}{\mu + 2G} e^{\pm i\mu x - Ky} \quad (6.3)$$

For an impermeable wall ( $G=0$ ), we get in  $x > 0$

$$\phi = 2V \int_0^{\infty} \frac{ka(k)e^{-\lambda(k)x}f(k,y)}{(k^2 + K^2)\lambda} + \frac{2\pi i VAK}{\mu} e^{i\mu x - Ky} \quad (6.4)$$

and for normal incident ( $\beta=0$ ),  $\phi$  reduces to

$$\phi_j = \pm 2V \int_0^{\infty} \frac{ka(k)e^{\mp kx}f(k,y)}{(k^2 + K^2)(k - 2iG)} dk \pm \frac{2\pi i VAK}{k + 2G} e^{\pm iKx - Ky} \quad (6.5)$$

The hydrodynamic pressure distribution on the porous plate surface is

$$p = -2i\rho \omega V \left[ \int_0^{\infty} \frac{2ka(k)f(k,y)}{(k^2 + K^2)(\lambda - 2iG)} dk + \frac{2\pi AK e^{-Ky}}{\mu + 2G} \right] e^{i\nu z} \quad (6.6)$$

In the particular when  $U(y) = e^{-Ky}$ , we have

$$\phi_j = \mp \frac{iV e^{-Ky \pm i\mu x}}{\mu + 2G} \quad (6.7)$$

For this particular case, the total average force on the part of the porous plate  $y > 0$ ,  $\frac{\pi}{2} \leq \nu z \leq \frac{\pi}{2}$  has an amplitude

$$|F| = \frac{\omega \rho V}{K} \frac{4}{\pi(\mu + 2G)} \quad (6.8)$$

The dimensionless force amplitude  $\left| \frac{K^3 \pi F}{4 \omega \rho V} \right|$  is shown in Figure (2) as a function of the parameter  $G/L$  for values of  $\nu/K = 0.0, 0.8, 1$ . It can be seen that for all values of  $\nu$  ( $0 \leq \nu/K \leq 1$ ), the force amplitude has its greatest value when the plate is impermeable and decreases as  $G/K$  increases indefinitely and when the plate becomes completely transparent to the fluid ( $G/K \rightarrow \infty$ ), the force reduces to zero as it should be based on physical tuition. In Figure (3) the force amplitude,  $\left| \frac{K^3 \pi F}{4 \omega \rho V} \right|$  also is plotted versus  $\nu/K$  for values of  $G/K = 0, 1, \infty$ . It can be seen that the force amplitude increases as  $\nu/K$  increases.

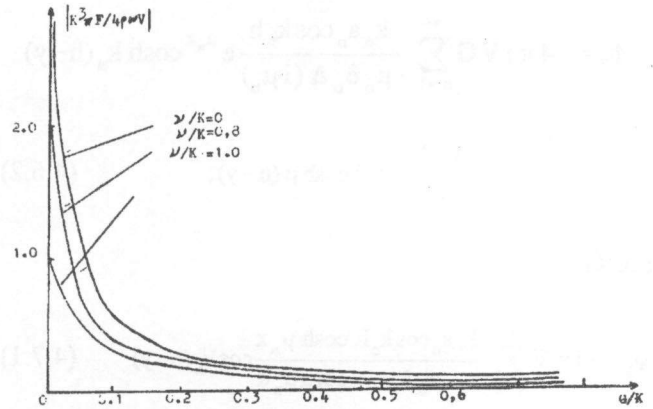


Figure 2. The force amplitude on the porous plate versus  $G/K$  for different values of  $\nu/K$  in the case of infinity channel.

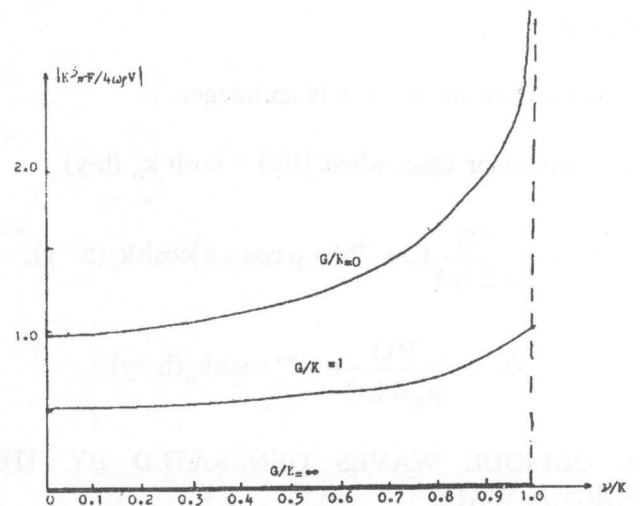


Figure 3. The force amplitude on the porous plate versus  $\nu/K$  for values of  $G/K=0, 1$  and  $\infty$  in the case of infinity channel.

As an interesting application of this section, let a time harmonic wave  $CV e^{i\mu x - Ky}$  incident obliquely from infinity to the porous plate which is kept fixed. The velocity potentials  $\phi_j$  satisfy (2.1), (2.2), (2.5) and

$$\left. \begin{aligned} \phi_{jx} &= \phi_{2x} \\ &= iG(\phi_2 - \phi_1) \end{aligned} \right\} \text{on } x=0. \quad (6.10)$$

Moreover

$$\phi_1 = (A_1 e^{i\mu x} + CV e^{-i\mu x}) e^{-Ky} \text{ as } x \rightarrow \infty, \quad (6.11)$$

$$\phi_2 = A_2 e^{-i\mu x - Ky} \text{ as } x \rightarrow -\infty \quad (6.12)$$

Here  $A_1, A_2$  (to be determined) are complex constants relating to the amplitude and phase of the reflected and transmitted waves, respectively. To solve this problem, let

$$\Psi_j = \phi_j - VCe^{-Ky - i\mu x} \quad (6.13)$$

These new functions satisfy equation (2.1) and have vanishing gradients as  $y \rightarrow \infty$ .

$$\text{on the porous plate } \Psi_{1x} = \Psi_{2x} \quad (6.14)$$

$$= iG\Psi_2 - \Psi_1 + iVC\mu e^{-Ky} \quad (6.15)$$

Therefore  $\Psi_j$  are the solutions of the boundary value problem treated above in this section with  $U(y)$  replaced by  $iC\mu e^{Ky}$ , Hence

$$\phi_1 = \frac{VC}{\mu + 2G} e^{-Ky + i\mu x} + VCe^{-Ky - i\mu x}, \quad (6.16)$$

$$\phi_2 = \frac{2GCV}{\mu + 2G} e^{-Ky + i\mu x}, \quad (6.17)$$

Since the wave energy is proportional to the amplitude square, we define the reflection and transmission coefficients  $R$  and  $T$  as the square of the ratio of the amplitudes of the reflected and transmitted waves to the amplitude of the incident wave respectively.

Therefore

$$R = \frac{\mu^2}{(\mu + 2G)^2}, T = \frac{4G^2}{(\mu + 2G)^2} \quad (6.18)$$

$$\text{and } R + T = \frac{\mu^2 + 4G^2}{(\mu + 2G)^2} \quad (6.19)$$

We should note here that for normal incidence results (6.18), (6.19) reduced to the corresponding results of Chwag and Dong [11]. The coefficients of reflection  $R$ , coefficient of transmission  $T$  and the sum  $R + T$  are plotted in Figure (4) versus  $G/K$  for values  $\nu/K = 0, 0.8$  and 1. We noted from this figure or by simple

differentiation of (6.19) the sum  $R + T$  reduces to a fixed minimum 0.5 irrespective of the value of  $\nu/K$ .

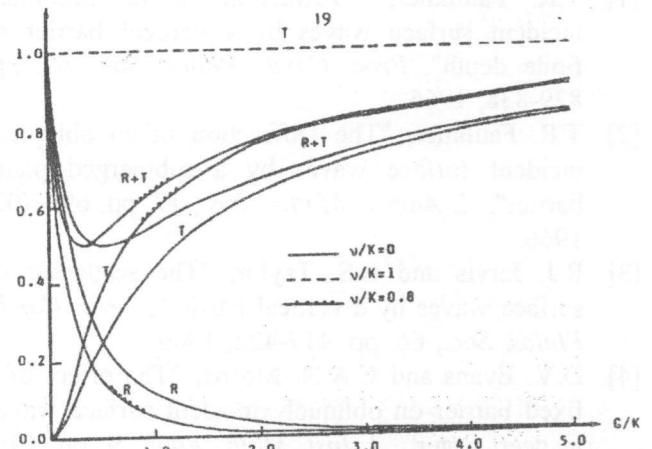


Figure 4. The coefficient of reflection  $R$  and the coefficient of transmission  $T$  versus  $G/K$  for different values of  $\nu/K$ .

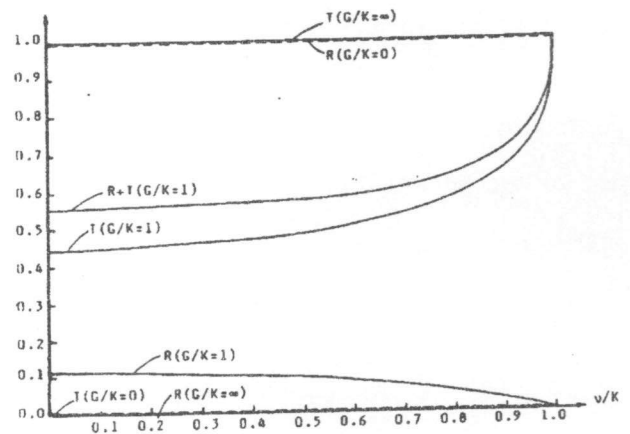


Figure 5. The coefficient of reflection  $R$  and the coefficient of transmission  $T$  versus  $G/K$  for different values of  $\nu/K$ .

In Figure (5),  $R, T$  and  $R+T$  are plotted versus  $\nu/K$  for values  $G/K = 0, 1, \infty$ .

We note at  $G = 0$ , the plate becomes impermeable, all incident wave energy will be reflected for all values of incident angles  $\beta$ . On the other hand as  $G \rightarrow \infty$  all incident wave energy will be transmitted ( $T=1, R=0$ ) again for all values of  $\beta$ .

For  $G/K = 1$  we see  $R$  decreases and  $T$  increases as  $\nu/K$  increases.

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