CONTRACTION RATIOS FOR MEASURING WEIRS AND FLUMES

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ABSTRACT

Two approaches for the design of weirs and flumes are developed on the bases of real flow assumptions. The energy losses between the approach and control sections are accounted for. Theoretical equations are presented to correlate area and width ratios to the Froude number at the approaching section, the approaching velocity, and the velocity at the control section. These equations are represented in graphical forms to be used as design charts for approach section and control section of rectangular, parabolic, or triangular shapes. Study shows that the ratios calculated by the present approaches are always higher than those calculated using the ideal flow conditions.

Keywords: Hydraulics, Measuring flumes and weirs, Contraction ratios.

Notations

A Flow cross section area at the control section, m^2,
A^* Flow cross section area at the control section when the flow depth y equals the head, H, m^2,
A_o Flow cross section area at the approach section, m^2,
A_c Flow cross section area of critical flow, m^2,
b_o The bed width of the approaching channel, m,
C_V Coefficient of velocity at the control section,
C_{va} Coefficient of the approach velocity,
F Froude number at the control section,
F_o Froude number at the approach section,
g Gravitational acceleration, m/sec.^{2},
H Head on the weir crest, m,
H_o Total head on the crest, $H_o = H + \alpha_o v_o^2/2g$, m,
P Height of the crest, m,
Q The flow rate, m^3/sec.,
T Top width of flow at the control section, m,
T^* Top width of flow at the control section when the flow depth y equals the head H, m,
T_o Top width of flow at the approaching section, m,
T_c Top width of critical flow cross section, m,
v Velocity of flow at the control section, m/sec,
v_o Velocity of flow at the approach section, m/sec,
v_c Velocity of critical flow, m/sec,
y Depth of flow at the control section, m,
y_o Depth of flow at the approach section, m,
y_c critical depth of flow, m,
y_h Hydraulic mean depth of flow at the control section, m,
y_{ho} Hydraulic mean depth of flow at the approach section, m,
α_o Energy correction coefficient.

INTRODUCTION

Weirs, specially broad-crested ones, and flumes are mainly constructed for measuring stream flow. These devices are characterized by some constriction of the flow at their control section which may be rectangular, parabolic, or triangular shape. This contraction of flow makes the cross sectional area and top width of the control section to be less than the cross sectional area and top width of the approaching channel. Hence, contraction ratio of both area and top width affects the design of such structures, since it directly determine the required width of the rectangular section, the parameter of the parabolic section, and the head angle of the triangular section. The contraction ratio of area and width are defined as the area and top width of the control section related to the area and top width of the approaching section, respectively.

The design of weirs and flumes should satisfy the following two conditions at the control section; (1) constriction must allow for wide range of measuring discharges without effecting the head-discharge relationship by tail water up to high submergence ratio, and (2) constriction should result a suitable value of the Froude number at the approaching section to pass acceptable amounts of sediments without any disturbance at the upstream water surface so that it can be accurately measured.

In the past, the design of discharge-measuring devices, in open channels, has been relatively difficult since design procedures have not been yet developed. Generally, designs have been chosen from a limited selection of previously calibrated devices, regardless the channel conditions in which they were constructed.

Recently, some approaches to the design of measuring flumes and weirs were developed [1, 2, 3, 4, 5]. These approaches are based on the assumption of ideal flow conditions, i.e., uniform velocity distribution, and no energy losses. Consequently, the depth of flow at the control section, y, is considered to be the critical depth or y=0.667 H. However, in real flow conditions, a part of the flow energy is lost due to the contraction of flow through such measuring structures. This causes the flow depth at the control section to be less than the critical depth, which is theoretically and experimentally confirmed [6, 7, 8]. For free flow over broad-crested weir of rectangular shape, it was theoretically found that the value of the critical depth, y_c equals to 0.667 $C_v^{4/3}$ H. The critical depth, y_c, has a minimum value of 0.529 H considering real flow conditions (C_v=0.84) and maximum value of 0.667 H considering ideal flow conditions (C_v=1.0). According to the former limits of the critical depth, the actual depth of flow at the control section ranges form 0.428 H to 0.667 H, [6]. This indicates that using ideal flow assumption may result in an increase of about 35% in the value of flow depth at the control section. Also, it was experimentally found that, the average value of the flow depth over broad-crested weir of rectangular section equals 0.485 H, [9]. This depicts that, ideal flow assumption causes an increase of 27% in the actual depth over the weir. From the above discussion, it is clear that, ideal flow assumption leads to inaccurate estimation of the values of the flow depth at the control section, which in turn affects the design of contraction ratios for area and width. Hence, procedure based on the ideal flow assumption is not recommended for weirs and flumes design.

In the present work, two approaches for the design procedures of weirs and flumes are developed using real flow considerations. The energy losses are accounted for using the velocity coefficient at the control section, C_v. Rectangular, parabolic, and triangular shapes, for the control section, are considered.

THEORETICAL STUDY

The objective of the proposed design, is to get an accurate estimation for the dimensions of the control section. The design is carried out using any of the following two approaches. However the two approaches are different, they produce the same dimensions for the control section.

The first approach

The first approach is based on the continuity equation, introducing the upstream Froude number to select a control section size and shape suitable to the flow conditions in the upstream channel. Referring to Figure (1), the discharge at the approach and the control sections are the same. Applying the continuity equation between these two sections, gives

$$\left(\frac{A}{A_o}\right)^3\left(\frac{T}{T_o}\right)^{-1}=\left(\frac{F_o}{F}\right)^2 \qquad (1)$$

where, $F_o = v_o/\sqrt{g y_{ho}}$, and $F = v/\sqrt{g y_h}$

The subscript "O" refers to the cross section in the approach channel at the gaging station where the head above the crest, H, is to be measured.

Exact values of A and T are not easily estimated for all control shapes. However, Bos [3] suggested a design based on the ratio A^*/A_o and T^*/T_o. Where A^* and T^* are the area and top width at the control section, respectively, assuming that the water surface elevation at the control section is the same as that in the approach channel as shown in Figure (2).

The control section may have a rectangular, a parabolic, or a triangular shapes, as shown in Figure (2). For each of these shapes the passing discharge Q=Constant x H^n, where n=1.5 for rectangular section, n=2.0 for parabolic section, and n=2.5 for triangular section. Control section of rectangular shape is widely used in practice. However, parabolic and triangular shapes are also used specially in narrow streams having small discharges.

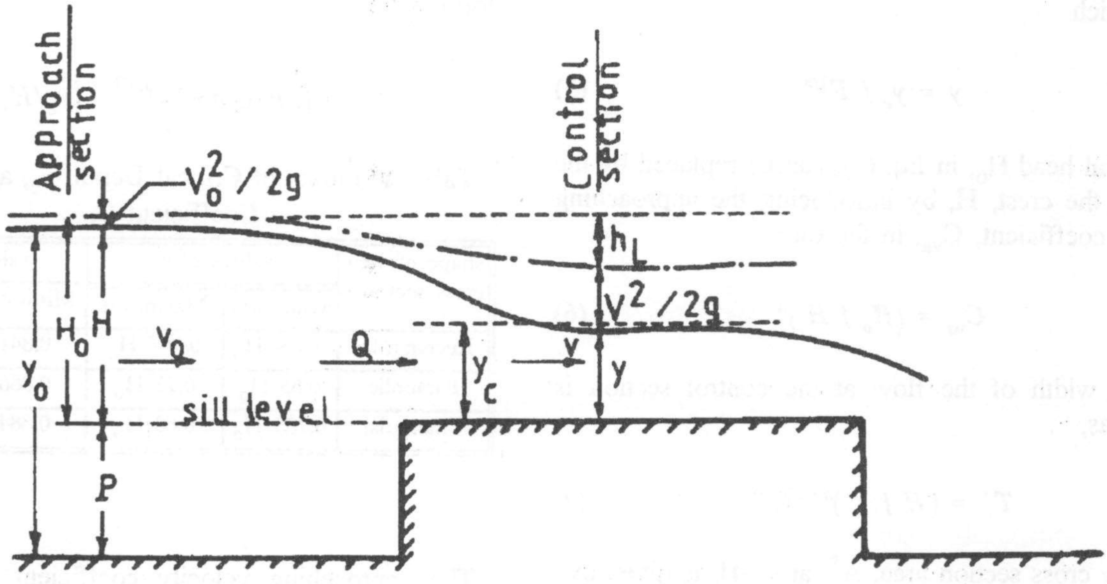

Figure 1. Definition sketch for flow over broad-crested weir.

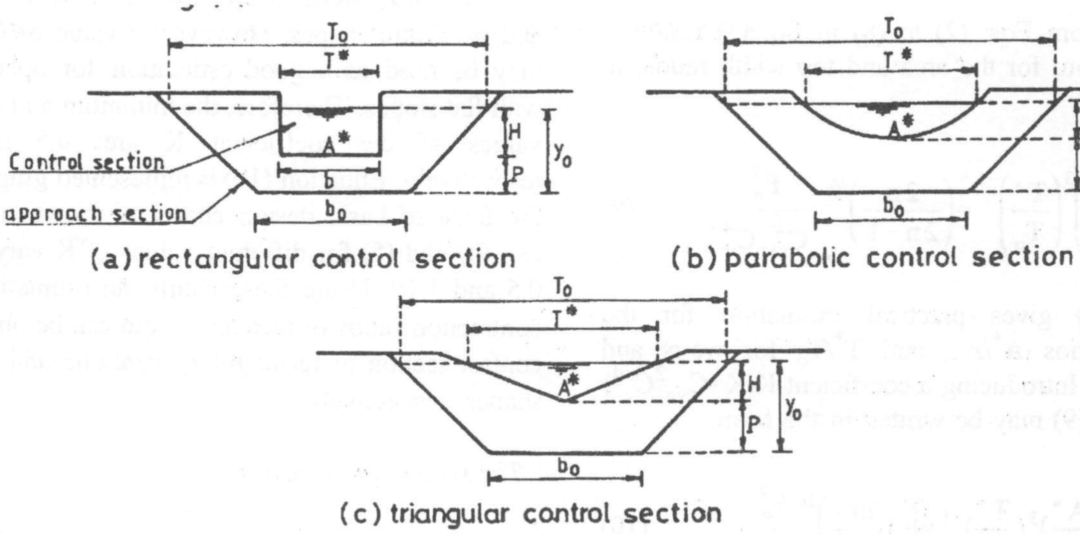

Figure 2. Definition sketch for the approach and control sections.

Extensive theoretical and experimental studies were carried out on broad-crested weirs of rectangular [6], parabolic [7], and triangular [8] sections. Accordingly, a general equation for the critical depth, in free flow conditions, can be written in the following form;

$$y_c = [(2n-1) / (2n)] \; C_v^{2/n} \; H_o \qquad (2)$$

where, n=1.5, 2.0, and 2.5 for rectangular, parabolic, and triangular shapes, respectively.

Similarly a general relation for the flow cross sectional area, A, at the control section can be expressed as,

$$A = Ty / (n-0.5) \qquad (3)$$

The depth of flow at the control section, y, is related to the critical depth, y_c, using the continuity equation, since

$$A.v. = A_c.\, v_c\,, \qquad (4)$$

from which

$$y = y_c / F^{1/n} \tag{5}$$

The total head H_o, in Eq. (2), can be replaced by the head on the crest, H, by introducing the approaching velocity coefficient, C_{va}, in the form

$$C_{va} = (H_o / H)^n \tag{6}$$

The top width of the flow at the control section is defined as,

$$T^* = (H / Y)^{n-1.5} . T \tag{7}$$

The flow cross section area, A^*, at y =H, is given by

$$A^* = T^* H / (n-0.5) \tag{8}$$

Substituting from Eqs. (2) to (8) in Eq. (1) results a general equation, for the area and top width ratios, in the form

$$\left(\frac{A^*}{A_o}\right)^3 \left(\frac{T^*}{T_o}\right)^{-1} = \left(\frac{2n}{2n-1}\right)^{2n} \cdot \frac{F_o^2}{C_{va}^2 . C_v^4} \tag{9}$$

Equation (9) gives practical estimation for the contraction ratios A^*/A_o, and T^*/T_o for weirs and flumes design. Introducing a coefficient $K(K=C_{va}{}^2C_v{}^4)$ then equation (9) may be written in the form

$$\left(\frac{A^*}{A_o}\right)^3 \left(\frac{T^*}{T_o}\right)^{-1} = \left(\frac{2n}{2n-1}\right)^{2n} \frac{F_o^2}{K} \tag{10}$$

Equation (10) can be solved graphically with respect to values of the coefficient K. Coefficient K has minimum and maximum values, depending on the minimum and maximum values of the coefficients C_v and C_{va}. According to the studies, performed on broad-crested weirs of rectangular, parabolic and triangular sections [6,7,8], the minimum and maximum values of C_v are listed in Table 1, with respect to the minimum and maximum values of the critical depth y_c . Values of the coefficient C_v, between these critical limits, can be obtained using the general equation for C_v in the form.

$$C_v = \left[2n/(2n-1)\right]^{n/2} . \left(y_c/H_o\right)^{n/2} \tag{11}$$

Table 1. Limits of Critical Depth, y_c, and Velocity Coefficient C_v.

shape of weir cross section	values of y_c		values of C_v	
	Minimum	Maximum	Minimum	Maximum
Rectangular	0.529 H_o	0.667 H_o	0.841	1.0
Parabolic	0.65 H_o	0.75 H_o	0.866	1.0
Triangular	0.725 H_o	0.80 H_o	0.884	1.0

The approaching velocity coefficient C_{va} tends to unity for very small velocities. The coefficient C_{va} increases as the velocity increases. The triangular shape has a greater values of C_{va} compared with rectangular and parabolic shapes. However, a value of C_{va} =1.05 may be used as a good estimation for open channel with flat slopes. Therefore, the minimum and maximum values of the coefficient K are; 0.5 and 1.10, respectively. Equation (10) is represented graphically in the form of basic design charts as shown in Figures. (3), (4) and (5) for different values of K vary between 0.5 and 1.10. Using these charts, an estimation of the contraction ratios of area and width can be obtained for control section of rectangular, parabolic and triangular shapes, respectively.

The second approach

The second approach is based on the critical depth equation for open channel since,

$$Q^2/g = A_c{}^3/T_c \tag{12}$$

The total energy head is defined as

$$H_o = H + v_o{}^2/2g$$

from which

$$Q^2/2gA_o{}^2 H = C_{va}{}^{1/n} -1 \tag{13}$$

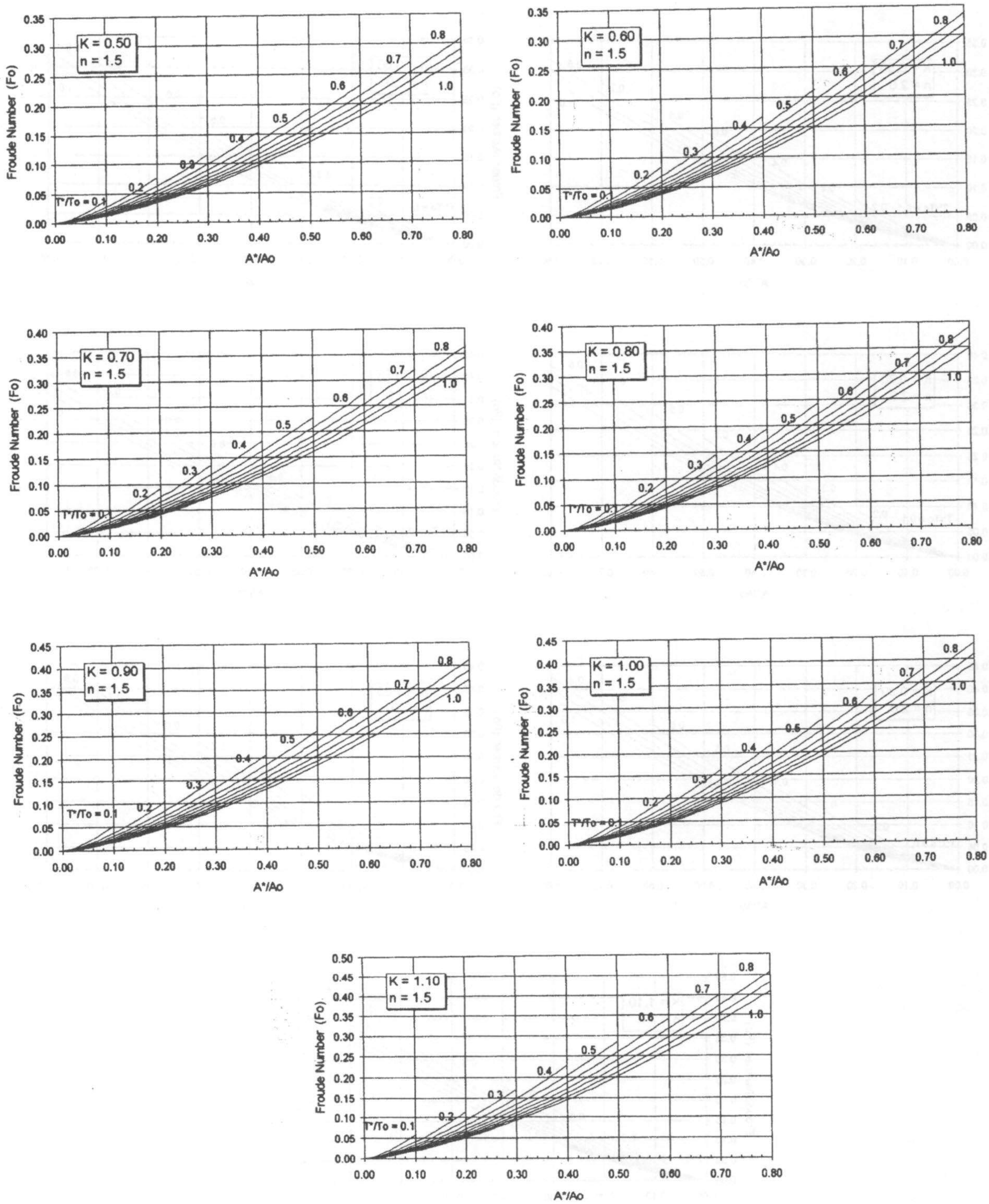

Fig. 3. Variation of A^*/A_o with Froude number F_o for different values of K, for rectangular control section (n = 1.5)

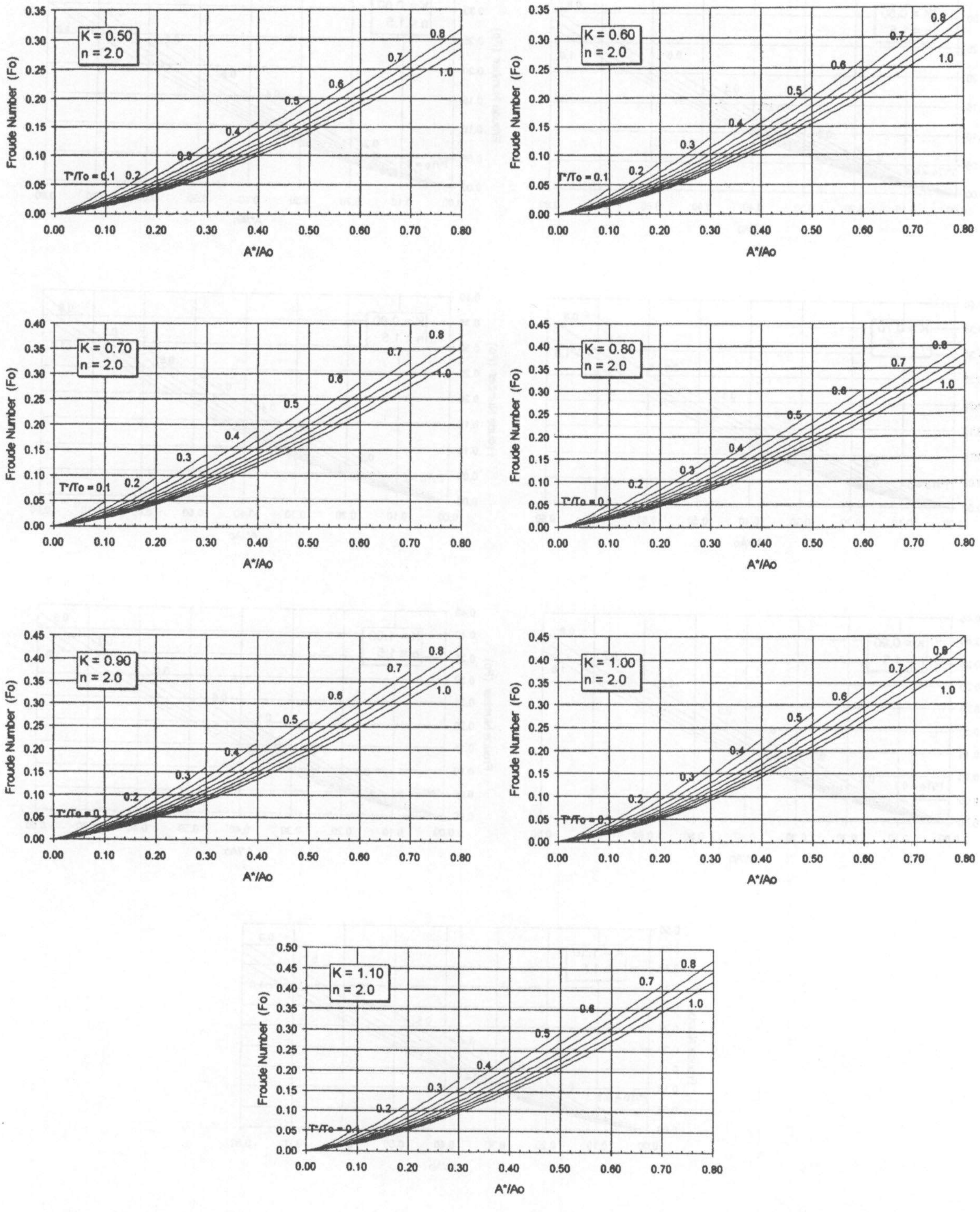

Fig. 4. Variation of A^*/A_o with Froude number F_o for different values of K, for parabolic control section (n = 2.0)

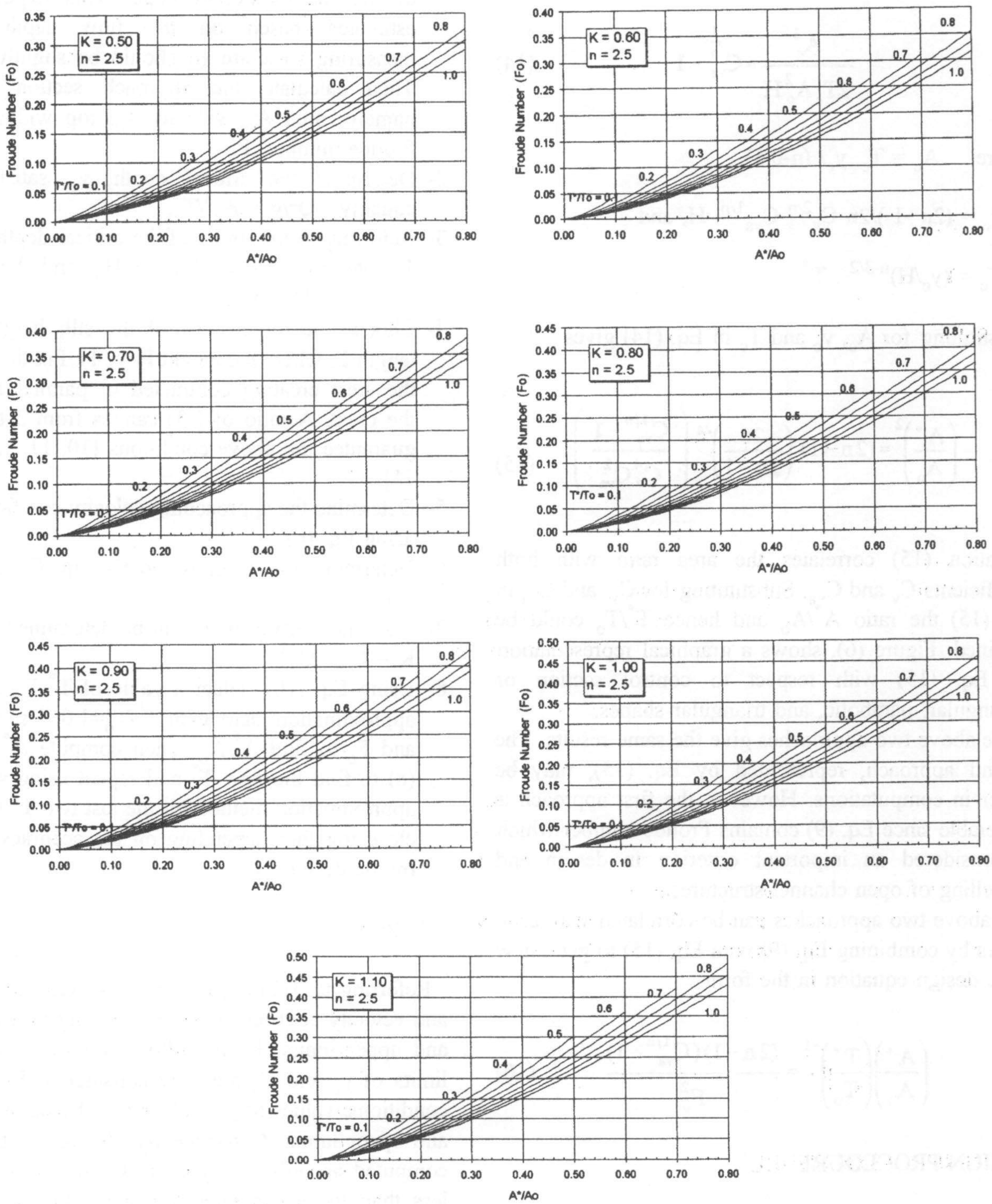

Fig. 5. Variation of A^*/A_o with Froude number F_o for different values of K, for triangular control section (n = 2.5)

Combining Eq. (12) with Eq. (13) gives,

$$\frac{A_c^3}{2T_c A_o^2 H} = C_{va}^{\frac{1}{n}} - 1 \qquad (14)$$

where $A_c = T_c . y_c / (n-0.5)$,

$y_c = (2n - 1)/2n\ C_v^{2/n}\ C_{va}^{1/n}. H$, and

$T_c = (y_c/H)^{n-3/2} . T^*$

Substituting for A_c, y_c and T_c in Eq. (14) gives

$$\left(\frac{A^*}{A_o}\right)^2 = (2n-1)\left(\frac{2n}{2n-1}\right)^{2n}\left(\frac{C_{va}^{1/n} - 1}{C_v^4 C_{va}^2}\right) \qquad (15)$$

Equation (15) correlates the area ratio with both coefficients C_v and C_{va}. Substituting for C_v and C_{va} in Eq. (15) the ratio A^*/A_o and hence T^*/T_o could be obtained. Figure (6), shows a graphical representation for Eq. (15) with respect to control section of rectangular, parabolic, and triangular shapes.

The above two approaches give the same results. The second approach, represented by Eq. (15), may be easier in computations. However, the first approach is preferable since Eq. (9) contains Froude number which is considered an important criterion in design and modelling of open channel structures.

The above two approaches can be correlated with each others by combining Eq. (9) with Eq. (15) to get a new basic design equation in the form

$$\left(\frac{A^*}{A_o}\right)\left(\frac{T^*}{T_o}\right)^{-1} = \frac{(2n-1)(C_{va}^{1/n} - 1)}{F_o^2} \qquad (16)$$

DESIGN PROCEDURE

Using Eq. (10) or Eq. (15) the values of contraction ratios, for broad-crested weir and flumes having; rectangular, parabolic, and triangular cross sections, may be carried out as follows:

1- Determine the upstream water depth, y_o corresponds to the maximum discharge. This depth can be estimated based on the flow depth without measuring structure (it should be slightly higher). Then calculate the approach section variables namely; area A_o, velocity v_o, top width T_o, and Froude number, F_o.

2- Determine the critical depth, y_c, satisfying the equality; $Q^2/g = A_c^3/T_c$.

3- Referring to the limits of the critical depth in Table 1, determine the total head H_o, and the head H, since $H = H_o - v_o^2/2g$.

4- Choose the dimensions of the sill; height P, and length L, where the height $P=y_o-H$. Fix the length of the crest creating contracted or parallel flow over the crest. A ratio of L/H ranges from 3 to 10 will guarantee the above conditions [10, 11, 12, 13, 14, 15].

5- Determine the approaching velocity coefficient, C_{va}, using Eq. (6).

6- Determine the velocity coefficient C_v using Eq. (11).

7- Now the coefficient K can be determined since $K = C_{va}^2\ C_v^4$.

8- Using Eq. (10), select a value of T^*/T_o. As a first approximation, start with $T^*/T_o=1.0$. Use this value and F_o to find A^*/A_o. Then compute A^*. Use Eq. (8) to find the new T^* and repeat using successive approximation method till the last two T^* values are close together depending on the accuracy assigned by the designers.

APPLIED EXAMPLE

Referring to Table 1, the values of critical depth, y_c, and velocity coefficient, C_v are ranging between lower and upper limits. In ideal flow conditions, the upper limits of y_c and C_v are only considered. In real flow conditions, values of y_c, and C_v vary between the lower and upper limits. Consequently, the contraction ratios computed according to the ideal flow assumption are less than those evaluated according to the real flow assumption. A comparative study for the design procedures, based on ideal and real flow assumptions, will be discussed through the following example since actual measurements are not available.

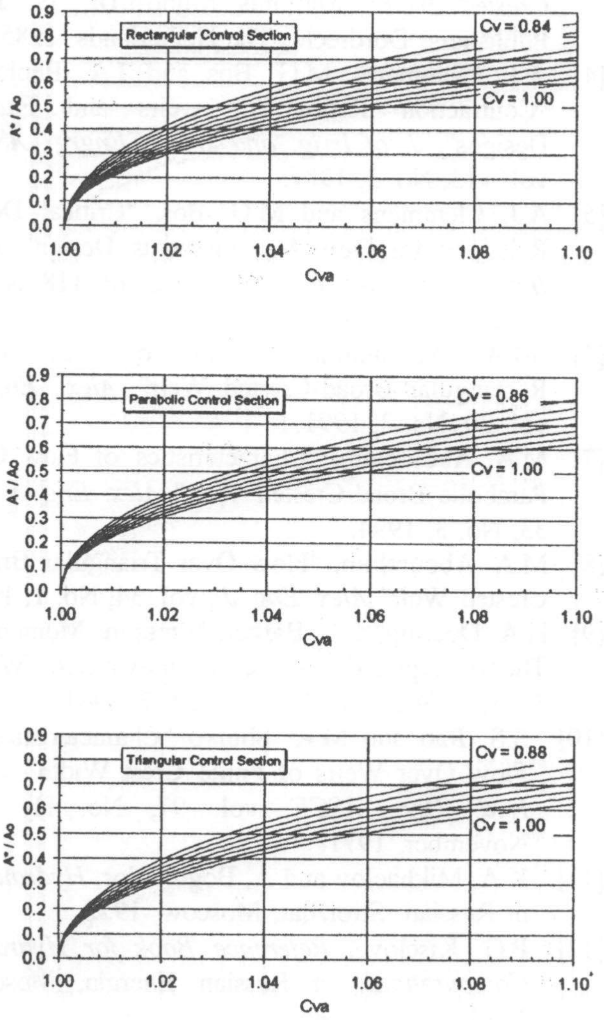

Figure 6. Contraction area ratio A^*/A_o as a function of coefficients and C_{va}.

Given the cross section properties at the approaching section, where the discharge Q=10.8 m^3/sec., bed width b_o = 6.0 m, water depth y_o = 2.0 m, and side slopes 3:2. Using the procedure, based on ideal flow assumption [Ref. 4], and the procedure based on the real flow assumption (the present study), evaluate the dimensions of a broad-crested weir if it has a cross section of rectangular, parabolic, and triangular shapes.

From the geometry of the approach section, A_o=18.0 m^2, T_o=12.0 m. Hence, v_o=0.6 m/sec, y_{ho}=1.5 m, F_o=0.1564, y_c=0.653 m., and the kinetic energy $\alpha_o v_o^2/2g$=0.018 m.

Considering the procedure based on real flow assumption,the solution is given in the tabulated form, Table (2). Considering the procedure based on ideal flow assumption, suggested in Ref. [4], the obtained contraction ratios are shown in Table 3, columns (1) and (4).

Table 2. Solution procedure based on real flow assumption.

Quantity	Rectangular shape	Parabolic shape	Triangular shape
Q, m^3/sec	10.80	10.80	10.80
b_o, m	6.0	6.0	6.0
y_o, m	2.0	2.0	2.0
A_o, m^2	18.0	18.0	18.0
v_o, m/sec	0.50	0.60	0.60
y_{ho}, m	1.50	1.50	1.50
F_o	0.1564	0.1564	0.1564
y_c, m	0.653	0.653	0.653
$v_o^2/2g$, m	0.018	0.018	0.018
n	1.50	2.00	2.50
y_c/H_o	0.529	0.65	0.725
H_o, m	1.234	1.005	0.90
H, m	1.216	0.987	0.882
C_{va}	1.0223	1.0368	1.0518
C_v	0.841	0.866	0.884
K	0.523	0.605	0.676
A^*/A_o	0.441	0.540	0.613
T^*/T_o	0.544	1.23	2.085

From Table (3) it is clear that both area and top width ratios calculated using ideal flow assumption are less than those obtained using real flow assumption. It was stated, in Ref. [4], that ideal flow assumption may cause 10% error in discharge prediction, but it has a poor effect on the design of weirs and flumes. However, as shown in Table (3), the ideal flow assumption causes a reduction of about 22-26% in the area ratio. This presents a high differences will affect in the function of the structure and can not be ignored in the design. This reduction leads to inaccurate measurements of channel discharges and also, this leads to rising the water surface, in the upstream side, which may cause unexpected damage for channel embankments.

Referring to Table (3), the values of T^*/T_o, calculated using both ideal and real flow assumptions, are accidentally found to be the same for the rectangular section. However, in other practical problems, a sensitive differences are found to exist.

Table 3. Comparison Between Ideal and Real Flow Assumptions.

shape of control section	A^*/A_o, Ref 4.	A^*/A_o, Authors	% dev.	T^*/T_o, Ref. 4	T^*/T_o, Authors	% dev.
Rectangular	0.349	0.441	26.0	0.544	0.544	0.0
Parabolic	0.433	0.540	24.7	1.140	1.230	7.6
Triangular	0.501	0.613	22.4	1.883	2.085	10.7

CONCLUSIONS

1. The design procedure based on ideal flow assumption can not be recommended for weirs and flumes construction.
2. A practical design procedure, for contraction ratios of area and top width, is presented in this paper. The procedure is based on the real flow assumption. The procedure may be used for weirs and flumes with control section of rectangular, parabolic, and triangular shapes, where more accurate results can be obtained.

The design enables to determine the width of rectangular control section since $T^*=A^*/H$. For parabolic section, the parameter of the parabolic section can be determined since it equals $T^{*2}/8H$. As for triangular control section, the head angle θ can estimated since $T^* = 2\ H \tan \theta/2$.

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