CALCULATION OF SOLID ANGLES SUBTENDED BY A CIRCULAR DISK DETECTOR BY MONTE CARLO METHOD

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ABSTRACT

By using a Monte-Carlo (MC) method, the average solid angles subtended by a circular disk detector of radius R coaxial to a circular disk source of radius S are calculated. The generated data are for S/h ranging from 0.1 to 10 and R/h from 0.1 to 6, where h is the distance between the source and detector. The generated average solid angles have values ranging from 0.0006 to 5.2499. The data were fitted to a simple analytical expression. The results were compared with published data.

Keywords: disk-to-disk geometry, Monte-Carlo, Solid angle.

INTRODUCTION

In some nuclear and radiation experiments, the accurate knowledge of the solid angle subtended by the detector with regard to the source is required. The most common simple case is the solid angle subtended at a point isotropic source by a coaxial detector of a circular aperture of radius R located at a certain distance h away from the source. In this case, the solid angle Ω_o is given by

$$\Omega_o = 2\pi\left[1 - \frac{1}{\sqrt{1 + (R/h)^2}}\right] \quad (1)$$

For any shape of source and detector, the solid angle Ω is given by an integration in the form

$$\Omega = \frac{1}{4\pi A_s}\int_{A_s} dA_s \int_{A_d} \frac{\cos\alpha}{r^2}\, dA_d \quad (2)$$

where A_s is the area of a plane isotropic source, r represents the distance between the two differential areas dA_s and dA_d of the source and detector respectively and α is the angle between the normal to the surface element dA_d and the distance r.
The integration given by Eq. (2) can be solved analytically in very few cases. For example, in the case of a disk source parallel to a coaxial circular detector, Masket (1957) and Ruffle (1967) obtained an expression for Ω involving elliptic integrals. After some approximations to make the original geometry integrable, Gardner and Verghese (1971), Selim and Abbas (1994) have found an analytical expression to calculate the solid angle subtended by a circular disk from a non-axial point source. In addition to their models, tables are given for the average solid angles subtended by a circular detector disk from a coaxial circular isotropic source disk for R/h and S/h ranging from 0.1 to 3.0. Later, Gardner et al (1980) have fitted the tabulated results to a polynomial expression. The MC technique is a general method which can be used to calculate the solid angles with complicated geometries. Such method was used by different authors, Williams (1966), Carchon et al (1975) and Wielopolski (1977).
In the present work, a MC program was developed to estimate the average solid angle subtended by circular detector disks from coaxial parallel circular source disks. The obtained data for R/h ranging from 0.1 to 6 and S/h from 0.1 to 10 were fitted to a very simple empirical expression.

MONTE-CARLO CALCULATIONS

By the MC method, the particle position of birth and the direction of emission are determined by using random numbers. It can be then checked whether or not the particle direction intercepts the detector. The solid angle can be calculated from the ratio of the number of particles hitting the detector to the number of those

emitted by the source.
Consider now an isotropic point source at O` of a distance ρ from the axis OZ of the detector, which located by a distance h above the source, as in Figure (1).

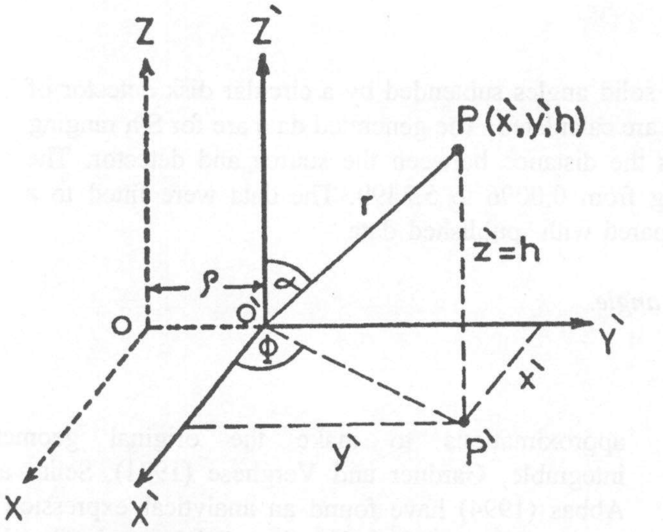

Figure 1. Source-to-detector geometry.

It is clear that the angles α and ϕ are spherical coordinates in a system with Z`-axis as a polar axis. The direction α of the emitted particle varies randomly and can be determined according to

$$\cos\alpha = 1 - (RN)_1$$

where $(RN)_1$ is a random number uniformly distributed in the interval (0,1). It is also obvious that ϕ is uniformly distributed in the interval $(0,2\pi)$ and given by

$$\phi = 2\pi \,.\, (Rn)_2$$

where $(RN)_2$ is another random number.
The coordinates of the point P (Figure (1)) with respect to the frame O` are given by

$$\left.\begin{aligned} x` &= r.\sin\alpha\cos\phi, \\ y` &= r.\sin\alpha\sin\phi, \\ z` &= r.\cos\alpha \end{aligned}\right\} \quad (3a)$$

As we show from Figure (1), the point P intersects the detector plane at z`=h. Accordingly, the equations (3a) can be written in the form

$$\left.\begin{aligned} x`/h &= \tan\alpha\cos\phi, \\ y`/h &= \tan\alpha\sin\phi \end{aligned}\right\} \quad (3b)$$

For simplicity, we consider now that the source O` locates at an arbitrary distance ρ from the axis OZ along the Y-axis. In this case the x- and y-coordinates of the point P are

$$\left.\begin{aligned} x/h &= \tan\alpha\cos\phi, \\ y/h &= \tan\alpha\sin\phi + \rho/h \end{aligned}\right\} \quad (3c)$$

with respect to the frame O.
Let us now take the case of a homogeneous surface disc source of radius S, whose center is located at O` and parallel to the disc detector. The coordinates of a random point on the source is given by

$$x_o = r`.\sin\Psi \quad \text{and} \quad y_o = r`.\cos\Psi$$

with respect to the frame O`, where

$$\Psi = 2\pi\,.(RN)_3 \quad \text{and} \quad r` = S.\sqrt{(RN)_4}$$

Again, $(Rn)_3$ and $(RN)_4$ are two random numbers uniformly distributed in the intervals (0,1). Therefore, the coordinates of the point P for a disc source, whose center is located at a point on Y-axis of distance ρ from OZ-axis, are given by

$$\left.\begin{aligned} x/h &= \tan\alpha\cos\phi + x_o/h, \\ y/h &= \tan\alpha\sin\phi + y_o/h + \rho/h \end{aligned}\right\} \quad (3d)$$

with respect to the frame O.
When the center of the source is located on the axis of the detector, we put $\rho/h = 0$.
We can now decide whether the emitted particle hits the detector by comparing the value $\sqrt{(x/h)^2 + (y/h)^2}$ with R/h, where R is the detector radius. The solid angle Ω is given by

$$\Omega = 2\pi\frac{N}{N_o}$$

Table 1. Values of the fit parameters a and b, and the R.M.S. relative deviation of calculated solid angles (by Eq. 4) from the Monte Carlo data for R/h ≤ 1.6.

R/h	ln a	b	correlation coefficient r	R.M.S. relative (%): a and b given in this table	R.M.S. relative (%): a and b from Eqs 5 and 6
0.1	-0.370	1.880	0.99997	2.90	3.1
0.3	-0.470	1.907	0.99998	1.00	2.5
0.5	-0.641	1.928	0.99999	0.95	1.4
0.7	-0.848	1.977	≈ 1	0.40	2.5
1.0	-1.260	2.072	0.99998	0.94	4.0
1.3	-1.690	2.167	0.99996	1.20	3.4
1.6	-2.200	2.340	0.99992	3.20	3.5

Table 2. Values of the fit parameters a and b for R/h ≥ 1.9.

R/h	First region: ln a	First region: b	First region: r	Second region: ln a	Second region: b	Second region: r
1.9	-3.021	2.771	0.99995	-2.122	2.123	0.9998
2.2	-3.520	2.889	0.99994	-2.405	2.145	0.9996
2.5	-3.960	2.977	0.99998	-2.742	2.205	0.9995
3.0	-4.671	3.176	0.99990	-3.340	2.328	0.9997
3.5	-5.301	3.250	0.99995	-3.795	2.398	0.9999
4.0	-5.800	3.241	0.99983	-4.582	2.637	0.9991
4.5	-6.240	3.202	0.99987	-5.487	2.937	0.9993
5.0	-6.602	3.150	0.99987	-6.460	3.253	0.9992
6.0	-7.263	3.121	0.99981	-8.717	4.031	0.9997

Table 3. R.M.S. deviation of the calculated values (using Eq. 4) from the Monte Carlo results.

R/h		1.9	2.2	2.5	3.0	3.5	4.0	4.5	5.0	6.0
R.M.S.(%)	a and b from Table 2	0.6	0.5	0.4	1.1	0.7	1.4	1.1	1.0	1.4
	a and b from Eqs. 8 and 9	4.0	0.9	2.0	1.2	1.4	2.1	1.8	1.9	1.9

Table 4. R.M.S.-error (%) of the calculated solid angles (by Eq. 4) from the tabulated values by Gardner and Verghese.

R/h	R.M.S. rel. deviation (%)	
	a and b from Tables 1 and 2	a and b according to the given expressions
0.1	1.0	1.8
0.2	1.6	2.7
0.3	1.3	3.0
0.4	2.2	2.8
0.5	2.3	2.3
0.6	2.0	1.4
0.7	1.7	1.0
0.8	3.3	0.7
0.9	5.3	2.8
1.0	2.2	1.2
1.2	2.3	1.4
1.4	2.7	2.6
1.6	1.6	3.7
1.8	0.9	1.7
2.0	2.0	1.4
2.2	1.7	1.0
2.4	0.9	0.8
2.6	0.5	0.5
2.8	0.7	0.5
3.0	0.9	0.6

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where N is the number of particles hitting the detector and N_o is the number of the emitted particles from the source in the upper hemisphere.
In order to have a reasonable level of statistical errors and to save the computation time, different numbers of histories (N_o) for each case was taken. It is known that the solid angle increases as S/h decreases at constant R/h, whereas at constant S/h, it increases as R/h increases. For small values of Ω, large number of histories was taken and vice versa. The minimum number of histories was 10^4 (for R/h =6 and S/h = 0.1), and the maximum one was $3x10^5$ (for R/h = 0.1 and S/h = 10). These numbers of trajectories given relative statistical errors of 0.1% and 2.8% respectively.

For 10^5 histories, the IBM (486-Dx66) compatible computer time for each fixed geometry was 9 sec.

RESULTS AND DISCUSSION

Using the previous MC method, values of the average solid angle subtended at an isotropic circular source by a coaxial parallel disk detector were generated for a range of S/h from 0.1 to 10 and R/h from 0.1 to 6. The generated values are ranging from a minimum of 0.0006 to a maximum of 5.2499.
At constant R/h value, the variation of Ω (R/h, S/h) with S/h was expressed by the following simple empirical equation

$$\Omega\,(R/h, S/h) = \frac{\Omega_o}{a\,(S/h)^b + 1} \quad (4)$$

where the parameters a and b are only a function of R/h, and Ω_o is given by Eq. (1). It can be observed that the solid angle Ω(R/h, S/h) tends to zero as S/h becomes infinity and it approaches Ω_o as S/h tends to zero.

The expression (4) was investigated using the average solid angles generated by the MC program. In Figure (2) are plots of $\ln\left[\frac{\Omega_o}{\Omega\,(R/h, S/h)} - 1\right]$ versus ln(S/h), for three selected R/h-values. Within the statistical errors of the MC calculations and for R/h ≤ 1.6, the points lie very close to a series of straight lines of different intercepts and gradients. For R/h > 1.6, the results give no one straight line for the whole range of S/h, but it was observed that the results can be represented by two different straight lines for each value of R/h.

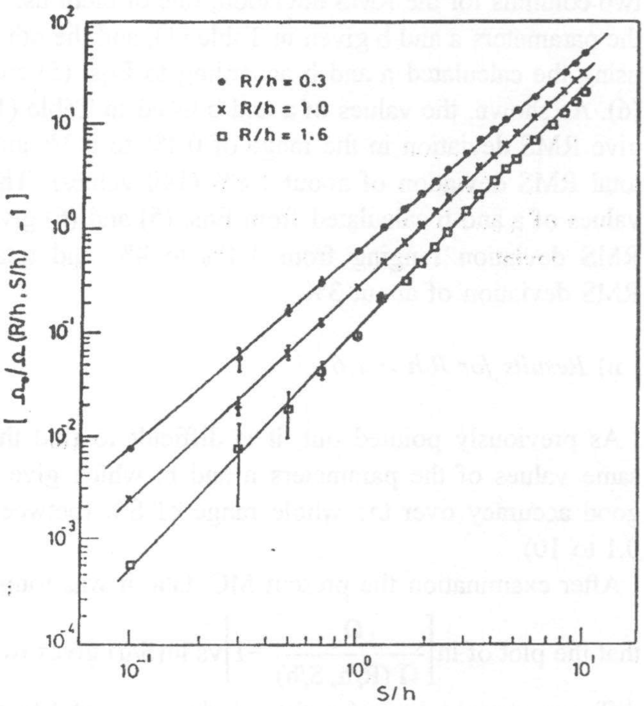

Figure 2. The variation of $\left[\frac{\Omega_o}{\Omega\,(R.h, S/h)} - 1\right]$ with S/h at three R/h-values.

i) *Results for R/h ≤ 1.6*

At each R/h-value and by using the linear least-squares method, the generated data were fit to Eq. (4). The obtained values of a and b (and the corresponding correlation coefficient r) are given in Table (1). The dependence of a and b on R/h was investigated and it was found that a and b can be represented by

$$\ln a = -\,(0.481\,R/h + 0.607)^{2.5} \quad (5)$$

with correlation coefficient r= 0.9994, and

$$\frac{\ln a}{b} = -\,(0.368\,R/h + 0.398)^2 \quad (6)$$

with r = 0.999

In order to examine the accuracy of the formulas (4), (5) and (6), the RMS relative deviation of the values calculated by Eq. (4) from the generated MC values was calculated. For each R/h-value, Table (1) contains two columns for the RMS deviation, one of them using the parameters a and b given in Table (1), and the other using the calculated a and b according to Eqs. (5) and (6). As shown, the values of a and b listed in Table (1) give RMS deviation in the range of 0.4% to 3.2% and total RMS deviation of about 1.8% (140 values). The values of a and b calculated from Eqs. (5) and (6) give RMS deviation ranging from 1.4% to 4% and total RMS deviation of about 3%.

ii) *Results for R/h > 1.6*

As previously pointed out, it is difficult to find the same values of the parameters a and b, which give a good accuracy over the whole range of S/h (between 0.1 to 10)

After examination the present MC data, it was found that the plot of $\ln\left[\frac{\Omega_o}{\Omega\,(R/h,\,S/h)}-1\right]$ vs ln(S/h) gives two different straight lines for the whole range of S/h at each R/h-value. It was also observed that the boundary of the two straight lines is not a fixed value, but it increases as R/h increases as follows.

$$(S/h)_b = 6.947 - 5.73/(R/h) \qquad (7)$$

Accordingly, the whole range of S/h will be divided into two regions

- the first region for $(S/h) \leq (S/h)_b$,
- the second region for $(S/h) > (S/h)_b$

The MC data have been fitted to the empirical formula (4) and the constants a and b were obtained for each R/h-value. Two different sets of the constants were obtained. These sets and the correlation coefficient r for each R/h are listed in Table (2).

Consequently, alternative functions of R/h were investigated and from a least square linear regression analysis of a and b, the following expressions were obtained

$$\left.\begin{aligned} \ln a &= 13.84 - 14.84\,(R/h)^{0.2}, \\ \frac{\ln a}{b} &= 0.5214 - 1.1635\,\sqrt{R/h} \end{aligned}\right\} \qquad (8)$$

for the first region, and

$$\left.\begin{aligned} \ln a &= -\,[0.1815\,(R/h)^2 + 1.4494]^{1.053}, \\ \frac{\ln a}{b} &= -\,[\,0.6955\,\ln(R/h) + 0.5465]^{4/3} \end{aligned}\right\} \qquad (9)$$

for the second region.

Again, in order to evaluate the validity of the previous expressions, the calculated solid angles were compared with both the MC data and the tabulated results given by Gardner and Verghese (1971).

According to Eq. (4) and using a and b given in Table (2), the RMS dispersion of the MC points about the calculated values is 0.98% (180 points), whereas the dispersion increases to a value of 2% when the parameters a and b were calculated from Eqn. (8) and (9). Table (3) lists the RMS errors for each R/h-value.

The RMS spread of Gardner-Verghese data (for a range of S/h and R/h from 0.1 to 3.0) about the calculated values using the given expressions is 1.9%, and the deviation has a value of 2% using a and b given in Tables (1) and (2), the in-between values of a and b were obtained by interpolation. Values of RMS errors are listed in Table (4) for each R/h-value.

From the previous discussion, it is evident that the results have a good agreement with other published data. The comparison is satisfactory and it can be allowed to conclude that the proposed empirical expressions are valid to calculate Ω(R/h, S/h) without serious discrepancies, in the given ranges of R/h and S/h. In addition, the MC results have relative small statistical errors and short computer time. Finally, our MC-program is also constructed to calculate the average solid angles in the case of non-axial disk source. Such calculations are presently being investigated.