A ONE DIMENSIONAL HEART MODEL

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ABSTRACT

A one-dimensional heart model is proposed in this paper. It consists of a segment representing the (S-A) node and three long cables representing atrial, purkinje, and ventricular fibers. A time delay is introduced between the last atrial and the first purkinje potentials to account for the (A-V) node effect. Parameters of an ionic model of a heart cell are estimated to give the recorded action potential pattern as well as the speed of conduction for each region. Cable equation is solved for each region. A hypothetical one-dimensional electrocardiographic potential (ECG) is proposed as an output of the model. The obtained pattern of this (ECG) has the main features of the well known (ECG) pattern.

Keywords: Atrium, Purkinje fiber, Ventricle, Sino-atrial node, Atrio-ventricular node, Endocardium, Epicardium.

INTRODUCTION

The conduction system of the heart is a specialized neuromuscular set of tissues responsible for pulse generation, transmission, and distribution to each cell of the myocardium. It includes the sinoatrial (S-A) mode, atrial tracts, atrio-ventricular (A-V) node, bundle of His, right and left bundle branches, and their ramifications of subendocardial purkinje fibers.

The electric pulse is normally generated at the (S-A) node and then transmitted to the (A-V) node through the atrial fibers. The (A-V) node delays the pulse for about 70 msec. Then a rapid transmission of the pulse is effected through the bundle of His and the two bundle branches till it reaches the intramyocardial purkinje fibers. Finally, the pulse crosses the myocardial wall in an almost lateral pattern [1].

Each of the above neuromuscular tissues has its own electrical characteristics; such as membrane capacitance, membrane ionic conductance, intracellular resistivity, and fiber dimensions. These characteristics are reflected on the electrical performance and therefore affect both the action potential pattern and the pulse conduction speed.

THEORETICAL BASIS

In this work we offer a one-dimensional model of the heart, where each part is represented by an active long cable. As shown in Figure (1), the (S-A) node is represented by a one segment. The atrium is represented by a cable 4 cm long. The (A-V) node is represented by a delay of 70 msec. The group of purkinje fibers including the bundle of His, the two bundle branches, and the intramyocardial purkinje terminals are represented by a single purkinje cable of length 8 cm. The model is terminated with a myocardial cable 1.5 cm long representing the ventricular wall.

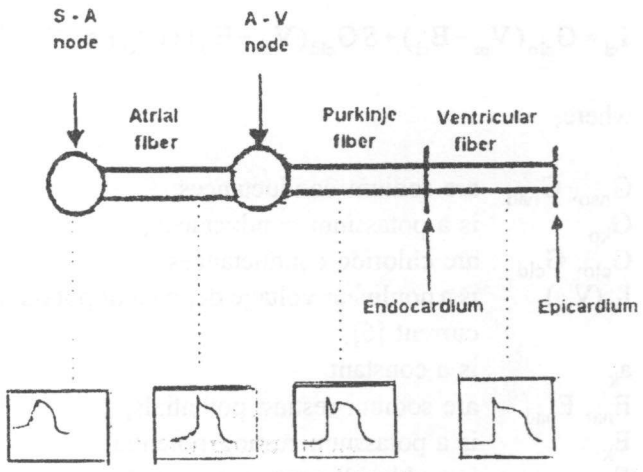

Figure 1. A hypothetical one-dimensional heart model. The action potential pattern as recorded from each region of mammalian heart.

To study propagation of the membrane potential (V_m) in one-dimension along a uniform fiber, the cable equation should be considered. This equation relates (V_m) as a function of time (t) and space (x) as follows [2];

$$\frac{a}{2\rho_i}\frac{\partial^2 V_m}{\partial x^2} = C_m\frac{\partial V_m}{\partial t} + J_m \qquad (1)$$

where a is the fiber radius, ρ_i is the intracellular resistivity, C_m is the membrane capacitance per unit surface area, and J_{io} is the ionic membrane current per unit area.

The cell-to-cell junction resistance [3] is neglected in our analysis. Also the extra cellular resistivity is considered too small to be calculated in our analysis [2].

The ionic current (J_{io}) is a voltage and time dependent function. It is the sum of all ionic currents, namely sodium, potassium, and chlorine. Several mathematical models could be used to calculate (J_{io}) such as the (MNT) model [4] for purkinje fibers and the (BR) model [5] for ventricular fibers.

In this work the current components are calculated according to a simplified ionic model of a heart cell previously offered by the author [6] as follows;

$$I_{na} = G_{nao}(V_m - E_{na}) + SG_{nad}(V_m - E_{nad})\left(\frac{t}{\tau_{Na}}\right)^3 e^{-t/\tau_{Na}} \qquad (2)$$

$$I_K = G_{Ko}(V_m - E_K) + S\alpha_K I_K(V_m)(t/t_K)\, e^{-t/t_K} \qquad (3)$$

$$I_{cl} = G_{clo}(V_m - E_{cl}) + SG_{cld}(V_m - E_{cl})(t/t_{cl})\, e^{-t/t_{cl}} \qquad (4)$$

where,

G_{nao}, G_{nad}	are sodium conductances,
G_{ko}	is a potassium conductance,
G_{clo}, G_{cld}	are chloride conductances,
$I_k(V_m)$	is a nonlinear voltage dependent potassium current [6],
a_k	is a constant,
E_{na}, E_{nad}	are sodium resting potentials,
E_k	is a potassium resting potential,
E_{cl}	is a chloride resting potential,
θ_{Na}, θ_k, θ_{cl}	are time constants for sodium, potassium, and chloride gates respectively,
S	is a time switch that has a value of unity at the beginning of depolarization process when the membrane potential exceeds its threshold value.

The ionic current components as derived by equations (2-4) are enough to reconstruct the action potential pattern for (S-A) node, atrial and ventricular fibers. To reconstruct the action potential of a purkinje fiber with a notch at the end of phase 1, we have to add a third component of slow sodium current (I_{nas}). This extra component is expressed by;

$$I_{nas} = SG_{nn}(V_n - E_{nn})\left(\frac{t}{\tau_{nn}}\right)^3 e^{-t/\tau_{nn}} \qquad (5)$$

where, G_{nn}, E_{nn}, and θ_{nn} are conductance, resting potential, and time constant of the slow sodium component respectively.

The cable equation (1) was solved by using the Crank-Nicolson method [7] for each region. Parameters of ionic currents were adjusted to give the required action potential pattern and speed of propagation according to Table (1).

A hypothetical one-dimensional electrocardiogram (ECG) was assumed to be obtained from the cross-wall ventricular potential by subtracting the membrane potential of the epicardium (the last ventricular potential V_{vepi}) and the endocardial purkinje membrane potential (the last purkinje potential $V_{pr\ endo}$). To add the contribution of atrial potentials, the last cell atrial membrane potential ($V_{at\,L}$) was subtracted from that of the first cell (V_{ato}). The results were reduced by a factor of 0.1 to be consistent with the fact that the effect of atrial potentials are small compared with that of the ventricles.

The proposed one-dimensional (ECG) were taken according to the following simple equation;

$$ECG = (V_{pr\,endo} - V_{v\,epi}) + 0.1(V_{at\,o} - V_{at\,L}) \qquad (6)$$

RESULTS AND CONCLUSIONS

The obtained parameters of the ionic current components for each region of our model are given in Table (2).

Fiber	Diameter (μm)	Intracellular resistivity ρ_i (Ω-cm)	Membrane capacitance C_m (μF/cm^2)	Spontaneous rhythm pulse-to-pulse (msec)	Max. depol. rate of rise (dv/dt) max (V/sec)	Action pot. duration (m/sec)	Conduction speed (m/sec)	Threshold pot. Vth (mV)
(S-A) node	---	---	[*] 1	[*] 1000	[9] 1	[9] 130	[8] 0.05	[9] -40
Atrial fiber	[10] 10	[*] 150	[12] 30	[*] 1200	[11] 80	[11] 200/250	[12] 0.8	[*] -70
Purkinje fiber	[11] 30	[14] 154	[14] 11.3	[*] 1300	[11] 610	[6] 400	[14] 3.5	[*] -70
Ventricular fiber	[11] 16	[15] 470	[15] 0.81	[*] 1500	[12] 350	[11] 200/300	[12] 1.0	[*] -70

* Estimated value

Table (1) : Some characteristic values measured from Atrial, Purkinje, and ventricular fibers

Fiber	C_m normalized	g_{Nao} g_{Nad} g_{NN} (nu/cm^2)	τ_{Na} τ_{NN} (msec)	g_{clo} g_{cld} (nu/cm^2)	τ_{cl} (msec)	g_{ko} (nu/cm^2) α_k	τ_k (msec)	E_{clo} E_{cld} (mv)	E_{Na} E_{Na} E_{NN} (mv)
S-A	1	0.0042 0.03 ---	15 ---	0.0025 0.06	30	0.004 0.25	200	-70 -40	+50 +30 ---
Atrium	1	0.004 1.0 ---	0.4 ---	0.001 0.04	10	0.004 0.25	450	-70 -20	+50 +30 ----
Purkinje	1	0.0033 25 1	0.2 25	0.001 0.4	2	0.00121 0.7	415	-70 -20	+50 +30 ----
Ventricle	1	0.0035 12 ---	0.3 ---	0.001 0.15	6	0.001 0.5	450	-70 -20	+50 +30 ----

Table (2) : Model constants for Atrial, Purkinje, and ventricular fibers

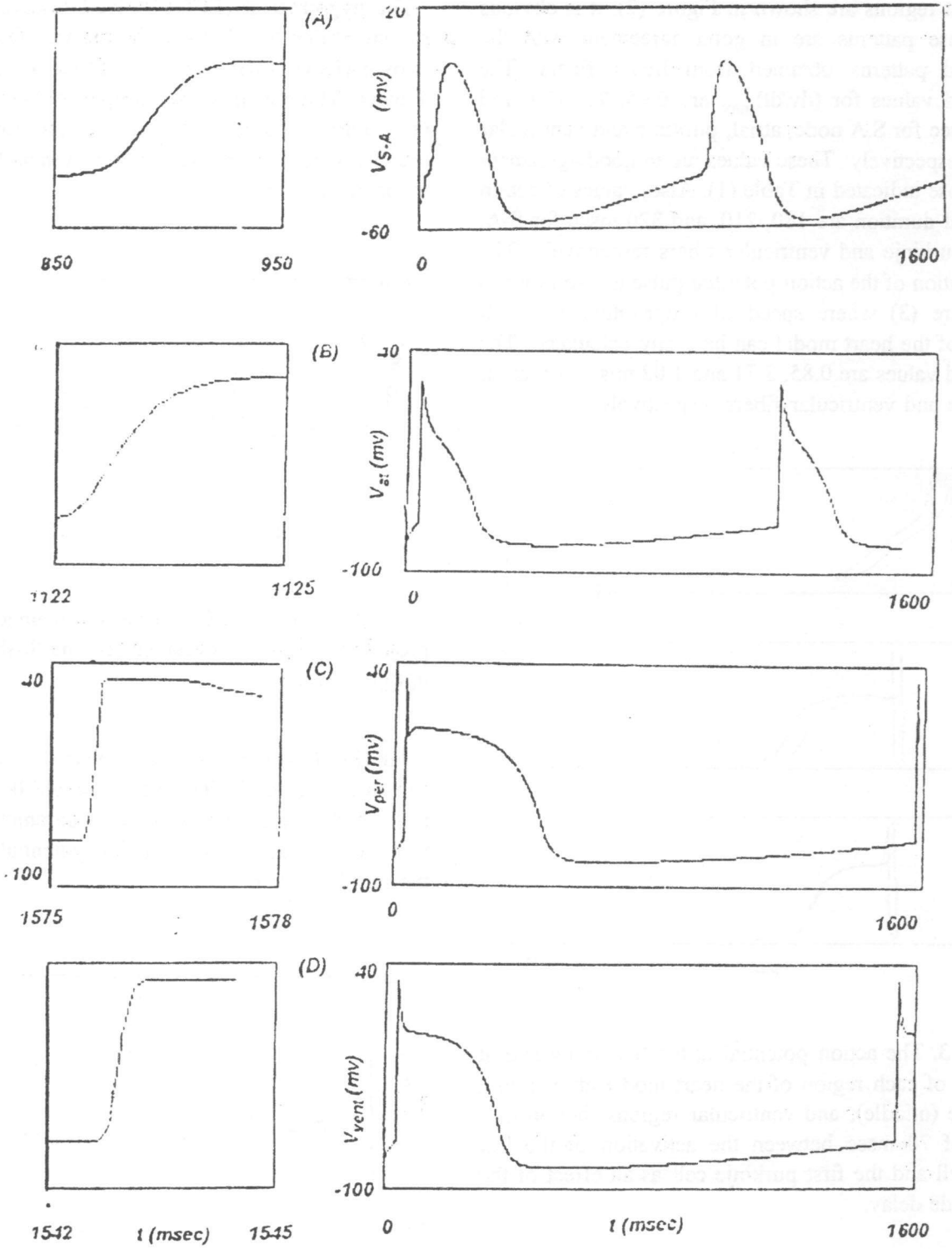

Figure 2. The action potential patterns obtained from the proposed model at different heart regions; a) S-A node, b) Atrial fiber, c) Purkinje fiber, and d) Ventricular fiber.

The reconstructed action potential patterns for different regions are shown in Figure (2), it is obvious that these patterns are in good agreement with the recorded patterns obtained from living fibers. The obtained values for $(dv/dt)_{max}$ are 0.95, 70, 770, and 380 V/sec for S.A node, atrial, purkinje and ventricular fibers respectively. These values are in good agreement with those indicated in Table (1). Also, values of action potential duration are 180, 210, and 320 msec for S-A, atrial, purkinje and ventricular fibers respectively. The propagation of the action potential pulse is also obvious in Figure (3) where speed of propagation for each region of the heart model can be easily calculated. The obtained values are 0.85, 3.71 and 1.02 m/sec for atrial, purkinje and ventricular fibers respectively.

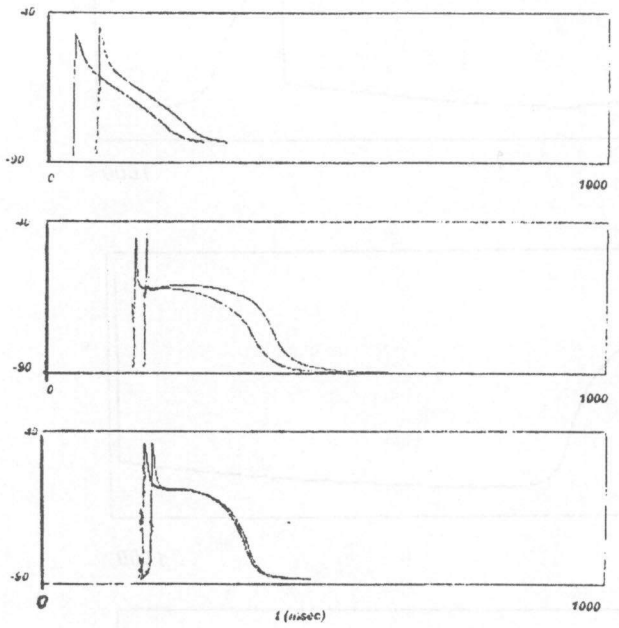

Figure 3. The action potential at the beginning and at the end of each region of the heart model atrial (top_, purkinje (middle), and ventricular regions (bottom). A delay of 70 msec between the activation of the last atrial cell and the first purkinje cell as an effect of the A-V node delay.

These results as good enough to give the characteristic shape of the action potential for each region specially when we consider the fact that the difference between the recorded data is considerably large [11].

The hypothetical (ECG) derived from our model is shown in Figure (4) it shows the main features of the known (ECG), namely, P wave, QRS complex, and T-wave. The obtained normal upright T-wave is due to the longer action potential duration of the intramyocardial purkinje fiber than that the working myocardial fiber.

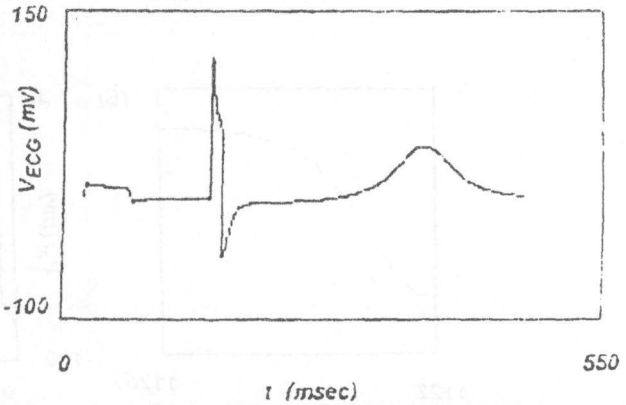

Figure 4. The (ECG) pattern obtained from the proposed model in normal conditions. It shows normal upright T-wave.

An (ECG) pattern with an inverted T-wave is also shown in Figure (5). This pattern could be obtained by a slight change of any of the ionic parameters in a way that tends to decrease the action potential duration of the purkinje fiber.

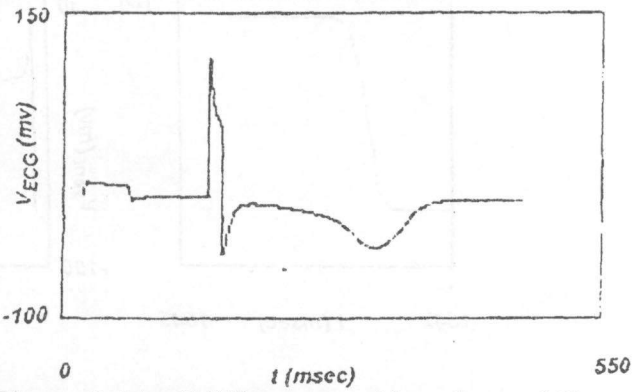

Figure 5. The (ECG) pattern with an inverted T-wave. This pattern was obtained by increasing the potassium conductivity of purkinje fiber by only 2%.

From the above results we may conclude the following:

1- The simplified ionic model of the heart cell can be used to express the electric activity of any fiber of the heart by only adjusting its parameters.
2- The proposed one-dimensional heart model can be used to study the normal electrical events as well as the effect of any regional change or defects within the heart at the ionic level.
3- The T-wave pattern is very sensitive to the change of any of the ionic parameters which may explain the non-specific T-wave changes of the normal (ECG) pattern.

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