POTENTIAL OF GPS FOR VERTICAL CONTROL

Sayed El Naghi and M. M. Rashad El -Din Moustafa
Transportation Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt

ABSTRACT

The Global Positioning System (GPS) represents the most effective new technique in modern surveying. The present work is devoted mainly to demonstrate the use and potential of GPS for vertical control in surveying works. The main objective of the present study is to investigate the reliability and potential of the GPS technique as compared to the classic technique for performing the vertical control. The prospects for vertical control using GPS data should be thoroughly investigated, for its importance in many surveying purpose. The Potential of the Global Positioning System Data for vertical control is presented and thoroughly investigated. The Sources of errors in GPS is introduced.

Keywords: GPS, Potential vertical control, Height determination, Height differences, Reference ellipsoid.

1. INTRODUCTION

Although an accurate method for height determination, classical leveling is costly, time consuming, laborious, and tedious, especially when applied in areas with large height differences, between points that are large distances apart, mountainous terrain, or restricted because of the lack of inter visibility. The advent of the Global Positioning System (GPS) has alleviated such issue [1] and offered a considerable solution for this problem.

In their orbits, GPS satellites positions are computed with respect to a reference ellipsoid, that best approximates globally the shape of the earth in a purely mathematical concept. This reference ellipsoid, adopted by the GPS since January 1987, is the world Geodetic System of 1984 which known by (WGS84) [2].

2. PRINCIPLES

Heights, as already known by surveyors and other users which are used within the standard engineering operational projects or routine mapping missions, are related to the Geoid rather than the ellipsoid. The former being an invisible physical surface which is difficult to the consistently located and to be mathematically represented. However, it represents the actual irregular shape of the earth, and defined as the equipotential surface approximated by the Mean Sea Level (M.S.L) under equilibrium with the gravity field of the earth.

There is, therefore, a lot of sense in using the arthometric height, the height of a point with respect to the geoid measured along the slightly curved plumb line normal to the geoid, as shown in Figure (2) [3].

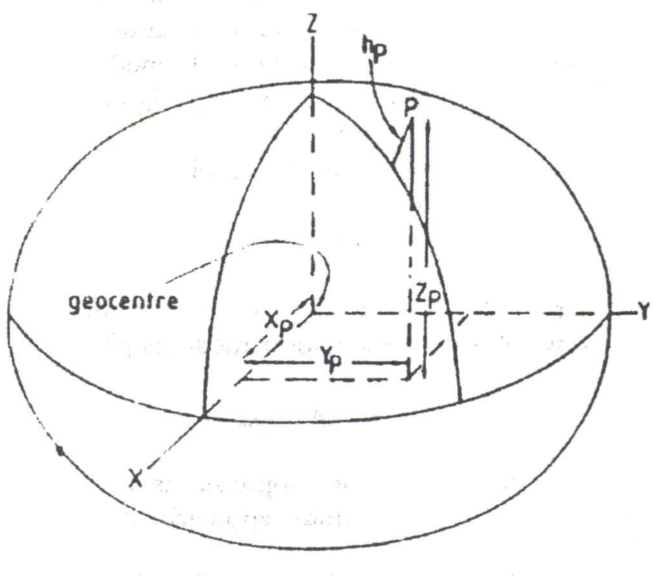

Figure 1. A point P shown relative to a geocentric ellipsoid and its cartesian coordinate system (XYZ).

The relation between the GPS - derived ellipsoidal height (h) and the geoid - referenced orthometric height (H) involves the Geoid Undulation (N), the geoid - ellipsoid separation known as the geoidal height. It is shown in a gross exaggeration in Figure (2).

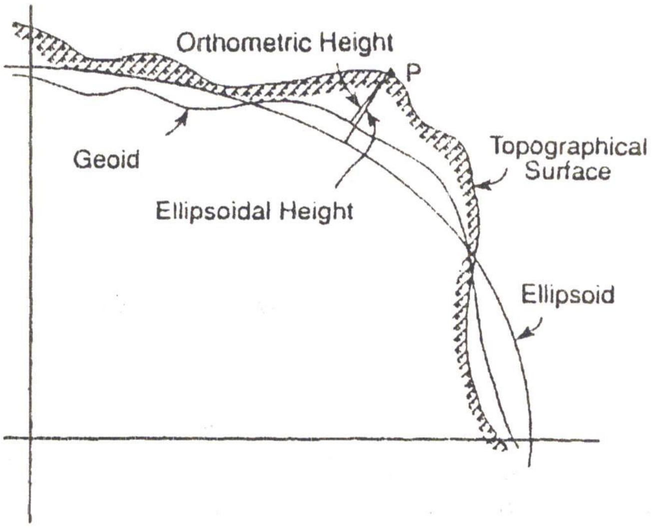

Figure 2. Part of that meridian of geocentric ellipsoid containing point P.

When the geoid undulation is accurately evaluated, the orthometric height can then be easily determined by subtracting the undulation from the ellipsoidal height. This can be performed either by the single point (absolute) approach, Figure (3-a) [4]:

In which the mathematical model is given by :

$$H = h - N \quad (1)$$

or by the differential (relative) approach, Figure (3-b), in which the mathematical model is given by:

$$\Delta H = \Delta h - \Delta N \quad (2)$$

Obviously, the relative approach is much more precise than the absolute approach due to the followings:

1. As the difference in the ellipsoidal height, measured simultaneously by the GPS data between two points, are much more accurate than the absolute ellipsoidal height at either of the terminals because of the presence of the same systematic errors at the terminals which cancel the difference.
2. ΔN is much more precise than N at either of the terminals. The precision required for H will depend upon the purpose for which the heights are being used. Some tasks will only require H for a few meters, in such the constraints on the determinations of h and N can be relaxed. For the highest order requirements, the precision to which Δh can be found, limits the precision of ΔH and dictates the precision requirements for ΔN which need to match is precision so that the precision of ΔH will not be seriously eroded [5].

For simplicity, h and H in Eq (1) and Figure (3) are considered to be along the common vertical but really, H is normal to the geoid while h is normal to the ellipsoid. The angle between the normal to the geoid and the normal to the ellipsoid is commonly referred to as The Deflection Of The Vertical which does not exceed 30 arc seconds in most areas and therefore its effect can be easily ignored compared to present uncertainties of geoid undulation estimates.

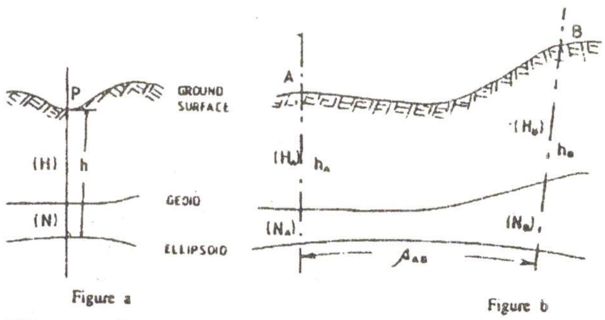

Figure 3. Relationship between ellipsoidal, orthometric, and geoidal heights for relative heighting.

Obviously, the errors in evaluating H or ΔH totally depend mainly upon the accuracy of the parameters used in its evaluation; Viz. the ellipsoidal height and the geoid undulation [3,4].

3. GPS - DERIVED ELLIPSOIDAL HEIGHTS

The results of many tests and operational projects have clearly shown that GPS survey methods can efficiently replace classical horizontal terrestrial survey methods comparable accuracies have also been achieved for GPS - derived ellipsoidal height differences. Such differences may be obtained with uncertainties approaching (0.2 cm + 0.01 - 0.1 ppm). These uncertainties depend on the significance of error sources. Such errors associated with GPS data can be minimized by adhering to appropriate specifications and procedures [3].

4- ERROR'S EFFECT ON GPS -HEIGHTING

Various kinds of errors affect GPS -derived ellipsoidal heights. In the last few years, research works have been directed towards evaluating and modeling of such errors. Next, errors affecting GPS-Heighting are introduced in some detail together with different studies in this concern.

4-1 Effect of the Geometry of the Satellite Configuration

Santerre (1989), conducted an investigation of the impact of GPS Satellite sky distribution on the propagation of errors in accurate relative positioning, by studying the behavior of covariance matrix, the confidence ellipsoid, and correlation coefficients in a least squares solution as function of satellite sky distribution, station coordinates, clock and tropospheric zenith delay. It was found that even if the system is fully operational, unmodelled errors will still significantly affect the final solution [7].

4-2 Effect of the Orbital Errors

The error in Satellites orbit may be defined by its three components :

- Along-track, the direction of motion.
- Radial, the direction from satellite to earth, and
- Across-track (perpendicular to the other two).

Beutler et al (1989) found out that the along track component has the greatest impact among others in height determination. In the particular case of a single satellite passing through the zenith of a ground station, it was found that an along track error of 1" in the plane of the observer as viewed from the ground station will result in a rotation of a network by 1" about an axis perpendicular to the orbit plane affecting the height component. It was also found that the error is maximum when the direction of the baseline is the same as the direction of the orbit plane [5,6].

The height error may be expressed as [6]:

$$e_{\Delta h} = \cos\,(A_{Z_s} - A_{Z_b})\,\frac{\Delta s}{\rho}\,b \qquad (3)$$

where:

$e_{\Delta h}$: the magnitude of the height error

A_{Z_s}: the azimuth of the orbital plane of the satellite

A_{Z_b}: the azimuth of the baseline

Δs: the along - track error

b: the baseline length

ρ: range of the satellite

Table 1. shows the effect of orbital uncertainty on the baseline height differences.

Table 1. Effect of Orbit Uncertainties on baseline height difference.

Δs	rho	Height Error
20 m	20 000 km	1 ppm
40 m	20 000 km	2 ppm

4.3 Effect of Troposphere

The Troposphere is generally the major source of error in height determination. Beutler et al (1987 b) reported that an error of 1mm in the zenith distance of the relative tropospheric refraction will cause a height error of approximately 2.9 mm [6].

Two methods have been suggested by Grant (1987) to minimize error in the heights due to a deferential residual error in the tropospheric correction between two stations. The first is to model such error at each station as a time invariant error in the solution.

The success of this method will depend on how stable the troposphere was during the observing session. The second method is to model the residual error at each station and at each epoch using a Kalman filter. The success of this method will depend on how well the dynamic model reflects the changing troposphere [5].

In addition, kouba (1987) has adapted experience with VLBI measurements to introduce a model that express the effect of wet troposphere on baseline height differences as [6]:

$$\sigma_{\delta h} = S^2 \sqrt{\frac{1 - \exp^{-\frac{b^2}{d^2}}}{b^2}} \quad (4)$$

where,

$\sigma_{\delta h}$: the error in height difference in ppm.

S: constant, taken as 80 mm.

b: baseline length

d: the correlation distance usually taken as 30 km.

The effect is shown in Figure (4).

4.4 Effect of Ionosphere

The effect of the ionospheric delay reaches its maximum when the satellite is near the horizon and is a minimum when the satellite is at the zenith. Depending upon the separation of the two receivers and the stability of the ionosphere, double differecing phase measurements between station sites will tend to cancel most of the ionospheric delay. As the ionospheric delay is frequency dependent, measurements made simultaneously on both L1 and L2 frequencies will eliminate most or almost all of the ionospheric correction. However a residual error in the relative ionospheric delay between two stations will be reflected as an error in the GPS height difference. This would tend to occur over long baseline, particularly those oriented north - south [5,6,7].

4.5 Effect of Antenna

The antenna phase center may also be dependent on the vertical angle to a satellite and this will affect the height determination. Mitchell et al (1990), stated that the whole error can only be better antenna design. Accurately measuring the height of the antenna phase center before and after data collection will probably reduce this effect [5,9].

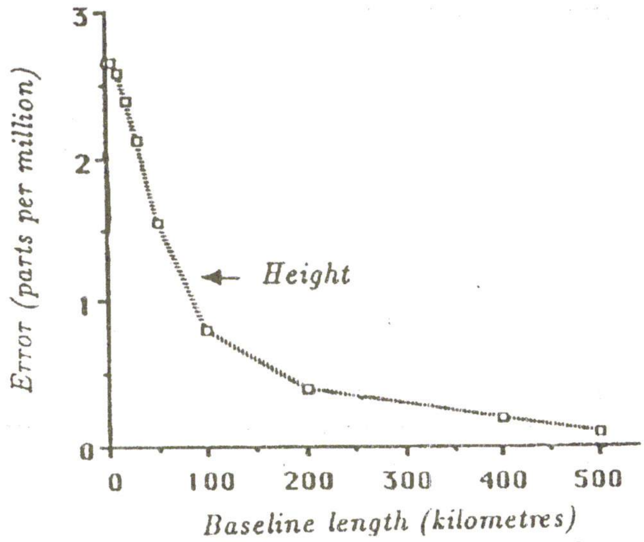

Figure 4. The effect of modeling errors of wet tropospheric refraction on height difference.

4.6 Effect of Multipath and Imaging:

Multipath and Imaging, also has an effect that depends on the antenna design and the location of the reflecting surface in the vicinity of the receiver, which is variable from site to another. Therefore, it is not possible to determine the magnitude of such effect. Multipath effects are site and antenna independent and therefore will not cancel out when double differenced between receivers. However, Tranquilla (1988) has found that, due to its cyclic nature, if observation periods are kept longer, it will tend to randomize. On the other hand, Mitchell et al (1990) stated that its effects can be reduced by the use of a well designed antenna which minimizes interference and possibly incorporating an absorbent ground plane to cut out signal reflection [5,6].

4.7 Effect of Timing

The satellite and receiver clock errors for differential positioning have three components:

- an epoch offset from the Universal Coordinated Time (UTC)
- an epoch difference between the two receivers
- a time rate difference between the two receivers and the satellites.

An epoch offset from the UTC, common to both receivers, will result in the satellite ephemerides being interpolated for incorrect time. King et al (1985) stated that, the receiver clocks need to be synchronized to UTC within 7 milliseconds for a base line error below 1 ppm. If the two receiver quartz clocks are synchronized to each other within 3 microseconds, that error reduced below 1 cm. The satellite and receiver clock errors are eliminated by differencing in the solution for baseline components [5].

5- THE GPS HEIGHTING STUDY GROUP:

Bar charts of the most important errors affecting baseline height differences derived by GPS is shown in Figure (5) for two baseline of length 5 and 50 kilometers. The tropospheric contribution (σ_{trop}) is estimated using Eq. (4) The value 2ppm for ionospheric delay (σ_{iono}) is estimated using CERN networks reported in Santerre (1989).

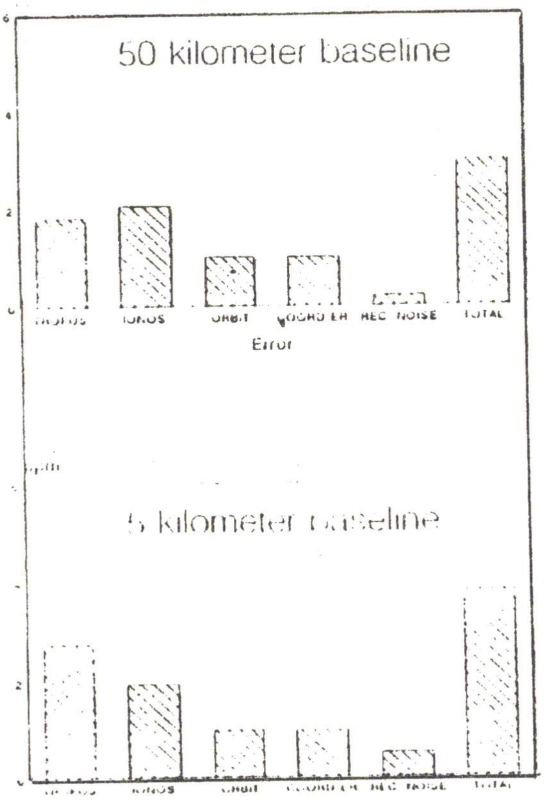

Figure 5. The estimated magnitude of errors (in parts per million) of height differences.

The satellite ephemeris is assumed to have an along track error of 20 m and the effect on height (σ_{orb}) is estimated using the Eq. (3). The effect of (σ_{cord}) is estimated from a computer simulation carried out by Holloway (1988).

Receiver noise and any residual errors (σ_{nois}) is estimated as 5 mm irrespective of baseline length. The total uncertainty of each baseline height ($\sigma_{\Delta h}$) is then calculated from [6].

$$\sigma^2_{\Delta h} = \sigma^2_{trop} + \sigma^2_{ion} + \sigma^2_{orp} + \sigma^2_{cord} + \sigma^2_{nois} \quad (5)$$

In Dec. 1990, another simulation has been conducted by the GPS Heighting Study Group. The simulations performed in this study were carried out, again, on two baseline, 5 and 50 km. long, using the full 18 satellite constellation for an assumed network.

Figure (6) shows that when the ambiguities are not resolved, the dominant height error is the tropospheric error and is proportionally much the same for both baseline. The receiver noise error is constant and therefore has a much greater influence on the shorter line. The other errors are proportionally very similar to each other. Figure (7) shows that by resolving the ambiguities correctly the error in easting coordinates is improved dramatically. The total receiver noise error has also been improved. The dominant error for the height component is still the residual tropospheric error with the error in the ionosphere and the fixed station coordinates also being significant.

It is also apparent that the total height error is not improved whether it was possible for the ambiguities to be resolved or not, ever though the error in the easting and northing components are improved [5].

6- EXPERIENCED PRECISION OF GPS-HEIGHTING

Estimates of precision of GPS-derived ellipsoidal height differences have been obtained by many researchers. These estimations are usually quoted as errors in height over baseline length in parts per million -A selection is shown in.

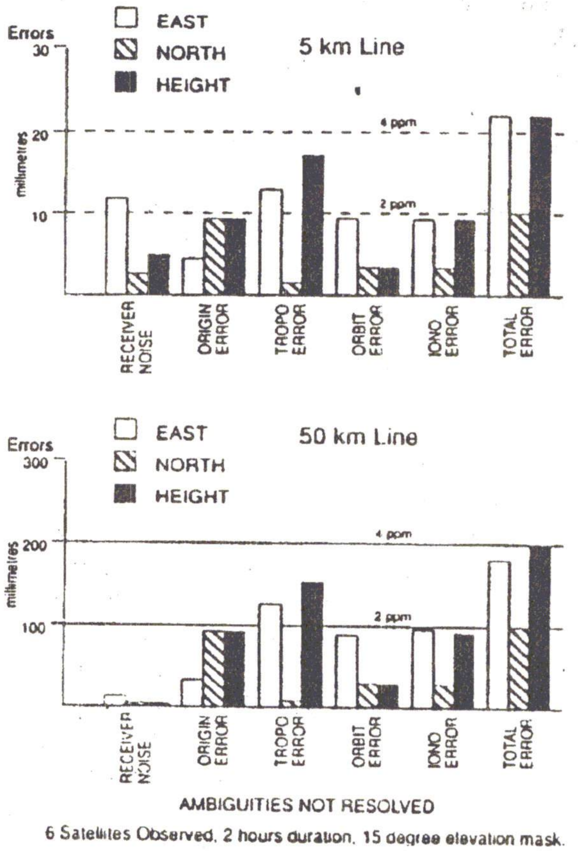

Figure 6. Simulated errors in 5 km band 50 km baseline.

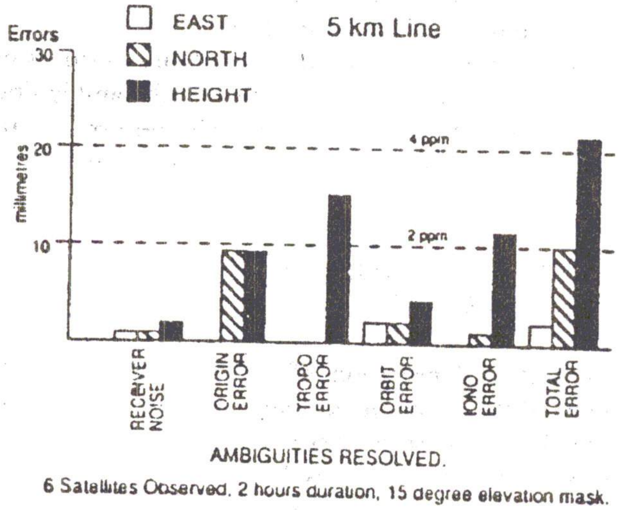

Figure 7. Simulated error in 5 km baseline,

Table 2.

Precision	Author
about 1.6 ppm	Engelis and Rapp, 1984
3 ppm	Schwarz and Sederis, 1985
2 to 3 ppm	Schwarz et al, 1987
to 3.2 ppm	Holloway, 1988
± (0.5 cm + 1 to 2 ppm)	Zilkoski and Hothem, 1988
1 to 3 ppm	Kearsley, 1988b
to 3.5 ppm	Leal, 1989
1 to 2.5 ppm	Kleusberg,1990
1 ppm	Abou-Beih, 1993

7.1 Geopotential Models

The global geoid can be represented by means of geopotential models, i.e. mathematical models in the form of spherical harmonics. The coefficients of the various terms in the series are determined using a combination of satellite orbit analysis, terrestrial gravity and N measured by satellite altimetry over the ocean. The geopotential model for N is expressed as [5,6]:

$$N = \frac{KM}{GR} \sum_{n_{max}}^{n=2} \sum_{m=0}^{n} P_{nm}(\cos\theta) \quad (8)$$
$$(C^{*}n,m \cos m\lambda + S^{*}n,m \sin m\lambda)$$

Where :

R: is the radius of the spherical model of the earth.

G: is the mean gravity of the earth.

KM: is the geocentric gravitational constant times the earth mass.

θ,λ: are polar distance, longitude.

$C^{*}n,m$, $S^{*}n,m$: are fully normalized potential coefficients.

Pn,m (cos θ): are legendre polynomials.

n,m: are degree, order of the spherical harmonic term.

7.2. Astrogeodetic Levelling

Astro-geodetic levelling results are normally related to a local ellipsoid.

Therefore, to transform GPS derived (h) into orthometric height (H), the separation between the geoid and a geocentric ellipsoid datum is needed.

Even though they are useful in establishing the transformation parameters between a local earth model and the World Geodetic Datum (WGS84), values of N evaluated from Astro-Geodesy are of limited use for transforming GPS heights into orthometric heights. Schwarz et al (1987) stated that the astrogeodetic data base is not dense enough, and the costs to upgrade it would be prohibitive and only gravimetric methods will therefore be the major method among others in evaluating the geoid-ellipsoid separation. Furthermore, Mitchell et al (1990) stated that the accuracy obtained from such method is not satisfactory promising as very few Laplace observations are made today, and the number must be expected to decrease since GPS is becoming much more popular [5,8].

8. PROPOSED TECHNIQUE

In recent years, the term Gravimetric methods has been expanded to include like least squares collection, mentioned above, which can use measurements other than gravity anomalies Schwarz et al (1987) used certain data types for the determination of N ; namely a geopotential model, point or mean gravity anomalies, and a detailed digital elevation model, and introduced a solution that can be written as :

$$N = N_{GM} + N_{\Delta g} + N_h \qquad (9)$$

where :

N_{GM}: is the contribution of the geopotential model.

$N_{\Delta g}$: is the contribution of the gravity anomalies.

N_h: is the contribution of the ellipsoidal heights.

Similarly, geoid height differences can be written as:

$$\Delta N = \Delta N_{GM} + \Delta N_{\Delta g} + \Delta N_h \qquad (10)$$

Figure (8) shows the different contributions for a typical geoid of 100 km length in mountainous terrain. H must be noted that N_{GM} changes very smoothly over a distance of 100 km while $N_{\Delta g}$ represents regional and local geoid features, and N_h, which changes rapidly specifically in mountainous terrain and usually has small amplitudes, represents wavelength features below 20 km that are caused by the topography [8].

Figure (9). shows the error, in ppm for each of the three components. The total error in is clearly governed by the ϵ_{GM} and the other two components are rather smaller in comparison. Finally the gravimetric determination of can be done with an accuracy of about 3ppm for distances between 10 and 100 km which makes it compatible with the current accuracy of GPS-derived Δh [8].

9. CONCLUSION

The Potential of the Global Position System Data for vertical control is presented and thoroughly investigated. The sources of errors in GPS is introduced.

It is concluded that orthometric height differences can be determined, from the geoid - ellipsoid separation and the GPS - derived ellipsoidal heights using either of the following approaches:

i) Simply ignoring the geoid ellipsoid separation will give errors which may be up to 50 ppm.
ii) Using geoid maps, including those representing astrogeodetic results.
iii) Using geopotential models, which is inexpensive and suited to an accuracy of about 5 ppm.
iv) Using gravimetric determinations,assumed to exclude geopotential models alone but to include all combination methods covered earlier, would lead to high accuracy of about 2ppm.
v) By the geometric methods, i.e. interpolation, possibly with surface fitting, between other points at which GPS observations have been made -and possibly in combination with methods referred to in (iii), may give an accuracy of up to 4ppm, or better over shorter distances.
vi) By a combinations of methods, most particularly, those at (iv) and (v) to achieve much better accuracy.

On the other hand, Zilkoski and Hothem (1989) recommended the following strategies when levelling networks are used in conjunction with GPS networks in orthometric height determination [2,9]:

All leveling data used to establish the heights

should be corrected for known systematic errors.
In addition, Dodson, A. H. and Gerrard, S.M.E. (1990) investigated leveling with GPS on test networks throughout England and Wales and concluded that the GPS- derived orthometric height differences can achieve accuracies as good as those produced by tertiary leveling over short distances and expected that equal accuracies can be maintained over longer baseline when taking a good care in processing the field data derived from the GPS. They suggested that, unlike traditional levelling, GPS heighting accuracy is less dependent upon distance. However, they suggested using the leveling at short distances, i.e. less than 1 km. Finally the results of many experiences with the GPS, indicate to a great extend the promising reliability and accuracy of this new technique for establishing a precise base for vertical control operation. Consequently, it can be safely recommeded to use the GPS as a reliable and accurate technique for vertical control, taking into consideration the effect of the arising errors in this techniques.

REFERENCES :

[1] O.M. Abou-Beih, (1993), *Towards The Determination of Accurate Orthometric Heights In Remote Areas From GPS Measurements and Gravity Data*, Published at Ain Shams University Bulletin, vol. 28, 1993.

[2] M.A. Baraka, H.A. Emam (1991), *Preliminary Investigation of Orthometric Height Determination In GPS Local Networks In Egypt*, published at Al-Azhar Engineering Second International Conference, December 21-24, 1991.

[3] B. Hofmann-Wellenhof, H. Lichtenegger and J. Collins (1992), *GLOBAL POSITIONING SYSTEM : Theory and Practice*", Springer-verlag Wien NewYork, 1992.

[4] A.H.W. Kearsley, (1988), *The Determination of the Geoid Ellipsoid Separation for GPS leveling* ", *the Australian Surveyor*, vol. 34, No.1, March, 1988.

[5] H.L. Mitchell, A.H.W. Kearsley,, J.R. Gilliland, V. Argeseanu, R.D. Holloway and P.J. Morgan, *GPS Heightening and the A.H.D.*, Technical Report, the University of Newcastle, N.S.W., Australia, December, 1990.

[6] C.L. Robinson, (1991) *Verification of Gravimetric Geoidal Models By A combination of GPS and Orthometric Heights*, M. Sc. E. Thesis, Department of Surveying Engineering Technical Report No. 152, University of New Brunswick, Canada, May 1991.

[7] R. Santerre, (1989) *GPS Satellite Sky Distribution: Impact on the Propagation of Some Important Errors in Precise Relative Positioning*, Ph. D. Dissertation, Department of Surveying Engineering Technical Report No.145, University of New Brunswick, Canada, May 1991.

[8] K.P. Schwarz, M.G. Sideris and R. Farsberg, (1987), Orthometric Heights Without Leveling *Journal of Surveying Engineering*, vol. 113, No.1, Feb.1987.

[9] D.B. Zilkoshi and L.D. Hothem, (1989), "GPS Satellite Survey and Vertical Control", *Journal of Surveying Engineering*, vol. 115, No. 2, May, 1989.

ACCURATE METHODS OF CURVE SETTING, ASSESSMENT AND COMPARISON

M. Rashad El-Din Moustafa
Dept., of Transportation Eng., Faculty of Engineering, Alexandria University, Egypt.

Magued N. Abdel-Malek and Abdel-Hamid Mohamed Allakany
Dept. of Eng., Math., and Physics, Faculty of Engineering, Alexandria University, Egypt.

ABSTRACT

The purpose of this paper is to assess and compare the field positioning methods of horizontal curves for first- grade roads and railway alignment. Classic and novice methods involving angular measurements are treated. Mathematical models for error analysis are developed; and comparison study is made using error ellipses. Keywords: Horizontal curves - Settingout - Accuracy - Error analysis

1 - INTRODUCTION

Accurate methods of curve setting-out usually involve angular measurements from one or more stations; either exclusively or associated with linear measurements. In this work, three methods of circular curve ranging are mainly investigated. The first and second procedures are the well known traditional methods. " Dual-theodolite method " and "Deflection angles or Rankine's method". The third is the less known novice method of "Optimum point method". The three methods are evaluated through a comparative study using the positional error accuracy determination employing error ellipses technique.

2 - SETTING OUT PROCEDURES

2.1 *Deflection Angles Method (DAM/Rankine's Method)*

A deflection angle to any point on the curve is the angle at the point of tangency T_1 between the back tangent and the chord from T_1 to that point. (see Figure (1)).

Rankine's method is based on the principle that "the deflection angle to any point on a circular curve is measured by one-half the angle subtended by the arc from T_1 to that point. It is assumed that the length of the arc is approximately equal to its chord. The last approximation is very reasonable when the radius (R) is equal or greater than 20 times the chord length.

Let T_1 I : rear tangent,
T_1 : point to curve (P.C.)
θ_1, θ_2, θ_3 , ...: the tangential angles or the angles which each of the successive chords $T_1 P_1$, $P_1 P_2$, $P_2 P_3$ etc.
makes with the respective tangent to the curve at T_1, P_1, P_2 etc.

Y_1, Y_2, Y_3, ...:total tangential angles or deflection angles to the point P_1, P_2, P_3, etc. (Punmia, 1975).
ι_1, ι_2, ι_3 ... = lengths of the chords $T_1 P_1$, $P_1 P_2$, $P_2 P_3$,

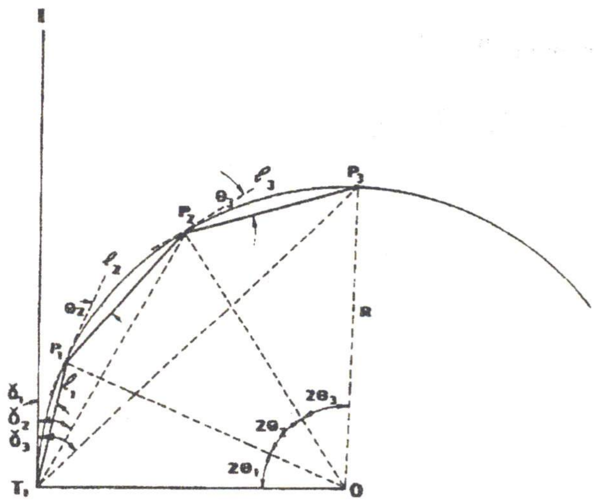

Figure 1. Rankine's method.

From the property of the circle.

angle $IT_1 P_1$ = half the angle $T_1 OP_1$
angle $T_1 OP_1 = 2\,\theta_1$

In case of chord $T_1 P_1$ = arc $T_1 P_1$

$$\text{Now, angle } \theta_1 = \frac{90\iota_1}{\pi R} \text{ (Degrees)},$$

$$\theta_2 = \frac{90\iota_2}{\pi R}$$

In general,

$$\theta_n = \frac{90\iota_n}{\pi R}, \text{ where, } \iota_n \text{ is normal chord.}$$

From the geometry of (Figure (1)), the deflection angle of the first point P_1 is equal to its tangential angle or

$$\tau_1 = \theta_1,$$

for the second point P_2

angle $IT_1 P_2$ = half the angle $T_1 OP_1$

i.e.

$$\text{angle } IT_1 P_2 = \theta_1 + \theta_2$$

$$\gamma_2 = \theta_1 + \theta_2$$

for point P_3.

$$\gamma_3 = \theta_1 + \theta_2 + \theta_3,$$

Generally for point P_n,

$$\gamma_n = \theta_1 + \theta_2 + \theta_3 + \dots + \theta_n$$

In case of equal chords

$$\theta_1 = \theta_2 = \theta_3 = \dots = \theta_n = \theta$$

then $\gamma_1 = \theta$
$\gamma_2 = 2\,\theta$
and $\gamma_n = n\,\theta$

2.2 Dual-Theodolite Method O.P.M.

In this method, two theodolites are used, one at T_1 and another at T_2. This method is used when the ground is unsuitable for chainage and is based upon the principle that "the angle between the tangent and the chord is equal to the angle which that chord subtends in the opposite segment" (see Figure (2)).

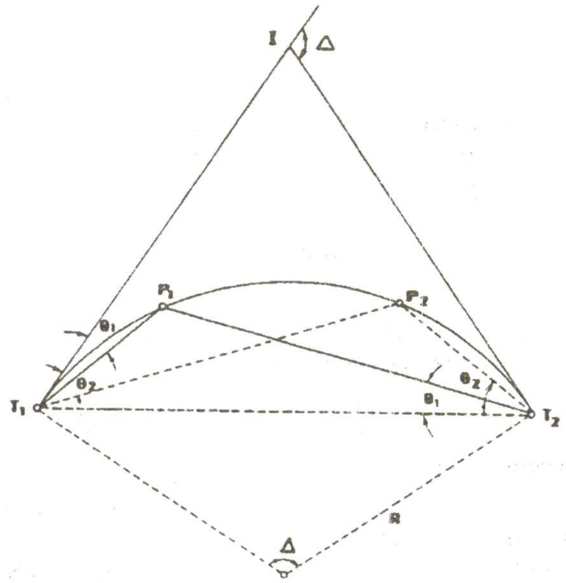

Figure 2. Two theodolites method.

Deflection angles to points on curve are the same as in the previous method. The actual steps of setting out for both methods can be found out in many text-books.

2.3 Optimum Point Method

In this method, horizontal curve is set out from a specific vantage point referred to as optimum point (OP). Many alternative vantage points can be chosen for setting- out purposes depending on field condition and economic consideration, all points within the curve vicinity are referred to a local co-ordinate system which has the (OP) as its origin [5].

Vantage Point

It is defined as the best point which can be chosen considering cost and/or field conditions for the ranging out of an entire curve. It is therefore not a fixed position. A reference point is chosen to which any vantage point can be referred and which can equally serve as a vantage point. Due its dual-role, this reference point is referred to as the optimum point (OP).

Choice of optimum point

The (OP) is the best vantage point which can be chosen for the purpose of ranging out an entire curve when theoretical and practical application are considered simultaneously. In other words to satisfy the above definition, the (OP) should conform to certain conditions as discussed below.

Figure (3) shows a circular curve AC_1 d_1 d_2 C B, A is the point of curvature (P.C) and B is the point of tangency (P.T.). The choice of (OP) is based on the following conditions:

1. The ratio of two distances from any pair of corresponding opposite points to the (OP) should be 1:1.
2. The sum of all distances involved in setting out should be minimum.
3. To be theoretically and practically versatile in application, there should be simple relationship between the OP and other curve parameters for any type of curve.

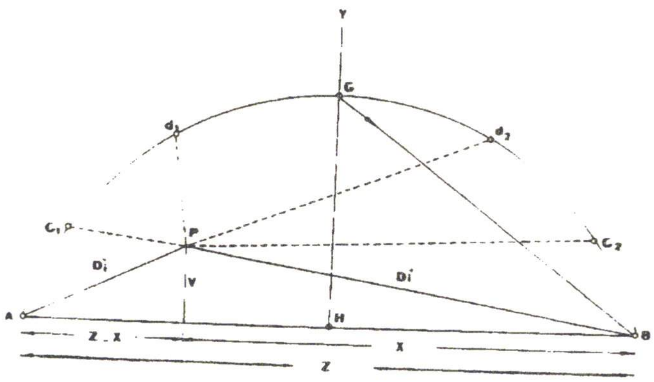

Figure 3. Determination of the optimum point.

To satisfy these conditions it has been shown [5] that the optimum point lies somewhere between G and H.

Two methods can be used in setting out according to optimum point method. These are summarized in the following.

2.3.1 *OPM by Radial Angles and Chord Lengths*

To set out the curve by this method (Figure (4)), the instrument is stationed at point (H) and angles "q" are turned out. The curve points are then fixed as the inter- sections of the rays and the chord lengths.

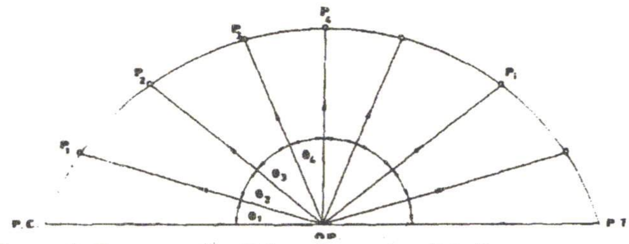

Figure 4. Concept of radial angles and radial distances.

It is very clear that, setting-out curves by using this approach is analogous to the method of deflection angles in case of choosing point (G) as the optimum point. In case of choosing point (H) as optimum point, double intersection maybe occur between the chords and the radial lines. Setting out curves by using deflection angles method (DAM) avoid this defect even in case of curves with greater deflection angles.

2.3.2 *OPM by Radial Angles and Radial Distances*

This involves the use of polar rays. The distance "d" to the curve point is calculated and the direction of the line defined by the angle "θ" the curve points are then located by measuring the distances along the radial lines. The chord length (ι) to be used for setting out procedure is usually known from job specification (it may be taken 20 m tape length). Angle "θ" and distance (d) are computed involving the solution of the respective triangles shown in Figure (5).

2.3.3 *Advantages of the Optimum Point Method*

1. Use of (OP) reduces the number of instrument stations.
2. In case of using (OPM), the location of any curve point is not strictly dependent on the preceding point.
3. The (OPM) technique is quite flexible in the

sense that its computations can be adapted for other methods of the curve ranging.

4. The (OPM) technique satisfies the different field conditions.

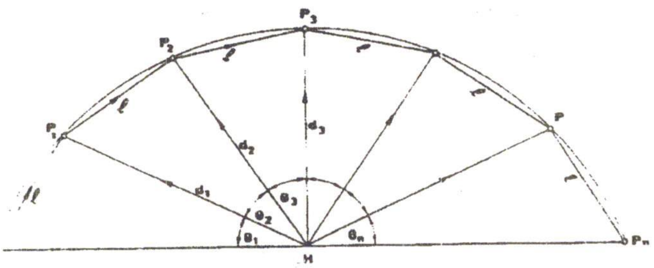

Figure 5. Concept of radial angles and chord lengths.

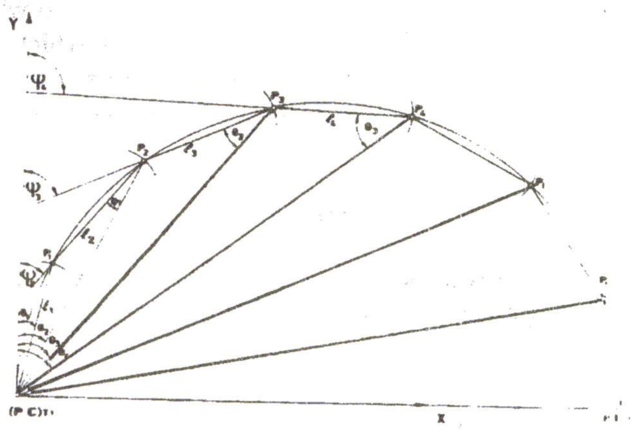

Figure 6. Deflection angles method coordinates system.

3- MATHEMATICAL MODEL FOR ERROR ANALYSIS

A brief description on the concepts of convariance, correlation, propagation of variances and covariances and Error Ellipse Technique is shown in [1].

The mathematical models for the three methods under study are derived in the following sections.

3.1 *Deflection Angles/Rankine's Method*

Referring to Figure (6) the mathematical model for error analysis is developed. Adopting the theodolite station T_1 is chosen as origin, and the Y-axis as the tangent at T_1.

Figure (6) shows a circular curve T_1, P_1, P_2,...P_n .. T_2 while ι_1, ι_2, ι_3,...ι_n ... are the chords joining the curve points. Let Ψ_1, Ψ_2, Ψ_3,...Ψ_n , etc be the corresponding angles between the above chords and the y-axis respectively.

Referring to Figure (6) the coordinates of the n^{th} point, on the curve, P_n (x_{pn}, Y_{pn}) are

$$x_{pn} = \iota_1 \sin \Psi_1 + \iota_2 \sin \Psi_2 + \iota_3 \sin \Psi_3 + ... + \iota_n \sin \Psi_n$$

$$y_{pn} = \iota_1 \cos \Psi_1 + \iota_2 \cos \Psi_2 + \iota_3 \cos \Psi_3 + ... + \iota_n \cos \Psi_n \quad (1)$$

For simplicity, let Eqs. (1) take the form

$$\left.\begin{array}{l} x_{pn} = \sum_{i=1}^{n} = \iota_i \sin \Psi_i \\ x_{pn} = \sum_{i=1}^{n} = \iota_i \cos \Psi_i \end{array}\right\} \quad (2)$$

Derivation of the mathematical model

According to the setting out technique, location of the point P depends on the quantities ι_i and Ψ_i (i=1,2,3,..,n) The relation between the bearing angles Ψ_i and the deflection angles θ_i, is given by

$$\Psi_i = \theta_{i-1} + \theta_i \quad (3)$$

the angles θ_i are measured separately with equal precision, then

$$\left.\begin{array}{l} \sigma_{\theta_1} = \sigma_{\theta_2} = \sigma_{\theta_3} = \sigma_{\theta_n} = \sigma_\theta \\ \sigma_{\theta_i} = \sigma_{\theta_j} = 0.0 \qquad \text{for all values of i and j} \end{array}\right\} \quad (4)$$

In case of adopting equal chords of length ι let σ_ι be the standard deviation of the measured chord, then

$$\left.\begin{array}{l} \iota_1 = \iota_2 = \iota_3 = \quad \iota_n = \iota \\ \sigma_{\iota_1} = \sigma_{\iota_2} = \sigma_{\iota_3} = \quad \sigma_{\iota_n} = \sigma_\iota \end{array}\right\} \quad (5)$$

Since these chords are measured independency then

$$\sigma_{\iota_i \iota_j} = 0.0$$

Differentiating Eqs. (3) partially with respect to the angles θ_i and substituting into variance-covariance propagation equations [1] gives:

$$\left.\begin{array}{l} \sigma^2_{\Psi_1} = \sigma^2_\theta \\ \sigma^2_{\Psi_2} = \sigma^2_{\Psi_1} = ... \qquad \sigma^2_{\Psi_n} = 2\sigma^2_{\theta}, \\ \sigma_{\Psi_1\Psi_2} = \sigma_{\Psi_2\Psi_3} = \sigma_{\Psi_3\Psi_4} = \sigma_{\Psi_n} - 1_{\Psi_{n-1}} = \sigma_\theta \end{array}\right\} \quad (6)$$

For all values of i and j, (except i = j+1), values of $\sigma_{\Psi 1\ \Psi 2} = 0.0$.

For circular curve with peg interval of length ι, the deflection angles θ_i and the bearing angles Ψ_i are given by:

$$\theta_i = i\ \frac{D}{2} \text{ and } \Psi_i = \frac{D}{2}(2i-1) \tag{7}$$

where D is the degree of the curve.

By differentiating equations (2) partially with respect to the quantities ι_i and Ψ_i and substituting into variances- covariances equations, taking equations (3,4,5,6 and 7) into consideration, the mathematical model of positioning error for a general point P_n on the curve is given by:

$$\left.\begin{aligned} \sigma^2_{x_{pn}} &= \sigma^2_{x_{p1}} + \sigma_i^2 m_i + 2g(n_i + A_i) \\ \sigma^2_{y_{pn}} &= \sigma^2_{y_{p1}} + \sigma_i^2 n_i + 2g(m_i + B_i) \\ \sigma_{xy_{pn}} &= \sigma_{xy_{p1}} + C_i(\sigma_i^2 - 2g) - gE_i \end{aligned}\right\} \tag{8}$$

where

$$m_i = \sum_{i=2}^{n} \sin^2 [\frac{D}{2}(2i-1)]$$

$$n_i = \sum_{i=2}^{n} \cos^2 [\frac{D}{2}(2i-1)]$$

$$A_i = \sum_{i=2}^{n} \sin^2 [\frac{D}{2}(2i-1)] \sin[\frac{D}{2}(2i-3)]$$

$$B_i = \sum_{i=2}^{n} \cos^2 [\frac{D}{2}(2i-1)] \cos[\frac{D}{2}(2i-3)]$$

$$C_i = \frac{1}{2}\sum_{i=2}^{n} \sin\ [D(2i-1)]$$

$$E_i = \sum_{i=2}^{n} \sin\ [\frac{D}{2}(2i-1)] \cos[\frac{D}{2}(2i-3)]$$

$$+ \cos\ [\frac{D}{2}(2i-1)] \sin[\frac{D}{2}(2i-3)]$$

and

$$g = \iota^2\ \sigma_\theta^{\ 2}$$

3.2 *Dual Theodolite Method*

Development of mathematical model

Choice of coordinate system:

Choosing the long chord T_1 T_2 and the perpendicular to it at point T_1 (P.C.) as the two axes x and y respectively (Figure (7).

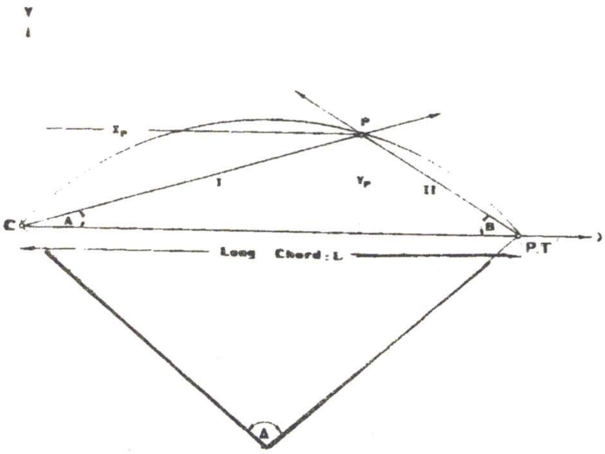

Figure 7. Two theodolites method Co-ordinates system.

The coordinates of the points T_1 (x_{t1}, y_{T1}) and T_2 (X_{T2}, y_{T2}) are (0,0) and (L,0) respectively, where L denotes the length of the long chord (T_1 T_2).

The coordinates of the point $P(x_p, y_p)$ are given by [7].

$$\left.\begin{aligned} X_p &= \frac{(Y_{T1} - Y_{T2}) + x_{T1}\cot B + X_{T2}\cot A}{\cot A + \cot B} \\ Y_p &= \frac{(x_{T2} - x_{T1}) + Y_{T1}\cot B + Y_{T2}\cot A)}{\cot A + \cot B} \end{aligned}\right\} \tag{9}$$

Substituting $X_{T1} = 0$, $Y_{T1} = 0$, $X_{T2} = L$ and $Y_{T2} = 0$ into Eqs. (9) we get

$$\left.\begin{aligned} X_p &= \frac{L\cot A}{\cot A + \cot B} \\ Y_p &= \frac{L}{\cot A + \cot B} \end{aligned}\right\} \tag{10}$$

Equations (9) may be simplified to

$$\left.\begin{aligned} X_p &= k\cot A \\ Y_p &= k \end{aligned}\right\} \tag{11}$$

where

$$k = \frac{L}{\cot A + \cot B}$$

To obtain variances and covariance for the positioning of point P, differentiating Eqs. (10) partially with respect to the two measured quantities A and B, substituting into variances and covariances equations gives:

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,(\mathrm{cosec}^4 A \cot^2 B\, \sigma^2_A + \mathrm{cosec}^4 B \cot^2 A\, \sigma^2_B \\ \sigma^2_{y_p} &= q\,(\mathrm{cosec}^4 A\, \sigma^2_A + \mathrm{cosec}^4 B\, \sigma^2_B) \\ \sigma_{xy} &= q\,[-\mathrm{cosec}^4 A \cot B\, \sigma^2_A + \mathrm{cosec}^4 B \cot A\, \sigma^2_B) \\ &+ \mathrm{cosec}^2 A\, \mathrm{cosec}^2 B\, \sigma_{AB} (\cot A - \cot B)] \end{aligned}\right\} \quad (12)$$

where

$$q = \frac{L^2}{(\cot A + \cot B)^4}$$

Eqs. (12) constitute a mathematical model which deter- mines the variances and covariance $\sigma^2_{x_p}, \sigma^2_{y_p}$ and σ_{xy_p} for the located point P.

Since the two orientation angles A and B are measured separately and error in measuring the first angle A causes no effect in measuring B, it follows that, there is no correlation between A and B.

i.e : $$\alpha_{AB} = 0.0 \quad (13)$$

A substitution of Eqs. (13) in Eqs. (12) gives

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,(\mathrm{cosec}^4 A \cot^2 B\, \sigma^2_A + \mathrm{cosec}^4 B \cot^2 A\, \sigma^2_B \\ \sigma^2_{y_p} &= q\,(\mathrm{cosec}^4 A\, \sigma^2_A + \mathrm{cosec}^4 B\, \sigma^2_B) \\ \sigma_{xy_p} &= q\,(-\mathrm{cosec}^4 B \cot A\, \sigma^2_B - \mathrm{cosec}^4 A \cot B\, \sigma^2_B) \end{aligned}\right\} \quad (14)$$

Further, the angles A and B are assumed to be measured with equal precision. This gives

$$\sigma_A = \sigma_B = \sigma \quad (15)$$

where

σ is the angular standard deviation.

Substituting from Eq. (15) into Eqs. (14) one gets

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,\sigma^2(\mathrm{cosec}^4 A \cot^2 B + \mathrm{cosec}^4 B \cot^2 A \\ \sigma^2_{y_p} &= q\,\sigma^2(\mathrm{cosec}^4 A + \mathrm{cosec}^4 B) \\ \sigma_{xy_p} &= q\,\sigma^2(\mathrm{cosec}^4 B \cot A - \mathrm{cosec}^4 A \cot B \end{aligned}\right\} \quad (16)$$

For the circular curve, the sum of the two angles A and B is equal to half the central angle (Δ) i.e. ($A + B = \frac{\Delta}{2}$) and this leads to

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,\sigma^2 \mathrm{cosec}^4 A \cot^2(\tfrac{\Delta}{2} - A) \\ &+ \mathrm{cosec}^4(\tfrac{\Delta}{2} - A) \cot^2 A] \\ \sigma^2_{x_p} &= q\,\sigma^2[\mathrm{cosec}^4 A + \mathrm{cosec}^4(\tfrac{\Delta}{2} - A)] \\ \sigma_{xy_p} &= q\,\sigma^2[\mathrm{cosec}^4(\tfrac{\Delta}{2} - A)\cot A \\ &- \mathrm{cosec}^4 A(\cot(\tfrac{\Delta}{2} - A] \end{aligned}\right\} \quad (17)$$

Replacing angle A with B in Eqs. (17) gives the variances and covariance for the corresponding opposite point p'.

On inspecting the resulting error equations (17) we find that

i. for curves having the central angle (Δ) less than 90, and standard deviation in x-direction (σ_x) is always greater than that in the y-direction (σ_y).
ii. For central angle ($\Delta = 90$), it is easy to prove that the error in x-direction σ_x is a constant value equal to ($L\ \sigma$).
iii. For the summit point, where $A = B = \theta$, position on determination referred to x and y directions is correlation-free. This can be proved as follows

$$\left.\begin{aligned} \sigma^2_x &= 2q(\mathrm{cosec}^4\theta)\cot\theta(\sigma^2) \\ \sigma^2_y &= 2q(\mathrm{cosec}^4\theta)(\sigma^2) \\ \sigma_{xy} &= 0.0 \end{aligned}\right\} \quad (18)$$

and if $\Delta = 90°$, then

$$\sigma_x{}^2 = \sigma_y{}^2 = 8\,q\,\sigma_2 \quad (19)$$

Eq. (19) shows that positioning determination of the summit point in case of ($\Delta = 90$) is accomplished with equal accuracy in the x and y directions.

Also, in case of $\sigma_A = \sigma_B$ it is easy to prove that, the orientation of the error ellipse does not depend on the precision of the angular measurements (σ).

It is evident from Eqs. (17), in order to determine the two corresponding points p, p' with same positioning accuracy (($\sigma_{x_p} = \sigma'_{x_p}$, $\sigma_{y_p} = \sigma'_{y_p}$ and $\sigma_{xy_p} = \sigma_{xy_{p'}}$), the two theodolites, used for laying out the curve, must be of the same precision. This was satisfied by the assumption given in Eq. (15).

3.3 *Optimum Point Method (Version I)*

Choice of coordinate system:

choosing the long chord and the perpendicular bisector as two arbitrary axes x and y respectively where point (OP) is the origin.

Let point P(x,y) be located on the curve, Figure (8), to establish the point (P), distance d is measured along the line making an angle θ with the x-axes. Distance d and angle θ are calculated by using simple coordinate geometry. The coordinates of point (P) are

$$\left.\begin{aligned} x &= d\cos\theta \\ y &= d\sin\theta \end{aligned}\right\} \qquad (20)$$

Variance and covariance for the positioning of point P can be obtained by differentiating Eqs. (20) partially with respect to the measured quantities d and θ, substituting into variance and covariance equations:

$$\left.\begin{aligned} \sigma_x^2 &= \cos^2\theta\,\sigma_d^2 + d^2\sin^2\theta\,\sigma_\theta^2 \\ \sigma_y^2 &= \sin^2\theta\,\sigma_d^2 + d^2\cos^2\theta\,\sigma_\theta^2 \\ \sigma_{xy} &= \cos\theta\sin\theta(\sigma_d^2 - d^2\sigma_\theta^2) \end{aligned}\right\} \qquad (21)$$

Examining the resulting equations (21) the following remarks can be given:

i) Positioning accuracy for any located point on the curve depends on the precision of both angular and linear measurements.
ii) Linear measurements error causes the serious positioning error for points on the curve because such an error shifts the points away of the curve path i.e. radially inward or outward.

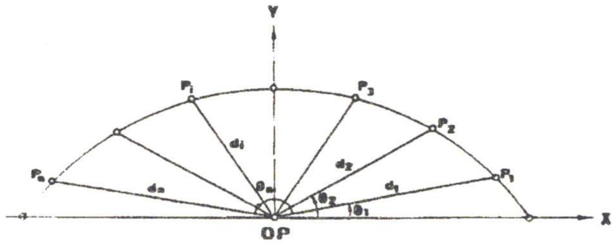

Figure 8. Setting out by optimum point (version I).

On the other hand, angular error causes an insignificant position error since it shifts the located point tangentially to the curve path.

3.4 *OPTIMUM POINT METHOD* (Version II)

In this method, the instrument is positioned at (OP') Figure (9). For determining variance and covariance for the locating points, the same procedure used in the previous methods is followed, Angles θ_1', θ_2', θ_3', ...θ_n' and distance d_1', d_2', d_2', ... d_n' are measured to establish the points P_1', P_2', P_3',... P_n' on the curve respectively. The mathematical model for obtaining the expected variance and covariance can be given by replacing θ and d with θ' and d', in equation (21) respectively, then we get.

$$\left.\begin{aligned} \sigma_x'^2 &= \cos^2\theta'\,\sigma_d'^2 + d'^2\sin^2\theta'\,\sigma_\theta'^2 \\ \sigma_y'^2 &= \sin^2\theta'\,\sigma_d'^2 + d'^2\cos^2\theta'\,\sigma_\theta'^2 \\ \sigma_{x'y} &= \sin\theta'\cos\theta'(\sigma_d'^2 - d'^2\sigma_\theta'^2) \end{aligned}\right\} \qquad (22)$$

4- ANALYSIS OF RESULTS

Error ellipses were constructed according to the above mathematical models for the cases under study. Variations were adopted in degree of curve and observational accuracy. The cases chosen here include only curves with degree 6°, at observational accuracy σ (length) = 0.005 m, σ (angle) = 10" Figures (10-a), (10-b), (10-c), and (10-d) show the distribution of error ellipses.

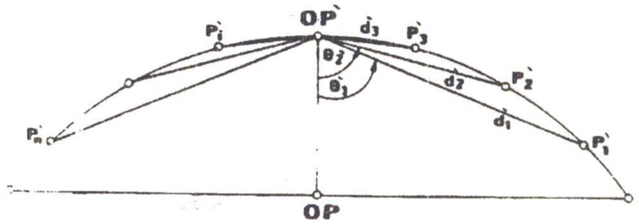

Figure 9. Setting out by optimum point (version II).

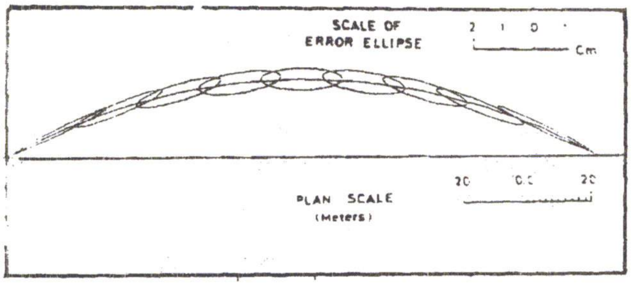

Figure 10-a.. Two theodolites method.

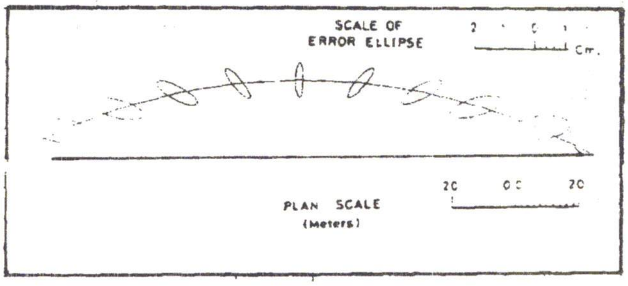

Figure 10-b. Optimum point method (I).

Graphical representation of error ellipses provides an illustration of the accuracy with which any point on a curve is positioned. Visual inspection of the distribution of error ellipses along the curve gives an invaluable aid for judging of setting out accuracy.

Radial component of error, σ_r, which can be scaled from the error ellipse graph is the most significant component of standard deviation. It gives a reliable indication about the accuracy of setting out along the curve. On the other hand, a large error component on the direction of the curve (tangential component) would be insignificant since the located point would still lie on the curve.

The maximum, minimum and average values of the radial error component σ_r among points distributed along the curve could be used as very convenient criteria to indicate homogeneity of positioning accuracy for different methods under investigation.

5- CONCLUSION

The methods under study can be assessed as follows:

5.1 *Rankine's Method*

i) Located points by using this method are dependent on each other since they are to be set out in chain. Accumulation of error can be observed as we move away from the starting point of the curve. Major axes of the error ellipses on the curve run approximately along the line joining this point with the starting point.
ii) Symmetrically positioned points are located with different accuracy.

5.2 *Dual Theodolite Method*

i) Major axes of the error ellipses are tangential to the curve path while minor axes are perpendicular which is ideal for positioning accuracy.
ii) For curves with larger radii, larger sizes of the error ellipses can be observed.
iii) The method also shows adequate homogeneity of accuracy distribution since radial components (σ_r) are nearly equal along the curve.
iv) Symmetrically positioned points with respect to midpoint of the curve are located with the same positioning accuracy.
v) As measurement's precision increases, positioning accuracy for located points increases.

5.3 *Optimum Point Method*

5.3.1 *Version I*

The corresponding cases for optimum point method version I, where the instrument position lies on the mid- point of the long chord, are represented in figure (10-c). The following remarks can be given:

i. Symmetrical point about the perpendicular bisector of long chord are located with equal accuracy.

ii. Linear error is the most significant factor affecting the positioning accuracy of the located points.
iii. Increasing the angular precision, makes the error ellipses more slim. While increasing the linear precision makes the major axes shorter.
iv. In case of smaller radii, orientation of error ellipses changes in such a way that the major axis turns quickly perpendicular to the curve path as compared with cases of greater radii

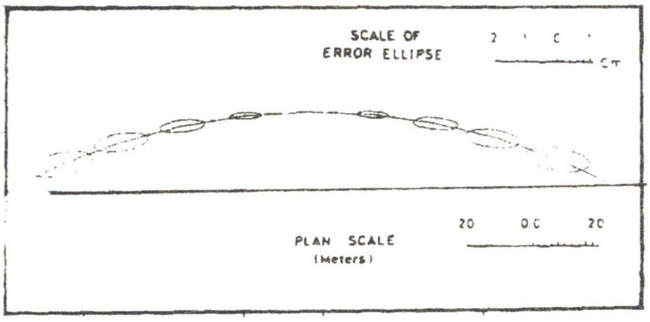

Figure 10-c. By optimum point (II).

5.3.2 *Version II*

i. Symmetrical points about the perpendicular bisector of long chord are located with equal accuracy.
ii. This method exhibits better positioning accuracy since the major axes of error ellipses go nearly tangential to the curve, especially in case of greater radii. Also it shows smaller radial error component s as compared with optimum point version I.
iii. As in the previous method (version I), increasing the precision of linear measurements causes a significant decrease in the length of major axes of the error ellipses and the increasing the angular precision leads to more slim ellipses which means a smaller radial component of error.

GENERAL

1. As expected, an increase in the precision of measurements used in setting out procedure, would increase the positioning accuracy. However, the effect is not always significant depending on the case.

2. For both cases of the optimum point method version I, where the instrument position lies on the mid-point of the long chord, and version II where the instrument position lies on the middle of the curve, the ellipse distribution is symmetrical with respect to the middle of the curve. Moreover there is no significant accumulation of error due to the fact that points on curve are located independently. Generally, version II is better than version I for the following reasons:

i) Error ellipses are generally oriented tangential to the curve in the case of version II and perpendicular to the curve in the case of version I. Therefore, an alignment using version II should be preferred due to the smaller radial error component σ_r.
ii) Linear error is the most significant factor contributing to the positioning error in case of version I while the angular error is significant in the case of version II.

3. Considering deflection angles method (Rankine), an accumulation of error can be observed as we move away from the starting point on the curve. It is clear that the effect of angular error in positioning accuracy for the first few points on curve is significant. Bad inter- section of position lines occurs in case of greater deflection angles at the end of the curve. On the other hand, this method exhibits small radial component in case of greater radii. Therefore, it is recommended to use this method in case of flat curves. Increasing angular precision causes a significant improvement in the positioning accuracy in case of long-flat curves.

4. In case of using two-theodolites method, symmetrical and more accurate positioning is achieved. The error ellipses are always oriented tangential to the curve. Also adequate homogeneity of positioning accuracy exists.

It could be noticed that, orientation of major axes of the error ellipses does not depend on the angular observational error but it depends only on curve configuration (i.e. its radius and deflection angle).

Finally to sum up:

Dual theodolite method can be categorized as the best method used for setting out circular curves since it shows an excellent point positioning as compared with other methods of setting out because of its indisputable merits over the other methods, it is worthwhile to recommend this method when high positioning accuracy is required. Optimum point method, version II, is the second best recommendation.

REFERENCES

[1] A.M. Allakany, "Evaluation of accuracy in curve setting out", *M.Sc. thesis*, Faculty of Engineering, Alexandria University, 1990.

[2] T.F. Hickerson, *Route surveys and design*, Fourth edition, McGraw-Hill, 568 pages, 1959.

[3] G.H. Igl, "Liability of a surveyor for error", *Surveying and mapping* (ACSM), Fall conversion, technical papers, September-October, pp. 24-29, 1976.

[4] C.F. Meyer, *Route surveying and design*, Fourth edition, International text book company, Pennsylvania (U.S.A.), 636 pages, 1969.

[5] O.C. Ojnnaka and N.K. Ndukwe, "Curve ranging by optimum point method", *Survey Review*, vol. 30, No. 1; pp 22-32, 1989.

[6] B.C. Punmia, *Surveying*, vol. 2, Standard Book House, Delhi, 1966.

[7] Richardus, P., *Project surveying*, North Holland Publishing Company, Amsterdam, 465 pages, 1975.

POTENTIAL OF GPS FOR VERTICAL CONTROL

Sayed El Naghi and M. M. Rashad El -Din Moustafa
Transportation Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt

ABSTRACT

The Global Positioning System (GPS) represents the most effective new technique in modern surveying. The present work is devoted mainly to demonstrate the use and potential of GPS for vertical control in surveying works. The main objective of the present study is to investigate the reliability and potential of the GPS technique as compared to the classic technique for performing the vertical control. The prospects for vertical control using GPS data should be thoroughly investigated, for its importance in many surveying purpose. The Potential of the Global Positioning System Data for vertical control is presented and thoroughly investigated. The Sources of errors in GPS is introduced.

Keywords: GPS, Potential vertical control, Height determination, Height differences, Reference ellipsoid.

1. INTRODUCTION

Although an accurate method for height determination, classical leveling is costly, time consuming, laborious, and tedious, especially when applied in areas with large height differences, between points that are large distances apart, mountainous terrain, or restricted because of the lack of inter visibility. The advent of the Global Positioning System (GPS) has alleviated such issue [1] and offered a considerable solution for this problem.

In their orbits, GPS satellites positions are computed with respect to a reference ellipsoid, that best approximates globally the shape of the earth in a purely mathematical concept. This reference ellipsoid, adopted by the GPS since January 1987, is the world Geodetic System of 1984 which known by (WGS84) [2].

2. PRINCIPLES

Heights, as already known by surveyors and other users which are used within the standard engineering operational projects or routine mapping missions, are related to the Geoid rather than the ellipsoid. The former being an invisible physical surface which is difficult to the consistently located and to be mathematically represented. However, it represents the actual irregular shape of the earth, and defined as the equipotential surface approximated by the Mean Sea Level (M.S.L) under equilibrium with the gravity field of the earth.

There is, therefore, a lot of sense in using the arthometric height, the height of a point with respect to the geoid measured along the slightly curved plumb line normal to the geoid, as shown in Figure (2) [3].

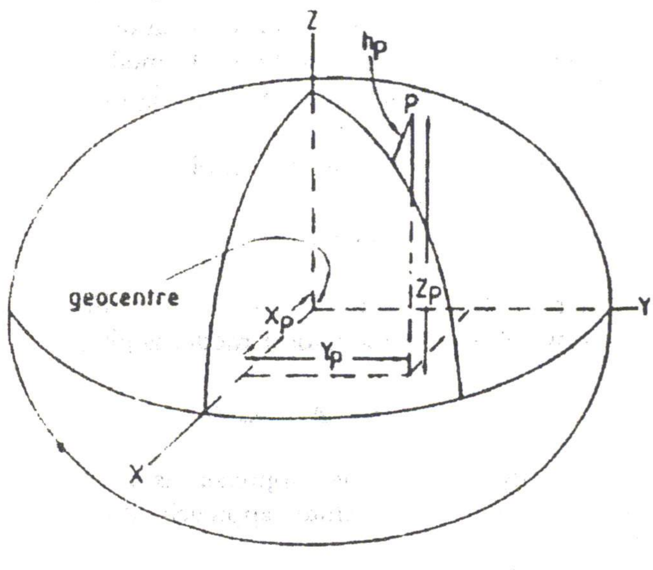

Figure 1. A point P shown relative to a geocentric ellipsoid and its cartesian coordinate system (XYZ).

The relation between the GPS - derived ellipsoidal height (h) and the geoid - referenced orthometric height (H) involves the Geoid Undulation (N), the geoid - ellipsoid separation known as the geoidal height. It is shown in a gross exaggeration in Figure (2).

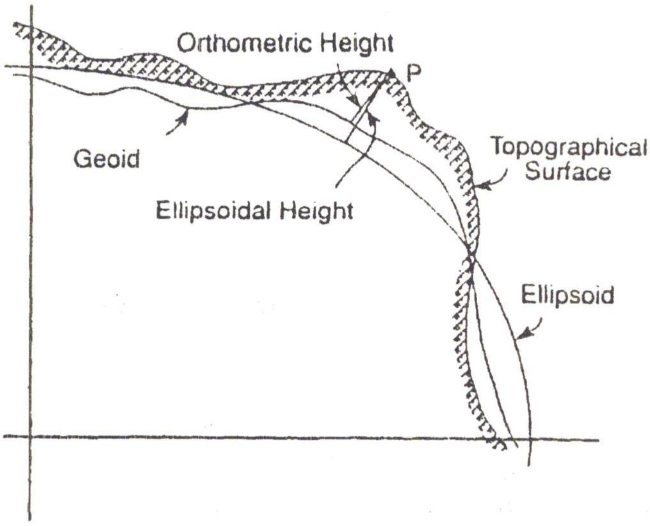

Figure 2. Part of that meridian of geocentric ellipsoid containing point P.

When the geoid undulation is accurately evaluated, the orthometric height can then be easily determined by subtracting the undulation from the ellipsoidal height. This can be performed either by the single point (absolute) approach, Figure (3-a) [4]:

In which the mathematical model is given by :

$$H = h - N \qquad (1)$$

or by the differential (relative) approach, Figure (3-b), in which the mathematical model is given by:

$$\Delta H = \Delta h - \Delta N \qquad (2)$$

Obviously, the relative approach is much more precise than the absolute approach due to the followings:

1. As the difference in the ellipsoidal height, measured simultaneously by the GPS data between two points, are much more accurate than the absolute ellipsoidal height at either of the terminals because of the presence of the same systematic errors at the terminals which cancel the difference.
2. ΔN is much more precise than N at either of the terminals. The precision required for H will depend upon the purpose for which the heights are being used. Some tasks will only require H for a few meters, in such the constraints on the determinations of h and N can be relaxed. For the highest order requirements, the precision to which Δh can be found, limits the precision of ΔH and dictates the precision requirements for ΔN which need to match is precision so that the precision of ΔH will not be seriously eroded [5].

For simplicity, h and H in Eq (1) and Figure (3) are considered to be along the common vertical but really, H is normal to the geoid while h is normal to the ellipsoid. The angle between the normal to the geoid and the normal to the ellipsoid is commonly referred to as The Deflection Of The Vertical which does not exceed 30 arc seconds in most areas and therefore its effect can be easily ignored compared to present uncertainties of geoid undulation estimates.

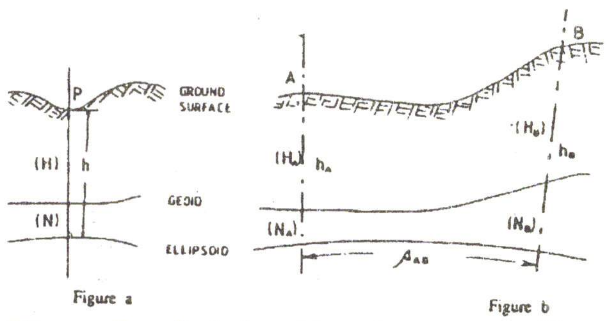

Figure 3. Relationship between ellipsoidal, orthometric, and geoidal heights for relative heighting.

Obviously, the errors in evaluating H or ΔH totally depend mainly upon the accuracy of the parameters used in its evaluation; Viz. the ellipsoidal height and the geoid undulation [3,4].

3. GPS - DERIVED ELLIPSOIDAL HEIGHTS

The results of many tests and operational projects have clearly shown that GPS survey methods can efficiently replace classical horizontal terrestrial survey methods comparable accuracies have also been achieved for GPS - derived ellipsoidal height differences. Such differences may be obtained with uncertainties approaching (0.2 cm + 0.01 - 0.1 ppm). These uncertainties depend on the significance of error sources. Such errors associated with GPS data can be minimized by adhering to appropriate specifications and procedures [3].

4- ERROR'S EFFECT ON GPS -HEIGHTING

Various kinds of errors affect GPS -derived ellipsoidal heights. In the last few years, research works have been directed towards evaluating and modeling of such errors. Next, errors affecting GPS-Heighting are introduced in some detail together with different studies in this concern.

4-1 Effect of the Geometry of the Satellite Configuration

Santerre (1989), conducted an investigation of the impact of GPS Satellite sky distribution on the propagation of errors in accurate relative positioning, by studying the behavior of covariance matrix, the confidence ellipsoid, and correlation coefficients in a least squares solution as function of satellite sky distribution, station coordinates, clock and tropospheric zenith delay. It was found that even if the system is fully operational, unmodelled errors will still significantly affect the final solution [7].

4-2 Effect of the Orbital Errors

The error in Satellites orbit may be defined by its three components :

- Along-track, the direction of motion.
- Radial, the direction from satellite to earth, and
- Across-track (perpendicular to the other two).

Beutler et al (1989) found out that the along track component has the greatest impact among others in height determination. In the particular case of a single satellite passing through the zenith of a ground station, it was found that an along track error of 1" in the plane of the observer as viewed from the ground station will result in a rotation of a network by 1" about an axis perpendicular to the orbit plane affecting the height component. It was also found that the error is maximum when the direction of the baseline is the same as the direction of the orbit plane [5,6].

The height error may be expressed as [6]:

$$e_{\Delta h} = \cos\,(A_{Z_s} - A_{Z_b})\,\frac{\Delta s}{\rho}\,b \qquad (3)$$

where:

$e_{\Delta h}$: the magnitude of the height error

A_{Z_s}: the azimuth of the orbital plane of the satellite

A_{Z_b}: the azimuth of the baseline

Δs: the along - track error

b: the baseline length

ρ: range of the satellite

Table 1. shows the effect of orbital uncertainty on the baseline height differences.

Table 1. Effect of Orbit Uncertainties on baseline height difference.

Δs	rho	Height Error
20 m	20 000 km	1 ppm
40 m	20 000 km	2 ppm

4.3 Effect of Troposphere

The Troposphere is generally the major source of error in height determination. Beutler et al (1987 b) reported that an error of 1mm in the zenith distance of the relative tropospheric refraction will cause a height error of approximately 2.9 mm [6].

Two methods have been suggested by Grant (1987) to minimize error in the heights due to a deferential residual error in the tropospheric correction between two stations. The first is to model such error at each station as a time invariant error in the solution.

The success of this method will depend on how stable the troposphere was during the observing session. The second method is to model the residual error at each station and at each epoch using a Kalman filter. The success of this method will depend on how well the dynamic model reflects the changing troposphere [5].

In addition, kouba (1987) has adapted experience with VLBI measurements to introduce a model that express the effect of wet troposphere on baseline height differences as [6]:

$$\sigma_{\delta h} = S^2 \sqrt{\frac{1 - \exp^{-\frac{b^2}{d^2}}}{b^2}} \tag{4}$$

where,

$\sigma_{\delta h}$: the error in height difference in ppm.
S: constant, taken as 80 mm.
b: baseline length
d: the correlation distance usually taken as 30 km.

The effect is shown in Figure (4).

4.4 Effect of Ionosphere

The effect of the ionospheric delay reaches its maximum when the satellite is near the horizon and is a minimum when the satellite is at the zenith. Depending upon the separation of the two receivers and the stability of the ionosphere, double differecing phase measurements between station sites will tend to cancel most of the ionospheric delay. As the ionospheric delay is frequency dependent, measurements made simultaneously on both L1 and L2 frequencies will eliminate most or almost all of the ionospheric correction. However a residual error in the relative ionospheric delay between two stations will be reflected as an error in the GPS height difference. This would tend to occur over long baseline, particularly those oriented north - south [5,6,7].

4.5 Effect of Antenna

The antenna phase center may also be dependent on the vertical angle to a satellite and this will affect the height determination. Mitchell et al (1990), stated that the whole error can only be better antenna design. Accurately measuring the height of the antenna phase center before and after data collection will probably reduce this effect [5,9].

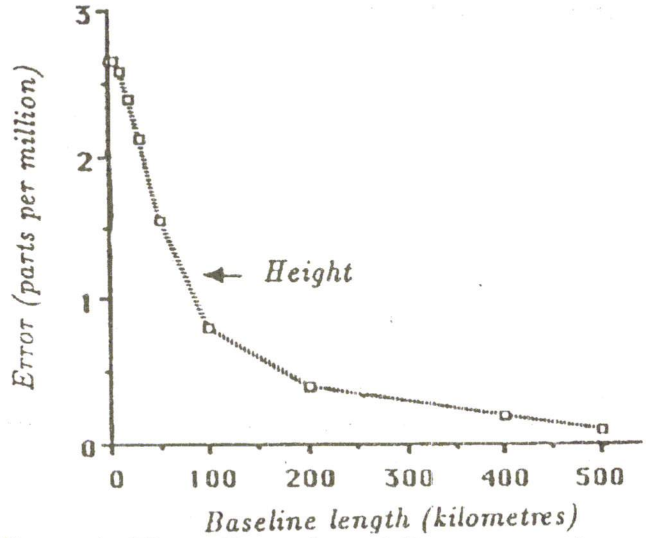

Figure 4. The effect of modeling errors of wet tropospheric refraction on height difference.

4.6 Effect of Multipath and Imaging:

Multipath and Imaging, also has an effect that depends on the antenna design and the location of the reflecting surface in the vicinity of the receiver, which is variable from site to another. Therefore, it is not possible to determine the magnitude of such effect. Multipath effects are site and antenna independent and therefore will not cancel out when double differenced between receivers. However, Tranquilla (1988) has found that, due to its cyclic nature, if observation periods are kept longer, it will tend to randomize. On the other hand, Mitchell et al (1990) stated that its effects can be reduced by the use of a well designed antenna which minimizes interference and possibly incorporating an absorbent ground plane to cut out signal reflection [5,6].

4.7 Effect of Timing

The satellite and receiver clock errors for differential positioning have three components:

- an epoch offset from the Universal Coordinated Time (UTC)
- an epoch difference between the two receivers
- a time rate difference between the two receivers and the satellites.

An epoch offset from the UTC, common to both receivers, will result in the satellite ephemerides being interpolated for incorrect time. King et al (1985) stated that, the receiver clocks need to be synchronized to UTC within 7 milliseconds for a base line error below 1 ppm. If the two receiver quartz clocks are synchronized to each other within 3 microseconds, that error reduced below 1 cm. The satellite and receiver clock errors are eliminated by differencing in the solution for baseline components [5].

5- THE GPS HEIGHTING STUDY GROUP:

Bar charts of the most important errors affecting baseline height differences derived by GPS is shown in Figure (5) for two baseline of length 5 and 50 kilometers. The tropospheric contribution (σ_{trop}) is estimated using Eq. (4) The value 2ppm for ionospheric delay (σ_{iono}) is estimated using CERN networks reported in Santerre (1989).

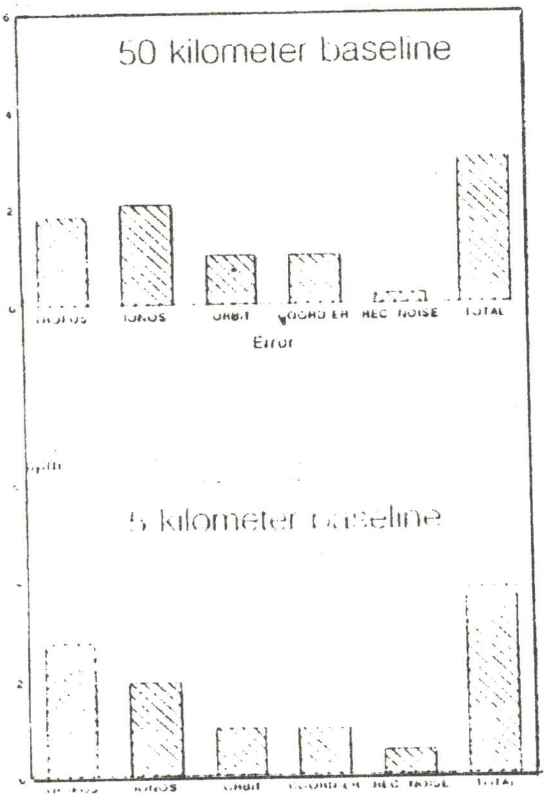

Figure 5. The estimated magnitude of errors (in parts per million) of height differences.

The satellite ephemeris is assumed to have an along track error of 20 m and the effect on height (σ_{orb}) is estimated using the Eq. (3). The effect of (σ_{cord}) is estimated from a computer simulation carried out by Holloway (1988).

Receiver noise and any residual errors (σ_{nois}) is estimated as 5 mm irrespective of baseline length. The total uncertainty of each baseline height ($\sigma_{\Delta h}$) is then calculated from [6].

$$\sigma^2_{\Delta h} = \sigma^2_{trop} + \sigma^2_{ion} + \sigma^2_{orp} + \sigma^2_{cord} + \sigma^2_{nois} \quad (5)$$

In Dec. 1990, another simulation has been conducted by the GPS Heighting Study Group. The simulations performed in this study were carried out, again, on two baseline, 5 and 50 km. long, using the full 18 satellite constellation for an assumed network.

Figure (6) shows that when the ambiguities are not resolved, the dominant height error is the tropospheric error and is proportionally much the same for both baseline. The receiver noise error is constant and therefore has a much greater influence on the shorter line. The other errors are proportionally very similar to each other. Figure (7) shows that by resolving the ambiguities correctly the error in easting coordinates is improved dramatically. The total receiver noise error has also been improved. The dominant error for the height component is still the residual tropospheric error with the error in the ionosphere and the fixed station coordinates also being significant.

It is also apparent that the total height error is not improved whether it was possible for the ambiguities to be resolved or not, ever though the error in the easting and northing components are improved [5].

6- EXPERIENCED PRECISION OF GPS-HEIGHTING

Estimates of precision of GPS-derived ellipsoidal height differences have been obtained by many researchers. These estimations are usually quoted as errors in height over baseline length in parts per million -A selection is shown in.

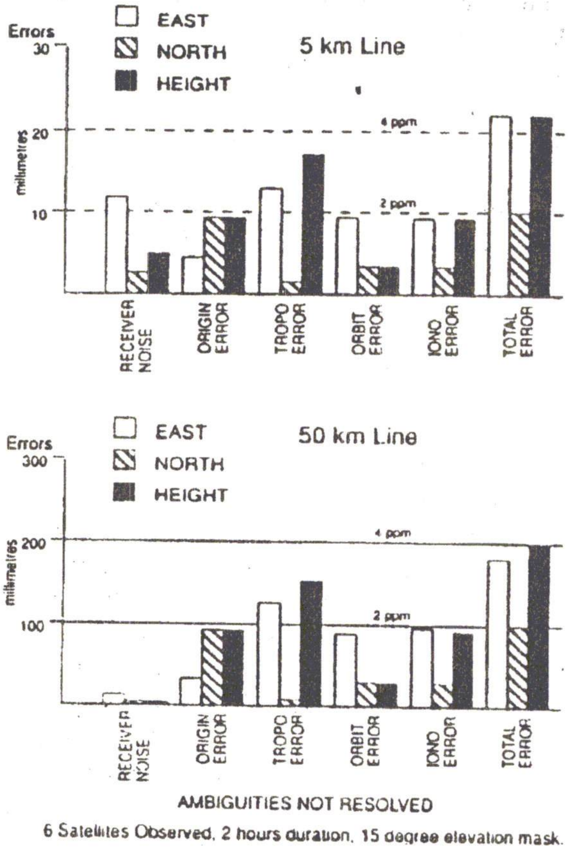

Figure 6. Simulated errors in 5 km band 50 km baseline.

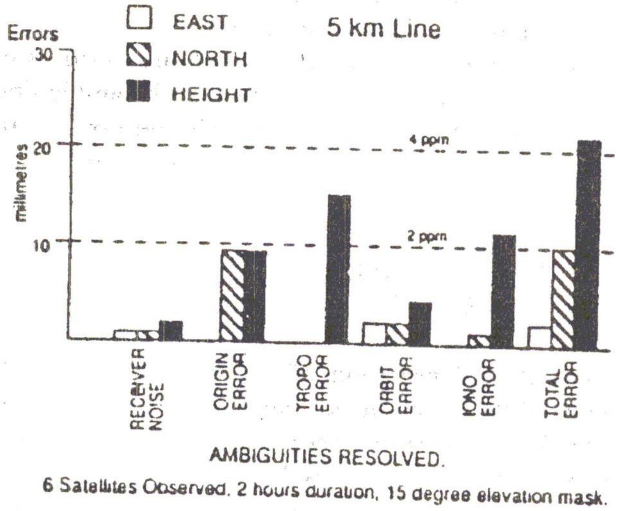

Figure 7. Simulated error in 5 km baseline,

Table 2.

Precision	Author
about 1.6 ppm	Engelis and Rapp, 1984
3 ppm	Schwarz and Sederis, 1985
2 to 3 ppm	Schwarz et al, 1987
to 3.2 ppm	Holloway, 1988
± (0.5 cm + 1 to 2 ppm)	Zilkoski and Hothem, 1988
1 to 3 ppm	Kearsley, 1988b
to 3.5 ppm	Leal, 1989
1 to 2.5 ppm	Kleusberg,1990
1 ppm	Abou-Beih, 1993

7.1 Geopotential Models

The global geoid can be represented by means of geopotential models, i.e. mathematical models in the form of spherical harmonics. The coefficients of the various terms in the series are determined using a combination of satellite orbit analysis, terrestrial gravity and N measured by satellite altimetry over the ocean. The geopotential model for N is expressed as [5,6]:

$$N = \frac{KM}{GR} \sum_{n_{max}}^{n=2} \sum_{m=0}^{n} P_{nm}(\cos\theta) \quad (C^{*}n,m \cos m\lambda + S^{*}n,m \sin m\lambda) \qquad (8)$$

Where :

R:	is the radius of the spherical model of the earth.
G:	is the mean gravity of the earth.
KM:	is the geocentric gravitational constant times the earth mass.
θ,λ:	are polar distance, longitude.
$C^{*}n,m$, $S^{*}n,m$:	are fully normalized potential coefficients.
Pn,m ($\cos\theta$):	are legendre polynomials.
n,m:	are degree, order of the spherical harmonic term.

7.2. Astrogeodetic Levelling

Astro-geodetic levelling results are normally related to a local ellipsoid.
Therefore, to transform GPS derived (h) into orthometric height (H), the separation between the geoid and a geocentric ellipsoid datum is needed.

Even though they are useful in establishing the transformation parameters between a local earth model and the World Geodetic Datum (WGS84), values of N evaluated from Astro-Geodesy are of limited use for transforming GPS heights into orthometric heights. Schwarz et al (1987) stated that the astrogeodetic data base is not dense enough, and the costs to upgrade it would be prohibitive and only gravimetric methods will therefore be the major method among others in evaluating the geoid-ellipsoid separation. Furthermore, Mitchell et al (1990) stated that the accuracy obtained from such method is not satisfactory promising as very few Laplace observations are made today, and the number must be expected to decrease since GPS is becoming much more popular [5,8].

8. PROPOSED TECHNIQUE

In recent years, the term Gravimetric methods has been expanded to include like least squares collection, mentioned above, which can use measurements other than gravity anomalies Schwarz et al (1987) used certain data types for the determination of N ; namely a geopotential model, point or mean gravity anomalies, and a detailed digital elevation model, and introduced a solution that can be written as :

$$N = N_{GM} + N_{\Delta g} + N_h \qquad (9)$$

where :

N_{GM}: is the contribution of the geopotential model.

$N_{\Delta g}$: is the contribution of the gravity anomalies.

N_h: is the contribution of the ellipsoidal heights.

Similarly, geoid height differences can be written as:

$$\Delta N = \Delta N_{GM} + \Delta N_{\Delta g} + \Delta N_h \qquad (10)$$

Figure (8) shows the different contributions for a typical geoid of 100 km length in mountainous terrain. H must be noted that N_{GM} changes very smoothly over a distance of 100 km while $N_{\Delta g}$ represents regional and local geoid features, and N_h, which changes rapidly specifically in mountainous terrain and usually has small amplitudes, represents wavelength features below 20 km that are caused by the topography [8].

Figure (9). shows the error, in ppm for each of the three components. The total error in is clearly governed by the ϵ_{GM} and the other two components are rather smaller in comparison. Finally the gravimetric determination of can be done with an accuracy of about 3ppm for distances between 10 and 100 km which makes it compatible with the current accuracy of GPS-derived Δh [8].

9. CONCLUSION

The Potential of the Global Position System Data for vertical control is presented and thoroughly investigated. The sources of errors in GPS is introduced.

It is concluded that orthometric height differences can be determined, from the geoid - ellipsoid separation and the GPS - derived ellipsoidal heights using either of the following approaches:

i) Simply ignoring the geoid ellipsoid separation will give errors which may be up to 50 ppm.
ii) Using geoid maps, including those representing astrogeodetic results.
iii) Using geopotential models, which is inexpensive and suited to an accuracy of about 5 ppm.
iv) Using gravimetric determinations,assumed to exclude geopotential models alone but to include all combination methods covered earlier, would lead to high accuracy of about 2ppm.
v) By the geometric methods, i.e. interpolation, possibly with surface fitting, between other points at which GPS observations have been made -and possibly in combination with methods referred to in (iii), may give an accuracy of up to 4ppm, or better over shorter distances.
vi) By a combinations of methods, most particularly, those at (iv) and (v) to achieve much better accuracy.

On the other hand, Zilkoski and Hothem (1989) recommended the following strategies when levelling networks are used in conjunction with GPS networks in orthometric height determination [2,9]:

All leveling data used to establish the heights

should be corrected for known systematic errors.
In addition, Dodson, A. H. and Gerrard, S.M.E. (1990) investigated leveling with GPS on test networks throughout England and Wales and concluded that the GPS- derived orthometric height differences can achieve accuracies as good as those produced by tertiary leveling over short distances and expected that equal accuracies can be maintained over longer baseline when taking a good care in processing the field data derived from the GPS. They suggested that, unlike traditional levelling, GPS heighting accuracy is less dependent upon distance. However, they suggested using the leveling at short distances, i.e. less than 1 km. Finally the results of many experiences with the GPS, indicate to a great extend the promising reliability and accuracy of this new technique for establishing a precise base for vertical control operation. Consequently, it can be safely recommeded to use the GPS as a reliable and accurate technique for vertical control, taking into consideration the effect of the arising errors in this techniques.

REFERENCES :

[1] O.M. Abou-Beih, (1993), *Towards The Determination of Accurate Orthometric Heights In Remote Areas From GPS Measurements and Gravity Data,* Published at Ain Shams University Bulletin, vol. 28, 1993.

[2] M.A. Baraka, H.A. Emam (1991), *Preliminary Investigation of Orthometric Height Determination In GPS Local Networks In Egypt*, published at Al-Azhar Engineering Second International Conference, December 21-24, 1991.

[3] B. Hofmann-Wellenhof, H. Lichtenegger and J. Collins (1992), *GLOBAL POSITIONING SYSTEM : Theory and Practice*", Springer-verlag Wien NewYork, 1992.

[4] A.H.W. Kearsley, (1988), *The Determination of the Geoid Ellipsoid Separation for GPS leveling* ", *the Australian Surveyor*, vol. 34, No.1, March, 1988.

[5] H.L. Mitchell, A.H.W. Kearsley,, J.R. Gilliland, V. Argeseanu, R.D. Holloway and P.J. Morgan, *GPS Heightening and the A.H.D.*, Technical Report, the University of Newcastle, N.S.W., Australia, December, 1990.

[6] C.L. Robinson, (1991) *Verification of Gravimetric Geoidal Models By A combination of GPS and Orthometric Heights*, M. Sc. E. Thesis, Department of Surveying Engineering Technical Report No. 152, University of New Brunswick, Canada, May 1991.

[7] R. Santerre, (1989) *GPS Satellite Sky Distribution: Impact on the Propagation of Some Important Errors in Precise Relative Positioning*, Ph. D. Dissertation, Department of Surveying Engineering Technical Report No.145, University of New Brunswick, Canada, May 1991.

[8] K.P. Schwarz, M.G. Sideris and R. Farsberg, (1987), Orthometric Heights Without Leveling *Journal of Surveying Engineering*, vol. 113, No.1, Feb.1987.

[9] D.B. Zilkoshi and L.D. Hothem, (1989), "GPS Satellite Survey and Vertical Control", *Journal of Surveying Engineering*, vol. 115, No. 2, May, 1989.