COMPARATIVE STUDY ON THE ACCURACY OF LAYOUT OF TRANSITION CURVES

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ABSTRACT

The methods of layout of transition curves are introduced and examined regarding positional accuracy. Error analysis is made and comparison is carried out on the basis of error-ellipses spatial distribution along the curve.

Keywords: Horizontal curves, Layout, Accuracy, Error analysis.

1 - INTRODUCTION

Transition curve is a curve of variable radius used for connection between a straight line and a circular curve. The radius of a transition curve varies from infinity at its tangent point with the straight to a minimum value at its tangent point with the circular curve (Figure ((1)).

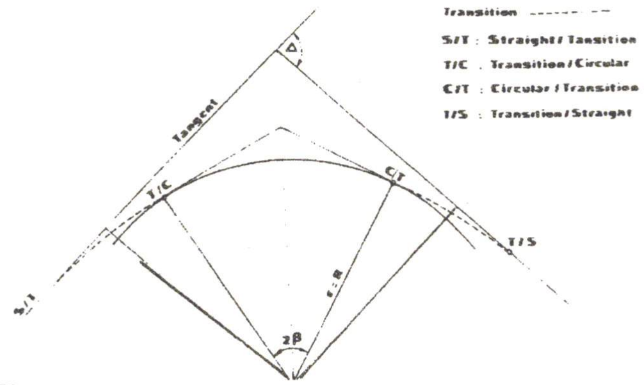

Figure 1. Circular curve with transition both ends.

Purposes of transition curve:

i. Allows superelevation to be introduced in a gradual manner.
ii. Used to introduce the radial force gradually thus minimizing its effect.
iii. Provides comfort to passengers at the turning without producing any shock or jerk.
iv. Allows higher speed to be provided at the turnings.
v. Less wear upon the turning gear.

Usually two transition curves are required to join a circular one with straights at both ends, but in some cases a circular curve joining two transitions is of zero length, so that the single circular is replaced by two transition curves having one common tangent point (see Figure (2)).

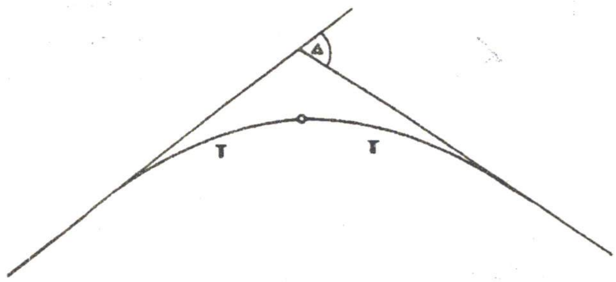

Figure 2. Two transitions instead of circular curve.

A vehicle of mass M (kg) travelling at speed v(ms) around a circular arc of radius R(m), is subjected to a radial acceleration (v^2/R) producing centrifugal force F. As F is dependent upon speed and the curvature is nil along the straight and maximum along the circular arc and thus at the tangent point, its effect is immediate.

The centrifugal force $F = \frac{mv^2}{R}$, must be resisted by either the rail, in case of a railway train, or the adhesion between the road and vehicle types, unless superelevation is applied where forces along the plan are equalised (see Figure (3)).

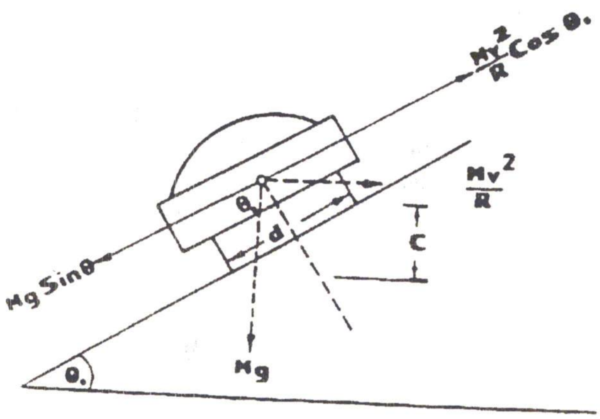

Figure 3. Centrifugal force and cant.

2- TYPES OF TRANSITIONS

Figures (4) shows the different types of plane curves used as transition curves.

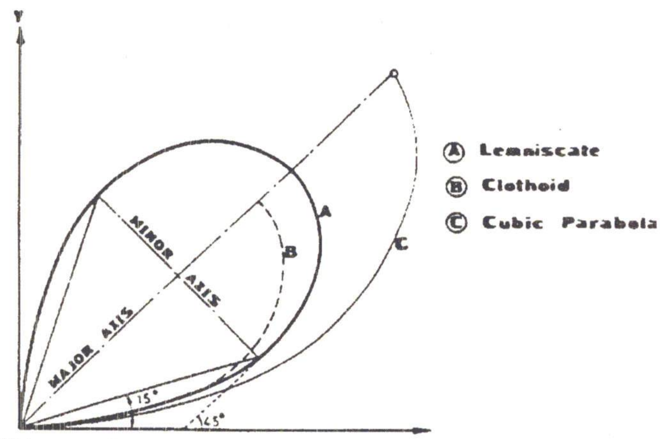

Figure 4. Various transition curves.

2.1 *The Ideal Transition Curve*

If the centrifugal force $F = \frac{Mv^2}{R}$ is to increase at a constant rate, it must vary with time and therefore, if speed is constant with distance.

$$F \,\alpha\, \iota \frac{Mv^2}{r}$$

$$\iota \,\alpha\, \frac{1}{r}$$

i.e. $\quad r\iota = RL = k$

where k is constant, R the radius of circular arc and L is the total length of the transition, from Figure (5)

$$\delta\iota = r\,\delta\Phi$$

$$\delta\Phi = \frac{\delta\iota}{r}$$

$$\delta\Phi = \frac{\iota\,\delta\iota}{k}$$

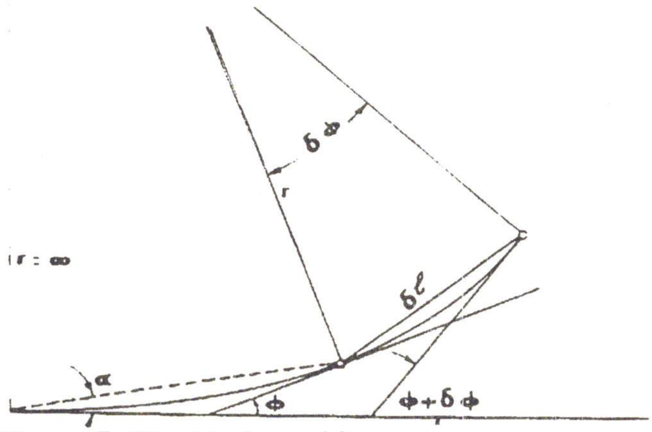

Figure 5. The ideal transition curve.

Integrating gives

$$\Phi = \frac{\iota^2}{2k} + C$$

But when $\iota = 0$, $\Phi = 0$ and thus $C = 0$

$$\Phi = \frac{\iota^2}{2RL}$$

This is an intrinsic equation of the clothoid (see Figure (6) to which the lemniscate and the cubic parabola are approximations often adopted when the deviation angle is small.

2.2 *The Clothoid*

For clothoid,

$$\Phi = \frac{\iota^2}{2RL}$$

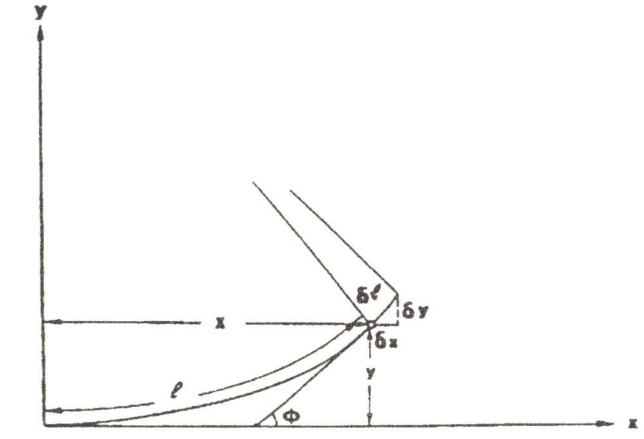

Figure 6. The clothoid.

where

ι = the length of the arc up to a given point of radius at that point, r
L= total length of the transition
R= the minimum radius of the transition
RL= $r\,\iota$ = k, a constant of the curve.

As the tangent angle Φ at any point is difficult to define, the curve can not be set out in this intrinsic form. In cartesian coordinates:

$$x = \iota\,(1 - \frac{\iota^4}{5x2!(2RL)^2} + \frac{\iota^8}{9x4!(2RL)^4} \text{ etc.,}$$

$$y = \iota\,(\frac{\iota^2}{3x(2RL)} - \frac{\iota^8}{7x3(2RL)^3} + \frac{\iota^{io}}{11x5!(2RL)^5} \text{ etc)}$$

The tangential angle (α)

$$\tan\alpha = \frac{y}{x} = \frac{\iota(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} + ...)}{1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} + ...}$$

Amount of shift (S) : From Figure (7).

$$PN = BF = y_{max}$$

$$= L\,(\frac{\phi_m}{3} - \frac{\phi_m}{42} + \frac{\phi_m}{1320})\text{etc}$$

$$= \frac{L^2}{6R} - \frac{L^4}{336R^3} + \frac{L^6}{42240R^5}$$

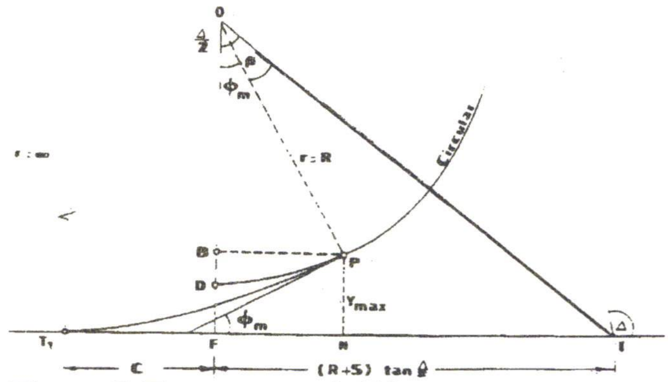

Figure 7. The amount of shift.

Shift (S) is given by

$$S = DF = BF - BD$$

$$= y_{max} - R\,(1 - \cos\Phi_m)$$

after expanding

$$S = \frac{L^2}{24R} - \frac{L^4}{2688R^3} + \frac{L^6}{506880R^5} \text{ etc.}$$

and for most practical purposes, $S = \frac{L^2}{24R}$

Tangent length (T_1 I) Figure (8),

$$T_1\,I = T_2\,I$$

$$= (R + S)\tan\frac{\Delta}{2} + C$$

Taking C equal L/2 approximately ($C \approx L/2$) then,

$$T_1\,I = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}$$

2.3 *The Bernoulli Lemniscate*

The polar equation is of the form

$$c^2 = a^2 \sin 2\alpha$$

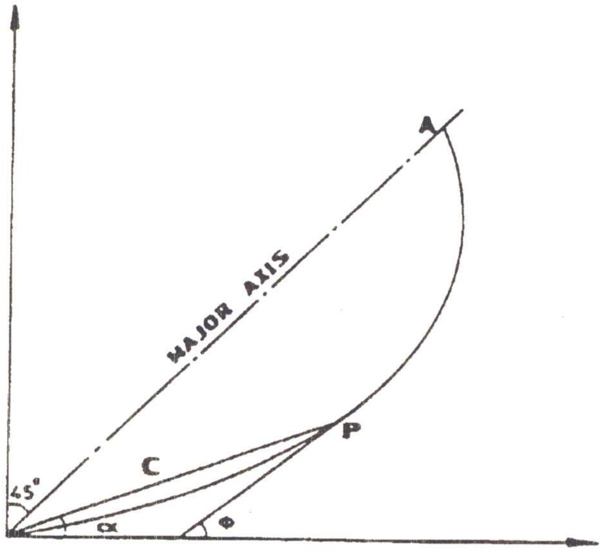

Figure 8. Bernoulli's lemniscate.

It is identical to the clothoid for deviation angle up to 60° , C is max when $\alpha = 45°$. If the lemniscate approximates to the circle, the polar equation becomes

$$c = 3\ R_c \sin 2\alpha$$

values of the constant "a" can be determined corresponding to the different values of α [6]

2.4 *The Cubic Parabola*

This type is preferable because of its simplicity. It is identical with the clothoid and lemniscate for deviation angle up to 12°. The radius of curvature reaches a minimum for deviation angle of 24° 06', and then increase. It is therefore not acceptable beyond this point (Figure (4)).

Equation of cubic parabola, is obtained by neglecting the second and sequent terms in x and y equations of clothoid, we get

$$x = l$$

$$y = \frac{l^3}{6RL}$$

Then, offsets from tangent for a cubic parabola are obtained from:

$$y = \frac{x^3}{6RL}$$

The equation of the cubic spiral is obtained by assuming the length of spiral nearly equal to its component on the tangent i.e.

$$l = x$$

then the equation of cubic spiral is

$$y = \frac{l^3}{6RL}$$

3 - SETTING-OUT OF TRANSITION CURVES

Setting-out may take one of the following two forms:

1- From the tangent point using either
i) Offsets from the tangent (Figure 9-a)
ii) Deflection angles from the tangent (figure 9-b)

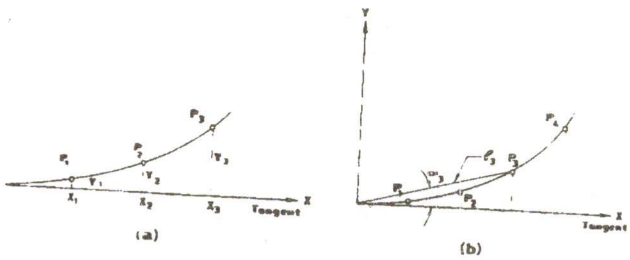

Figure 9. Setting-out transition curve.

2- From central point by polar rays using theodolite or EDM. This method involves the computation of the coordinates of points on the curve.

In either of these methods, the location of the tangent points is essential. If the setting-out is by deflection angles or offset from the tangent, then the location of the tangent point is a first priority. Provided that the maximum value of (Φ) is less than 12° the cubic parabola/spiral provides an adequate solution and the setting out computations are very much simplified.

3.1 *Offsets from the tangent method*

In case of setting-out transition curve by using this method, choosing an arbitrary length x_1 on the tangent, the corresponding offset y_1 is determined from the equation:

$$y = \frac{x}{6RL}$$

Having done offset y_1, point p_1 (x_1, y_1) on the curve is fixed. If $x_2 = 2x_1$ and $x_3 = 3x_1$ then $y_2 = 8y_1$ and $y_3 = 27y_1$ etc.

3.2 *Deflection angles method*

The values of the deflection angles a can be calculated from either following relations:

$$\alpha = \tan^{-1} \frac{x^2}{6RL} \text{ (in case of cubic parabola)}$$

$$\alpha = \frac{1}{2} \sin^{-1} \frac{C}{3R} \text{ (in case of lemniscate).}$$

$$\alpha = \frac{\iota^2}{6RL} \text{ (in case of clothoid)}$$

At the end the transition, $\iota = L$ and then (considering clothoid)

$$\alpha = \frac{206265}{6R} \text{ (seconds)}$$

$$\alpha = \frac{57.2958}{6R} \text{ (degrees)}$$

Check the point P that join a transition to the circular arc,

$$y_{max} = \frac{L^2}{6R} = 4\ s$$

where s = shift

4 - MATHEMATICAL MODEL FOR ERROR ANALYSIS

4.1 *Choice of the Coordinate system*

Since linear or angular measurements are used to lay out the curve from the tangent points, it is a first priority to locate the tangent points and consider the tangent as the x-axis and the perpendicular to it as the y-axis (see Figure (9-a,b))

4.2 *Offsets from the Tangent*

Choosing convenient peg interval along the transition, say ι, x and y coordinates for the curve points obtained from the (Clothoid) equation.

$$x_i = \iota_i \left(1 - \frac{\iota_i^4}{40R^2L^2} + \frac{\iota_i^8}{3456R^4L^4}\right)$$

$$y_i = \iota_i \left(\frac{\iota_i^2}{6RL} - \frac{\iota_i^6}{336R^4L^3} + \frac{\iota_i^{io}}{42240R^5L^5}\right)$$

where ι_i is the length of the transition to the considered i^{th} point on the curve. Positioning error for the ith point, located in the curve, can be obtained directly from the following forms given by [1].

$$\sigma_{xi} = \sigma_{\iota o} \sqrt{\frac{x_i}{l_o}}$$

$$\sigma_{yi} = \sigma_{\iota o} \sqrt{\frac{y_i}{l_o}}$$

while $\sigma_{xi\ yi}$ is zero.

4.3 *Deflection Angles From The Tangent*

Choosing convenient values of ι_i along the transition, the corresponding values of the deflection angles α_i are obtained from the clothoid equation.

$$\alpha_i = \frac{\iota_i}{6RL}$$

where R is the radius of the circular curve. For establishing the i^{th} point P_i (ι_i, α_i) on the curve, angle α_i is directed from the tangent and length ι_i is measured along the sight direction. The coordinates of point P_i are,

$$x_i = l_i \cos \alpha_i$$
$$y_i = l_i \cos \alpha_i \qquad (2)$$

Differentiating Eqs. (2) partially with respect to a and l_i, substituting into variances and covariances equations [6], then the variance and covariance for the x_i and y_i are

$$\left.\begin{aligned} \sigma_{x_i}^2 &= \sigma_{li}^2 \cos^2 \sigma_i + l_i^2 \sin^2 \alpha_i \sigma_\alpha^2 \\ \sigma_{y_i}^2 &= \sigma_{li}^2 \sin^2 \sigma_i + l_i^2 \cos^2 \sigma_i \sigma_\alpha^2 \\ \sigma_{xy_i}^2 &= \sigma_{li}^2 \sin \sigma_i \cos \alpha_i (\sigma_{\alpha i}^2 - l_i \sigma_\alpha^2) \end{aligned}\right\} \qquad (3)$$

where σ_{li} is the standard deviation of the distance l_i and it may be obtained from the following form

$$\sigma_{li} = \sigma_{lo} \sqrt{\frac{l_i}{l_o}} \qquad (4)$$

σ_α is the angular standard deviation, and l_o, σ_{lo} are the length, standard deviation of the tape used for lay out the curve respectively.

4.4 *Analysis of Results*

Transition curve of 100 ms length was used for the analysis using the two above-mentioned methods of alignment. (Figure 10-a,b) show the resulting spatial distribution of error ellipses for both methods of layout.

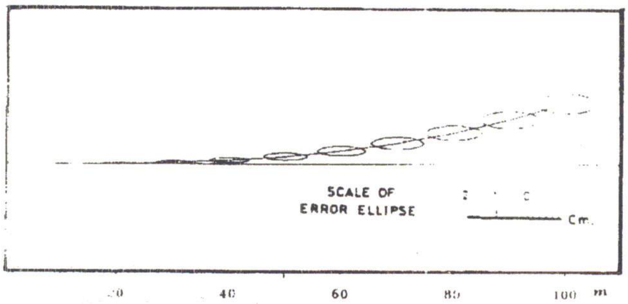

Figure 10-a. Transition curve by co-ordinates.

5 - ANALYSIS AND RECOMMENDATIONS

Visual inspection of the distribution of error ellipses along the curve gives an invaluable aid for judging the accuracy of setting-out.

It should be noted when examining radial component of error which can be scaled from error-ellipses is the most significant component of standard deviation.

Similar to all methods depending on alignment from the initial tangent point, the size of error-ellipses grow progressively away from that point.

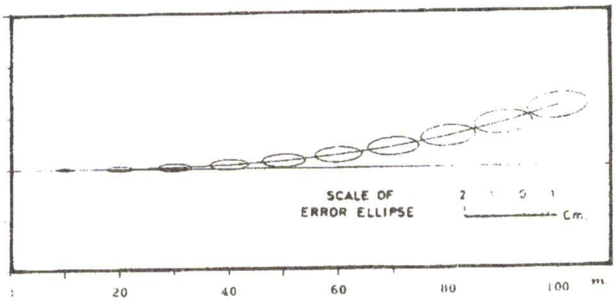

Figure 10-b. Transition curve by def. angles.

However error ellipses exhibited by both methods are approximately the same but the method of layout by offsets from the tangent gives ellipses which are slightly thinner in shape as compared to those given by the method of setting out by deflection angles.

It is therefore recommended to use the first method for setting-out transition curves because it provides better positioning accuracy as well as being faster and cheaper.

6- REFERENCES

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