ACCURATE METHODS OF CURVE SETTING, ASSESSMENT AND COMPARISON

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ABSTRACT

The purpose of this paper is to assess and compare the field positioning methods of horizontal curves for first- grade roads and railway alignment. Classic and novice methods involving angular measurements are treated. Mathematical models for error analysis are developed; and comparison study is made using error ellipses. Keywords: Horizontal curves - Settingout - Accuracy - Error analysis

1 - INTRODUCTION

Accurate methods of curve setting-out usually involve angular measurements from one or more stations; either exclusively or associated with linear measurements. In this work, three methods of circular curve ranging are mainly investigated. The first and second procedures are the well known traditional methods. " Dual-theodolite method " and "Deflection angles or Rankine's method". The third is the less known novice method of "Optimum point method". The three methods are evaluated through a comparative study using the positional error accuracy determination employing error ellipses technique.

2 - SETTING OUT PROCEDURES

2.1 *Deflection Angles Method (DAM/Rankine's Method)*

A deflection angle to any point on the curve is the angle at the point of tangency T_1 between the back tangent and the chord from T_1 to that point. (see Figure (1)).

Rankine's method is based on the principle that "the deflection angle to any point on a circular curve is measured by one-half the angle subtended by the arc from T_1 to that point. It is assumed that the length of the arc is approximately equal to its chord. The last approximation is very reasonable when the radius (R) is equal or greater than 20 times the chord length.

Let T_1 I : rear tangent,
T_1 : point to curve (P.C.)
θ_1, θ_2, θ_3 , ...: the tangential angles or the angles which each of the successive chords $T_1 P_1$, $P_1 P_2$, $P_2 P_3$ etc.
makes with the respective tangent to the curve at T_1, P_1, P_2 etc.

Y_1, Y_2, Y_3, ...:total tangential angles or deflection angles to the point P_1, P_2, P_3, etc. (Punmia, 1975).
ι_1, ι_2, ι_3 ... = lengths of the chords $T_1 P_1$, $P_1 P_2$, $P_2 P_3$,

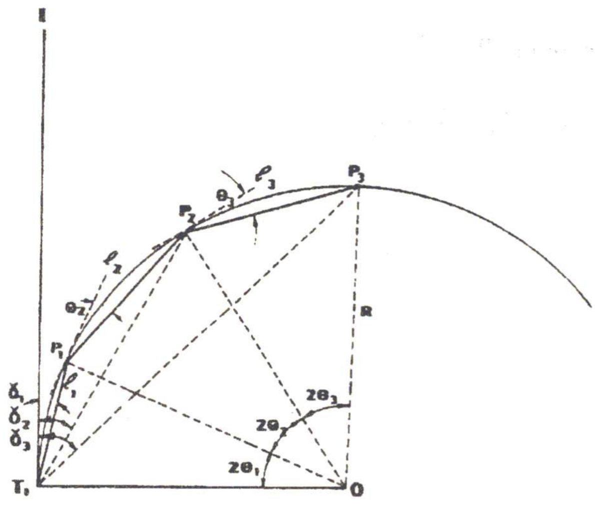

Figure 1. Rankine's method.

From the property of the circle.

angle $IT_1 P_1$ = half the angle $T_1 OP_1$
angle $T_1 OP_1 = 2\,\theta_1$

In case of chord $T_1 P_1$ = arc $T_1 P_1$

$$\text{Now, angle } \theta_1 = \frac{90\,\iota_1}{\pi R} \text{ (Degrees)},$$

$$\theta_2 = \frac{90\,\iota_2}{\pi R}$$

In general,

$$\theta_n = \frac{90\,\iota_n}{\pi R}, \text{ where, } \iota_n \text{ is normal chord.}$$

From the geometry of (Figure (1)), the deflection angle of the first point P_1 is equal to its tangential angle or

$$\tau_1 = \theta_1,$$

for the second point P_2

angle $IT_1 P_2$ = half the angle $T_1 OP_1$

i.e.

angle $IT_1 P_2 = \theta_1 + \theta_2$

$$\gamma_2 = \theta_1 + \theta_2$$

for point P_3.

$$\gamma_3 = \theta_1 + \theta_2 + \theta_3,$$

Generally for point P_n,

$$\gamma_n = \theta_1 + \theta_2 + \theta_3 + \dots + \theta_n$$

In case of equal chords

$$\theta_1 = \theta_2 = \theta_3 = \dots = \theta_n = \theta$$

then $\gamma_1 = \theta$
$\gamma_2 = 2\,\theta$
and $\gamma_n = n\,\theta$

2.2 Dual-Theodolite Method O.P.M.

In this method, two theodolites are used, one at T_1 and another at T_2. This method is used when the ground is unsuitable for chainage and is based upon the principle that "the angle between the tangent and the chord is equal to the angle which that chord subtends in the opposite segment" (see Figure (2)).

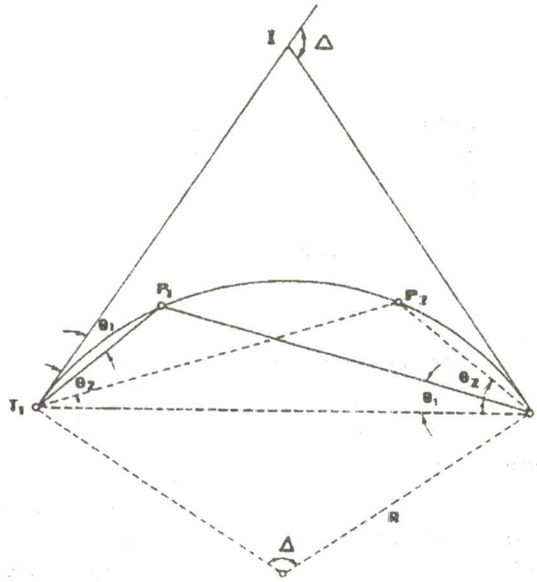

Figure 2. Two theodolites method.

Deflection angles to points on curve are the same as in the previous method. The actual steps of setting out for both methods can be found out in many text-books.

2.3 Optimum Point Method

In this method, horizontal curve is set out from a specific vantage point referred to as optimum point (OP). Many alternative vantage points can be chosen for setting- out purposes depending on field condition and economic consideration, all points within the curve vicinity are referred to a local co-ordinate system which has the (OP) as its origin [5].

Vantage Point

It is defined as the best point which can be chosen considering cost and/or field conditions for the ranging out of an entire curve. It is therefore not a fixed position. A reference point is chosen to which any vantage point can be referred and which can equally serve as a vantage point. Due its dual-role, this reference point is referred to as the optimum point (OP).

Choice of optimum point

The (OP) is the best vantage point which can be chosen for the purpose of ranging out an entire curve when theoretical and practical application are considered simultaneously. In other words to satisfy the above definition, the (OP) should conform to certain conditions as discussed below.

Figure (3) shows a circular curve AC_1 d_1 d_2 C B, A is the point of curvature (P.C) and B is the point of tangency (P.T.). The choice of (OP) is based on the following conditions:

1. The ratio of two distances from any pair of corresponding opposite points to the (OP) should be 1:1.
2. The sum of all distances involved in setting out should be minimum.
3. To be theoretically and practically versatile in application, there should be simple relationship between the OP and other curve parameters for any type of curve.

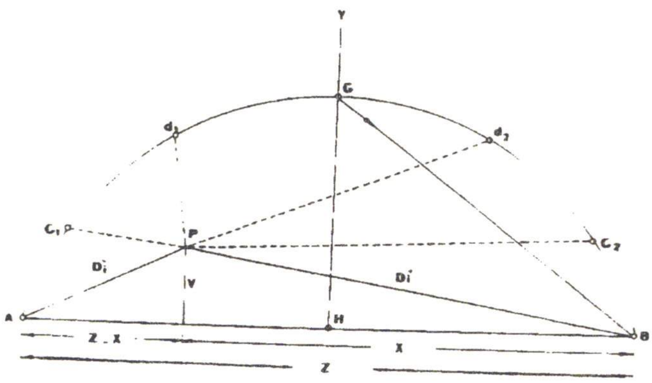

Figure 3. Determination of the optimum point.

To satisfy these conditions it has been shown [5] that the optimum point lies somewhere between G and H.

Two methods can be used in setting out according to optimum point method. These are summarized in the following.

2.3.1 *OPM by Radial Angles and Chord Lengths*

To set out the curve by this method (Figure (4)), the instrument is stationed at point (H) and angles "q" are turned out. The curve points are then fixed as the inter- sections of the rays and the chord lengths.

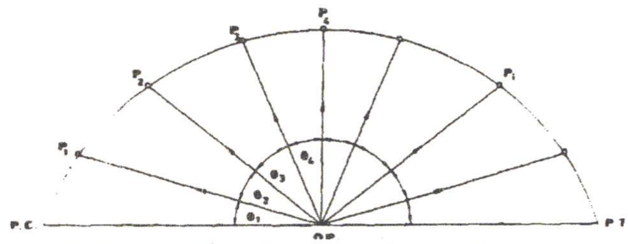

Figure 4. Concept of radial angles and radial distances.

It is very clear that, setting-out curves by using this approach is analogous to the method of deflection angles in case of choosing point (G) as the optimum point. In case of choosing point (H) as optimum point, double intersection maybe occur between the chords and the radial lines. Setting out curves by using deflection angles method (DAM) avoid this defect even in case of curves with greater deflection angles.

2.3.2 *OPM by Radial Angles and Radial Distances*

This involves the use of polar rays. The distance "d" to the curve point is calculated and the direction of the line defined by the angle "θ" the curve points are then located by measuring the distances along the radial lines. The chord length (ι) to be used for setting out procedure is usually known from job specification (it may be taken 20 m tape length). Angle "θ" and distance (d) are computed involving the solution of the respective triangles shown in Figure (5).

2.3.3 *Advantages of the Optimum Point Method*

1. Use of (OP) reduces the number of instrument stations.
2. In case of using (OPM), the location of any curve point is not strictly dependent on the preceding point.
3. The (OPM) technique is quite flexible in the

sense that its computations can be adapted for other methods of the curve ranging.

4. The (OPM) technique satisfies the different field conditions.

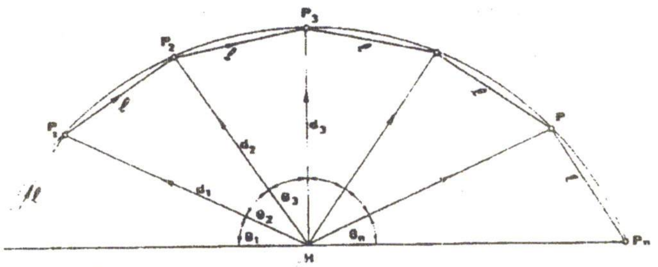

Figure 5. Concept of radial angles and chord lengths.

3- MATHEMATICAL MODEL FOR ERROR ANALYSIS

A brief description on the concepts of convariance, correlation, propagation of variances and covariances and Error Ellipse Technique is shown in [1].

The mathematical models for the three methods under study are derived in the following sections.

3.1 *Deflection Angles/Rankine's Method*

Referring to Figure (6) the mathematical model for error analysis is developed. Adopting the theodolite station T_1 is chosen as origin, and the Y-axis as the tangent at T_1.

Figure (6) shows a circular curve T_1, P_1, P_2,...P_n .. T_2 while ι_1, ι_2, ι_3,...ι_n ... are the chords joining the curve points. Let Ψ_1, Ψ_2, Ψ_3,...Ψ_n , etc be the corresponding angles between the above chords and the y-axis respectively.

Referring to Figure (6) the coordinates of the n^{th} point, on the curve, P_n (x_{pn}, Y_{pn}) are

$$x_{pn} = \iota_1 \sin \Psi_1 + \iota_2 \sin \Psi_2 + \iota_3 \sin \Psi_3 + ... + \iota_n \sin \Psi_n$$

$$y_{pn} = \iota_1 \cos \Psi_1 + \iota_2 \cos \Psi_2 + \iota_3 \cos \Psi_3 + ... + \iota_n \cos \Psi_n \quad (1)$$

For simplicity, let Eqs. (1) take the form

$$\left. \begin{array}{l} x_{pn} = \sum_{i=1}^{n} = \iota_i \sin \Psi_i \\ x_{pn} = \sum_{i=1}^{n} = \iota_i \cos \Psi_i \end{array} \right\} \quad (2)$$

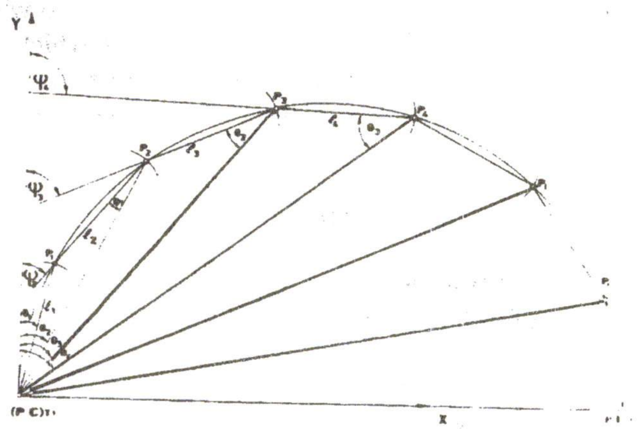

Figure 6. Deflection angles method coordinates system.

Derivation of the mathematical model

According to the setting out technique, location of the point P depends on the quantities ι_i and Ψ_i (i=1,2,3,..,n) The relation between the bearing angles Ψ_i and the deflection angles θ_i, is given by

$$\Psi_i = \theta_{i-1} + \theta_i \quad (3)$$

the angles θ_i are measured separately with equal precision, then

$$\left. \begin{array}{l} \sigma_{\theta_1} = \sigma_{\theta_2} = \sigma_{\theta_3} = \sigma_{\theta_n} = \sigma_\theta \\ \sigma_{\theta_i} = \sigma_{\theta_j} = 0.0 \qquad \text{for all values of i and j} \end{array} \right\} \quad (4)$$

In case of adopting equal chords of length ι let σ_ι be the standard deviation of the measured chord, then

$$\left. \begin{array}{l} \iota_1 = \iota_2 = \iota_3 = \quad \iota_n = \iota \\ \sigma_{\iota_1} = \sigma_{\iota_2} = \sigma_{\iota_3} = \quad \sigma_{\iota_n} = \sigma_\iota \end{array} \right\} \quad (5)$$

Since these chords are measured independency then

$$\sigma_{\iota_i \iota_j} = 0.0$$

Differentiating Eqs. (3) partially with respect to the angles θ_i and substituting into variance-covariance propagation equations [1] gives:

$$\left. \begin{array}{l} \sigma^2_{\Psi_1} = \sigma^2_\theta \\ \sigma^2_{\Psi_2} = \sigma^2_{\Psi_1} = ... \qquad \sigma^2_{\Psi_n} = 2\sigma^2_{\theta}, \\ \sigma_{\Psi_1\Psi_2} = \sigma_{\Psi_2\Psi_3} = \sigma_{\Psi_3\Psi_4} = \sigma_{\Psi_n}-1_{\Psi_{n-1}} = \sigma_\theta \end{array} \right\} \quad (6)$$

For all values of i and j, (except i = j+1), values of $\sigma_{\Psi 1\, \Psi 2} = 0.0$.

For circular curve with peg interval of length ι, the deflection angles θ_i and the bearing angles Ψ_i are given by:

$$\theta_i = i\,\frac{D}{2} \text{ and } \Psi_i = \frac{D}{2}(2i-1) \qquad (7)$$

where D is the degree of the curve.

By differentiating equations (2) partially with respect to the quantities ι_i and Ψ_i and substituting into variances- covariances equations, taking equations (3,4,5,6 and 7) into consideration, the mathematical model of positioning error for a general point P_n on the curve is given by:

$$\left.\begin{aligned} \sigma^2_{x_{pn}} &= \sigma^2_{x_{p1}} + \sigma^2_{\iota} m_i + 2g(n_i + A_i) \\ \sigma^2_{y_{pn}} &= \sigma^2_{y_{p1}} + \sigma^2_{\iota} n_i + 2g(m_i + B_i) \\ \sigma_{xy_{pn}} &= \sigma_{xy_{p1}} + C_i(\sigma^2_{\iota} - 2g) - gE_i \end{aligned}\right\} \qquad (8)$$

where

$$m_i = \sum_{i=2}^{n} \sin^2 [\tfrac{D}{2}(2i-1)]$$

$$n_i = \sum_{i=2}^{n} \cos^2 [\tfrac{D}{2}(2i-1)]$$

$$A_i = \sum_{i=2}^{n} \sin^2 [\tfrac{D}{2}(2i-1)] \sin[\tfrac{D}{2}(2i-3)]$$

$$B_i = \sum_{i=2}^{n} \cos^2 [\tfrac{D}{2}(2i-1)] \cos[\tfrac{D}{2}(2i-3)]$$

$$C_i = \frac{1}{2}\sum_{i=2}^{n} \sin\, [D(2i-1)]$$

$$E_i = \sum_{i=2}^{n} \sin\, [\tfrac{D}{2}(2i-1)] \cos[\tfrac{D}{2}(2i-3)]$$

$$+ \cos\, [\tfrac{D}{2}(2i-1)] \sin[\tfrac{D}{2}(2i-3)]$$

and

$$g = \iota^2\, \sigma_\theta^2$$

3.2 *Dual Theodolite Method*

Development of mathematical model

Choice of coordinate system:

Choosing the long chord T_1 T_2 and the perpendicular to it at point T_1 (P.C.) as the two axes x and y respectively (Figure (7).

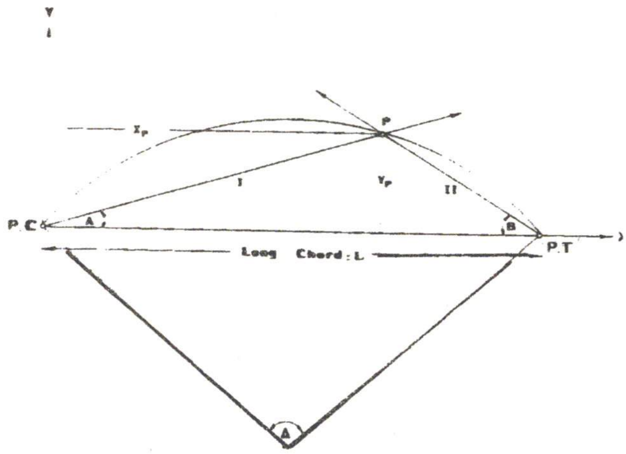

Figure 7. Two theodolites method Co-ordinates system.

The coordinates of the points T_1 (x_{t1}, y_{T1}) and T_2 (X_{T2}, y_{T2}) are (0,0) and (L,0) respectively, where L denotes the length of the long chord (T_1 T_2).

The coordinates of the point $P(x_p, y_p)$ are given by [7].

$$\left.\begin{aligned} X_p &= \frac{(Y_{T1} - Y_{T2}) + x_{T1}\cot B + X_{T2}\cot A}{\cot A + \cot B} \\ Y_p &= \frac{(x_{T2} - x_{T1}) + Y_{T1}\cot B + Y_{T2}\cot A)}{\cot A + \cot B} \end{aligned}\right\} \qquad (9)$$

Substituting $X_{T1} = 0$, $Y_{T1} = 0$, $X_{T2} = L$ and $Y_{T2} = 0$ into Eqs. (9) we get

$$\left.\begin{aligned} X_p &= \frac{L\cot A}{\cot A + \cot B} \\ Y_p &= \frac{L}{\cot A + \cot B} \end{aligned}\right\} \qquad (10)$$

Equations (9) may be simplified to

$$\left.\begin{aligned} X_p &= k\cot A \\ Y_p &= k \end{aligned}\right\} \qquad (11)$$

where

$$k = \frac{L}{\cot A + \cot B}$$

To obtain variances and covariance for the positioning of point P, differentiating Eqs. (10) partially with respect to the two measured quantities A and B, substituting into variances and covariances equations gives:

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,(\mathrm{cosec}^4 A \cot^2 B\, \sigma^2_A + \mathrm{cosec}^4 B \cot^2 A\, \sigma^2_B \\ \sigma^2_{y_p} &= q\,(\mathrm{cosec}^4 A\, \sigma^2_A + \mathrm{cosec}^4 B\, \sigma^2_B) \\ \sigma_{xy} &= q\,[-\mathrm{cosec}^4 A \cot B\, \sigma^2_A + \mathrm{cosec}^4 B \cot A\, \sigma^2_B) \\ &\quad + \mathrm{cosec}^2 A\, \mathrm{cosec}^2 B\, \sigma_{AB} (\cot A - \cot B)] \end{aligned}\right\} \qquad (12)$$

where

$$q = \frac{L^2}{(\cot A + \cot B)^4}$$

Eqs. (12) constitute a mathematical model which deter- mines the variances and covariance $\sigma^2_{x_p}, \sigma^2_{y_p}$ and σ_{xy_p} for the located point P.

Since the two orientation angles A and B are measured separately and error in measuring the first angle A causes no effect in measuring B, it follows that, there is no correlation between A and B.

i.e : $$\alpha_{AB} = 0.0 \qquad (13)$$

A substitution of Eqs. (13) in Eqs. (12) gives

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,(\mathrm{cosec}^4 A \cot^2 B\, \sigma^2_A + \mathrm{cosec}^4 B \cot^2 A\, \sigma^2_B \\ \sigma^2_{y_p} &= q\,(\mathrm{cosec}^4 A\, \sigma^2_A + \mathrm{cosec}^4 B\, \sigma^2_B) \\ \sigma_{xy_p} &= q\,(-\mathrm{cosec}^4 B \cot A\, \sigma^2_B - \mathrm{cosec}^4 A \cot B\, \sigma^2_B) \end{aligned}\right\} \qquad (14)$$

Further, the angles A and B are assumed to be measured with equal precision. This gives

$$\sigma_A = \sigma_B = \sigma \qquad (15)$$

where

σ is the angular standard deviation.

Substituting from Eq. (15) into Eqs. (14) one gets

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,\sigma^2(\mathrm{cosec}^4 A \cot^2 B + \mathrm{cosec}^4 B \cot^2 A \\ \sigma^2_{y_p} &= q\,\sigma^2(\mathrm{cosec}^4 A + \mathrm{cosec}^4 B) \\ \sigma_{xy_p} &= q\,\sigma^2(\mathrm{cosec}^4 B \cot A - \mathrm{cosec}^4 A \cot B \end{aligned}\right\} \qquad (16)$$

For the circular curve, the sum of the two angles A and B is equal to half the central angle (Δ) i.e. ($A + B = \frac{\Delta}{2}$) and this leads to

$$\left.\begin{aligned} \sigma^2_{x_p} &= q\,\sigma^2 \mathrm{cosec}^4 A \cot^2(\tfrac{\Delta}{2} - A) \\ &\quad + \mathrm{cosec}^4(\tfrac{\Delta}{2} - A)\cot^2 A] \\ \sigma^2_{x_p} &= q\,\sigma^2[\mathrm{cosec}^4 A + \mathrm{cosec}^4(\tfrac{\Delta}{2} - A)] \\ \sigma_{xy_p} &= q\,\sigma^2[\mathrm{cosec}^4(\tfrac{\Delta}{2} - A)\cot A \\ &\quad - \mathrm{cosec}^4 A(\cot(\tfrac{\Delta}{2} - A] \end{aligned}\right\} \qquad (17)$$

Replacing angle A with B in Eqs. (17) gives the variances and covariance for the corresponding opposite point p'.

On inspecting the resulting error equations (17) we find that

i. for curves having the central angle (Δ) less than 90, and standard deviation in x-direction (σ_x) is always greater than that in the y-direction (σ_y).
ii. For central angle ($\Delta = 90$), it is easy to prove that the error in x-direction σ_x is a constant value equal to ($L\ \sigma$).
iii. For the summit point, where $A = B = \theta$, position on determination referred to x and y directions is correlation-free. This can be proved as follows

$$\left.\begin{aligned} \sigma^2_x &= 2q(\mathrm{cosec}^4\theta)\cot\theta\,(\sigma^2) \\ \sigma^2_y &= 2q(\mathrm{cosec}^4\theta)(\sigma^2) \\ \sigma_{xy} &= 0.0 \end{aligned}\right\} \qquad (18)$$

and if $\Delta = 90°$, then

$$\sigma_x{}^2 = \sigma_y{}^2 = 8\,q\,\sigma_2 \qquad (19)$$

Eq. (19) shows that positioning determination of the summit point in case of (Δ = 90) is accomplished with equal accuracy in the x and y directions.

Also, in case of $\sigma_A = \sigma_B$ it is easy to prove that, the orientation of the error ellipse does not depend on the precision of the angular measurements (σ).

It is evident from Eqs. (17), in order to determine the two corresponding points p, p' with same positioning accuracy ($(\sigma_{x_p} = \sigma'_{x_p}, \sigma_{y_p} = \sigma'_{y_p}$ and $\sigma_{xy_p} = \sigma_{xy_p'})$, the two theodolites, used for laying out the curve, must be of the same precision. This was satisfied by the assumption given in Eq. (15).

3.3 *Optimum Point Method (Version I)*

Choice of coordinate system:

choosing the long chord and the perpendicular bisector as two arbitrary axes x and y respectively where point (OP) is the origin.

Let point P(x,y) be located on the curve, Figure (8), to establish the point (P), distance d is measured along the line making an angle θ with the x-axes. Distance d and angle θ are calculated by using simple coordinate geometry. The coordinates of point (P) are

$$\left.\begin{aligned} x &= d\cos\theta \\ y &= d\sin\theta \end{aligned}\right\} \qquad (20)$$

Variance and covariance for the positioning of point P can be obtained by differentiating Eqs. (20) partially with respect to the measured quantities d and θ, substituting into variance and covariance equations:

$$\left.\begin{aligned} \sigma_x^2 &= \cos^2\theta\,\sigma_d^2 + d^2\sin^2\theta\,\sigma_\theta^2 \\ \sigma_y^2 &= \sin^2\theta\,\sigma_d^2 + d^2\cos^2\theta\,\sigma_\theta^2 \\ \sigma_{xy} &= \cos\theta\sin\theta(\sigma_d^2 - d^2\sigma_\theta^2) \end{aligned}\right\} \qquad (21)$$

Examining the resulting equations (21) the following remarks can be given:

i) Positioning accuracy for any located point on the curve depends on the precision of both angular and linear measurements.
ii) Linear measurements error causes the serious positioning error for points on the curve because such an error shifts the points away of the curve path i.e. radially inward or outward.

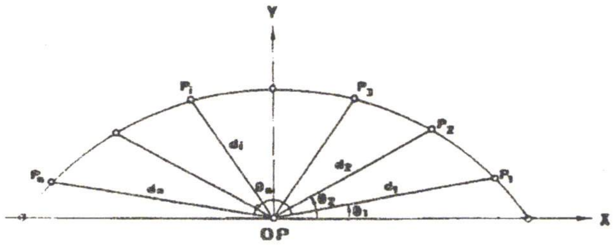

Figure 8. Setting out by optimum point (version I).

On the other hand, angular error causes an insignificant position error since it shifts the located point tangentially to the curve path.

3.4 *OPTIMUM POINT METHOD* (Version II)

In this method, the instrument is positioned at (OP') Figure (9). For determining variance and covariance for the locating points, the same procedure used in the previous methods is followed, Angles θ_1', θ_2', θ_3', ...θ_n' and distance d_1', d_2', d_2', ... d_n' are measured to establish the points P_1', P_2', P_3',... P_n' on the curve respectively. The mathematical model for obtaining the expected variance and covariance can be given by replacing θ and d with θ' and d', in equation (21) respectively, then we get.

$$\left.\begin{aligned} \sigma_x'^2 &= \cos^2\theta'\,\sigma_d'^2 + d'^2\sin^2\theta'\,\sigma_\theta'^2 \\ \sigma_y'^2 &= \sin^2\theta'\,\sigma_d'^2 + d'^2\cos^2\theta'\,\sigma_\theta'^2 \\ \sigma_{x'y} &= \sin\theta'\cos\theta'(\sigma_d'^2 - d'^2\sigma_\theta'^2) \end{aligned}\right\} \qquad (22)$$

4- ANALYSIS OF RESULTS

Error ellipses were constructed according to the above mathematical models for the cases under study. Variations were adopted in degree of curve and observational accuracy. The cases chosen here include only curves with degree 6°, at observational accuracy σ (length) = 0.005 m, σ (angle) = 10" Figures (10-a), (10-b), (10-c), and (10-d) show the distribution of error ellipses.

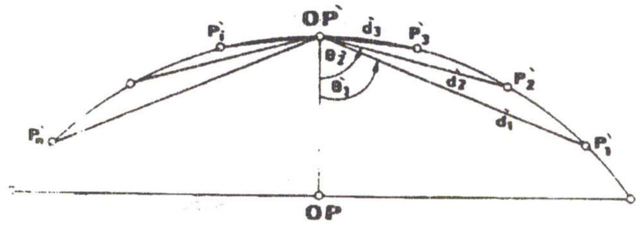

Figure 9. Setting out by optimum point (version II).

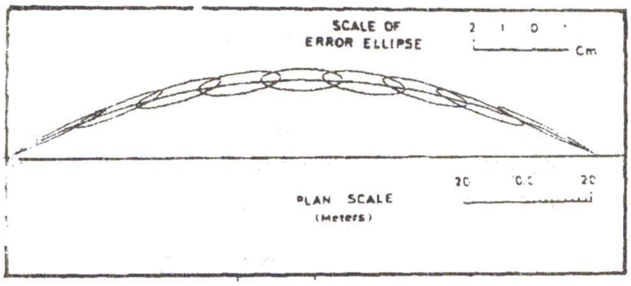

Figure 10-a.. Two theodolites method.

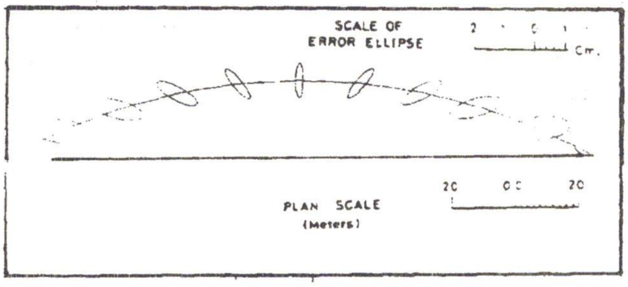

Figure 10-b. Optimum point method (I).

Graphical representation of error ellipses provides an illustration of the accuracy with which any point on a curve is positioned. Visual inspection of the distribution of error ellipses along the curve gives an invaluable aid for judging of setting out accuracy.

Radial component of error, σ_r , which can be scaled from the error ellipse graph is the most significant component of standard deviation. It gives a reliable indication about the accuracy of setting out along the curve. On the other hand, a large error component on the direction of the curve (tangential component) would be insignificant since the located point would still lie on the curve.

The maximum, minimum and average values of the radial error component σ_r among points distributed along the curve could be used as very convenient criteria to indicate homogeneity of positioning accuracy for different methods under investigation.

5- CONCLUSION

The methods under study can be assessed as follows:

5.1 *Rankine's Method*

i) Located points by using this method are dependent on each other since they are to be set out in chain. Accumulation of error can be observed as we move away from the starting point of the curve. Major axes of the error ellipses on the curve run approximately along the line joining this point with the starting point.
ii) Symmetrically positioned points are located with different accuracy.

5.2 *Dual Theodolite Method*

i) Major axes of the error ellipses are tangential to the curve path while minor axes are perpendicular which is ideal for positioning accuracy.
ii) For curves with larger radii, larger sizes of the error ellipses can be observed.
iii) The method also shows adequate homogeneity of accuracy distribution since radial components (σ_r) are nearly equal along the curve.
iv) Symmetrically positioned points with respect to midpoint of the curve are located with the same positioning accuracy.
v) As measurement's precision increases, positioning accuracy for located points increases.

5.3 *Optimum Point Method*

5.3.1 *Version I*

The corresponding cases for optimum point method version I, where the instrument position lies on the mid- point of the long chord, are represented in figure (10-c). The following remarks can be given:

i. Symmetrical point about the perpendicular bisector of long chord are located with equal accuracy.

ii. Linear error is the most significant factor affecting the positioning accuracy of the located points.

iii. Increasing the angular precision, makes the error ellipses more slim. While increasing the linear precision makes the major axes shorter.

iv. In case of smaller radii, orientation of error ellipses changes in such a way that the major axis turns quickly perpendicular to the curve path as compared with cases of greater radii

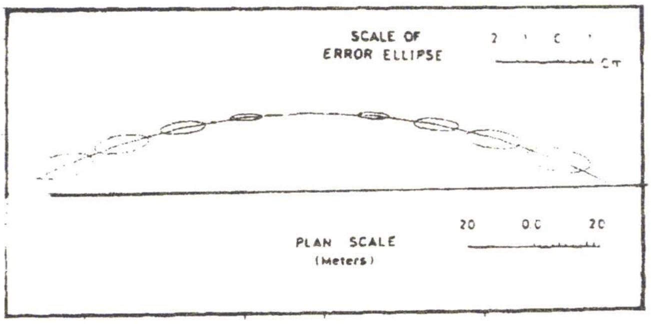

Figure 10-c. By optimum point (II).

5.3.2 *Version II*

i. Symmetrical points about the perpendicular bisector of long chord are located with equal accuracy.

ii. This method exhibits better positioning accuracy since the major axes of error ellipses go nearly tangential to the curve, especially in case of greater radii. Also it shows smaller radial error component s as compared with optimum point version I.

iii. As in the previous method (version I), increasing the precision of linear measurements causes a significant decrease in the length of major axes of the error ellipses and the increasing the angular precision leads to more slim ellipses which means a smaller radial component of error.

GENERAL

1. As expected, an increase in the precision of measurements used in setting out procedure, would increase the positioning accuracy. However, the effect is not always significant depending on the case.

2. For both cases of the optimum point method version I, where the instrument position lies on the mid-point of the long chord, and version II where the instrument position lies on the middle of the curve, the ellipse distribution is symmetrical with respect to the middle of the curve. Moreover there is no significant accumulation of error due to the fact that points on curve are located independently. Generally, version II is better than version I for the following reasons:

i) Error ellipses are generally oriented tangential to the curve in the case of version II and perpendicular to the curve in the case of version I. Therefore, an alignment using version II should be preferred due to the smaller radial error component σ_r.

ii) Linear error is the most significant factor contributing to the positioning error in case of version I while the angular error is significant in the case of version II.

3. Considering deflection angles method (Rankine), an accumulation of error can be observed as we move away from the starting point on the curve. It is clear that the effect of angular error in positioning accuracy for the first few points on curve is significant. Bad inter- section of position lines occurs in case of greater deflection angles at the end of the curve. On the other hand, this method exhibits small radial component in case of greater radii. Therefore, it is recommended to use this method in case of flat curves. Increasing angular precision causes a significant improvement in the positioning accuracy in case of long-flat curves.

4. In case of using two-theodolites method, symmetrical and more accurate positioning is achieved. The error ellipses are always oriented tangential to the curve. Also adequate homogeneity of positioning accuracy exists.

It could be noticed that, orientation of major axes of the error ellipses does not depend on the angular observational error but it depends only on curve configuration (i.e. its radius and deflection angle). Finally to sum up:

Dual theodolite method can be categorized as the best method used for setting out circular curves since it shows an excellent point positioning as compared with other methods of setting out because of its indisputable merits over the other methods, it is worthwhile to recommend this method when high positioning accuracy is required. Optimum point method, version II, is the second best recommendation.

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