DYNAMIC ANALYSIS OF FLOW IN A WIDE RECTANGULAR CHANNEL SUBJECT TO SEEPAGE LOSSES

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ABSTRACT

The problem of evaluating the flow profile for a wide channel subject to seepage into a permeable top layer is investigated in this paper. The top layer is underlined by a highly permeable aquifer of low piezometric pressure. The analysis considers the case in which the total channel discharge is lost, which results in zero channel discharge somewhere in the downstream direction. The authors present a solution technique based on the numerical integration method that accounts for both the impact and friction losses. A computer program involving details of the trial and error cycles of the technique is developed. A solved numerical example is provided. Solutions include the estimation of channel length downstream from a reference section and prediction of the water surface profile. The results are compared with those of another technique which ignores dynamic losses and assumes a straight line profile.

Keywords: Flow profile, Channel, Seepage losses, Friction losses.

INTRODUCTION

Many investigators have studied problems of canal seepage on different lines of approach (Harr 1962; and Kochina 1962). In some desert areas where the hydraulic conductivity of the top soil is high, channel water is subject to considerable seepage looses. Hathoot (1984 and 1989) presented approximate solution for some cases in which the total channel discharge is lost by introducing simplifying assumptions.

In an experimental study Hathoot et al. (1991) concluded that the water surface profile may be roughly considered a straight line in cases of seepage from wide rectangular channels. The case of seepage from wide rectangular channels into a top permeable layer underlain by a highly permeable aquifer of low piezometric head,

Figure (1), was investigated by Hathoot et al. (1992). In their analysis they considered a straight line water profile and neglected the dynamic effect of water flow as well as the friction looses. In this paper the authors present a solution to the afore-mentioned canal seepage problem on the basis of the dynamic equation of gradually spatially varied flow with decreasing discharge.

THEORY

The following assumptions are considered in the theoretical analysis of the present work:

1. The channel has a constant longitudinal slope.
2. The permeable top layer is homogeneous and isotropic.
3. The piezometric pressure of the lower aquifer is constant.
4. The surface of interface between the permeable top layer and the highly permeable aquifer is horizontal.
5. The water depth, and top layer thickness of a reference channel section are known in advance.

The dynamic equation for spatially varied flow with decreasing discharge (French 1985) is given by:

$$\frac{dy}{dx} = \frac{S_o - S_f - \left(\frac{\alpha Q}{gA^2}\right)\left(\frac{dQ}{dx}\right)}{1 - \left(\frac{\alpha Q^2}{gA^2 y_h}\right)} \qquad (1)$$

in which y = water depth; x = distance from the initial section; S_o = channel longitudinal slope; S_f = friction slope; α = energy correction factor; Q = channel discharge; g = acceleration due to gravity; A = water area; and y_h = hydraulic mean depth. For wide rectangular channels Eq.(1) may be put in the form :

$$\frac{dy}{dx} = \frac{S_o - S_f - \left(\frac{\alpha q}{gy^2}\right)\left(\frac{dq}{dx}\right)}{1 - \left(\frac{\alpha q^2}{gy^3}\right)} \tag{2}$$

in which q = discharge per unit channel width.

Equation (2) can be solved by a number of techniques. In gradually varied flow problems the solution procedure must begin at a control (French 1985). Therefore, for a given channel with lateral outflow, the existence of a critical flow section should be examined. Following the reasoning of Henderson (1966), critical flow occurs or dy/dx = 0 when the numerator of Eq.(2) is zero or

$$S_o - S_f - \left(\frac{\alpha q}{gy^2}\right)\left(\frac{dq}{dx}\right) = 0 \tag{3}$$

If the Chezy equation is used, the friction slope may be written as

$$S_f = \frac{q^2}{C^2 y^3} \tag{4}$$

in which C = Chezy's coefficient.
Substitution of Eq.4 in Eq.3 yields

$$S_o - \frac{q^2}{C^2 y^3} - \left(\frac{\alpha q}{gy^2}\right)\left(\frac{dq}{dx}\right) = 0 \tag{5}$$

For critical flow in wide channels we have:

$$q = \sqrt{\frac{gy^3}{\alpha}} \tag{6}$$

Substitution of Eq.6 in Eq.5 yields

$$S_o - \frac{g}{\alpha C^2} - \sqrt{\frac{\alpha}{gy}}\left(\frac{dq}{dx}\right) = 0 \tag{7}$$

Referring to Figure (1) the rate of outflow at a general section is

$$\frac{dq}{dx} = -q_s' = -K\left(1 + \frac{y - h_o}{D}\right) \tag{8}$$

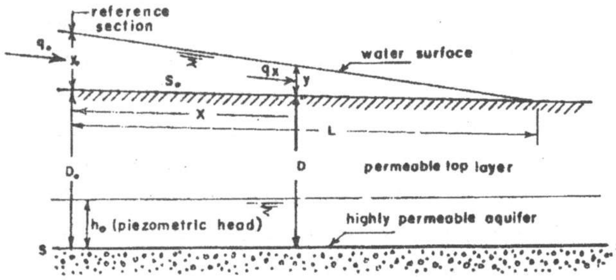

Figure 1. Longitudinal geological section.

in which q_s = seepage loss per unit channel length and width; K = hydraulic conductivity of top soil; h_o = the piezometric pressure head of the lower aquifer; and D = thickness of top layer at the section under consideration. As the longitudinal slope of channel, S_o, is constant we have

$$\frac{dq}{dx} = -K\left(1 + \frac{y - h_o}{D_o - xS_o}\right) \tag{9}$$

in which D_o = top layer thickness at the reference section, Figure (1).
Substitution of Eq.9 in Eq.7 yields

$$S_o - \frac{g}{\alpha C^2} + \sqrt{\frac{\alpha}{gy}}\, K \left(1 + \frac{y - h_o}{D_o - xS_o}\right) = 0 \tag{10}$$

Solving Eq.10 for x

$$x = \frac{1}{S_o}\left[D_o + \frac{y - h_o}{1 + \frac{S_o - \frac{g}{\alpha C^2}}{K\sqrt{\frac{\alpha}{gy}}}}\right] \tag{11}$$

By means of Eq.11 the location of the critical section, if one exists, down stream from the reference section can be estimated. In other words if x $\geq$ the channel length, Figure (1), there is no possibility of a critical section to occur, and vice versa. According to Hathoot et al. (1992), a rough estimate of the channel length, L, may be obtained from

$$S_o\left(\frac{q_o-LK}{Ky_o}\right)=1+\left(\frac{h_o}{y_o}+\frac{D_o}{LS_o}-1\right)\ln\left(1-\frac{LS_o}{D_o}\right) \qquad (12)$$

in which q_o = channel discharge at the reference section and y_o = water depth at the same section. As Eq.12 is implicit it should be solved through a trial-and-error procedure.

SOLUTION TECHNIQUE

Although spatially varied flow problems can be solved by a number of trial computational procedures the most effective solution technique is numerical integration combined with trial and error (French 1985 and Chow 1959). As shown in Figure (2) the drop in the water surface between section 1 and 2 is given by

$$\Delta y' = \Delta y + S_o \Delta x \qquad (13)$$

in which $\Delta y = y_1 - y_2$; and Δx = width of flow element. On the basis of Newton's second law of motion (French 1985), the equation of the numerical integration for spatially varied flow with decreasing discharge can be written as given by Chow (1959)

$$\Delta y' = \frac{\alpha q_1(V_1+V_2)\Delta V}{g(q_1+q_2)}\left(1-\frac{\Delta q}{2q_1}\right)+S_f\,\Delta x \qquad (14)$$

in which q_1 = discharge at section 1; V_1 = velocity at section 1; V_2 = velocity at section 2; $\Delta V = V_1 - V_2$; q_2 = discharge at section 2; $\Delta q = q_1 - q_2$; and S_f = the friction slope given by Eq.4 or its Mannings equivalent form. Equation 14 can be used to determine the flow profile and the channel length.

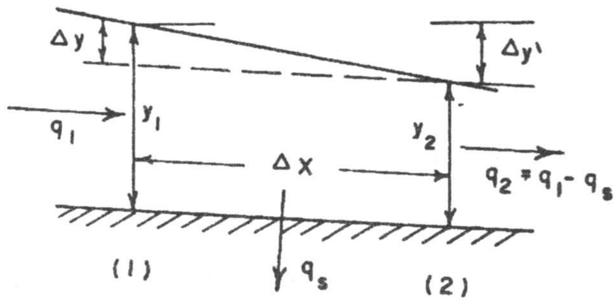

Figure 2. Definition sketch for flow element.

NUMERICAL EXAMPLE

It is required to determine the water surface profile and the channel length for the following data: h_o = 2.0m; $S_o = 2.0 \times 10^{-4}$; y_o = 1.0 m; D_o = 10.0 m; K = 4.0 m/day; Manning's coefficient n = 0.03; and α = 1.1.

SOLUTION

The possibility of the existence of a critical section should be checked first. In Table (1) are shown the results of application of Eq.11 considering different depths of flow. The approximate channel length is then evaluated by means of Eq.(12). The

Table 1. Estimation of the Location of Critical Section.

y(m)	$C = y^{1/6}/n$	x(km)
1.00	33.3333	50.0099
0.75	31.7728	50.0129
0.50	29.6966	50.0166
0.25	26.4567	50.0217
0.01	15.4719	50.0418

discharge q_o contained in Eq.12 may be calculated through the application of Manning's formula by assuming y_o to be the normal depth as follows:

$$q_o = \frac{(1.0)^{5/2}}{0.03}(0.0002)^{1/2}$$

$$= 0.4714\ m^3/s\ \textit{perm.wide}$$

Solving Eq.12 for the aforementioned data by trial and error yields: L = 12.3417 km. Comparison of the x value contained in Table (1) with the calculated value of L shows that there is no possibility for a critical section to occur along the channel reach under consideration. The Froude number at the reference section is

$$F = \frac{0.4714}{\sqrt{9.81(1.0)}}$$
$$= 0.1505 \ (<1.0)$$

Since no control section exists, it is evident that subcritical flow prevails along the channel reach. In subcritical flow, calculations should be carried out in the upstream direction. In this example the starting point is point i at which the discharge is zero and the final section is that at which the water depth, y = 1.0 m. Calculations are shown in Table (2). At point i, x_i = L = 12341.7 m and the top soil thickness is given by

$$D_i = D_o - X_i S_o \qquad (15)$$

D_i = 10.0 - 12341.7 (0.0002)
= 7.5317 m

The first upstream section 1, Figure (3), is chosen close to point i with Δx_i as small as 10.0 m, say. For $y_1 = 0$; $y_i = 0$; and $D_{av} = (D_i + D_1)/2.0$, seepage between section 1 and point i can be calculated from:

$$q_s = q'_s \, \Delta x_i \qquad (16)$$

in which q'_s is the seepage per unit channel length and width, Eq.8, q_s itself is the channel discharge at section 1 such that $q_s = q_1$. The water depth at section 1 can be roughly evaluated by applying Manning's formula as:

$$y_1 = \left[\frac{nq_1}{\sqrt{S_o}}\right]^{0.6} \qquad (17)$$

with y_1 from Eq.17 and $y_i = 0$; the average water depth is given by:

$$y_{av} = \frac{y_1 + y_i}{2}$$
$$= \frac{y_1}{2}$$

Application of Eq.16 with $y_{av} = y_1/2$ and D_{av} as before produces new q_s and hence q_1 values. The above mentioned procedure is repeated till rational values of y_1 and q_1 are obtained. After this, calculations for section 2, which is ΔX upstream from section 1, Figure (3), are carried out. A rough estimate of the discharge at section 2 is given as

$$q_2 = q_1 + q_s \frac{\Delta x}{\Delta x_i} \qquad (18)$$

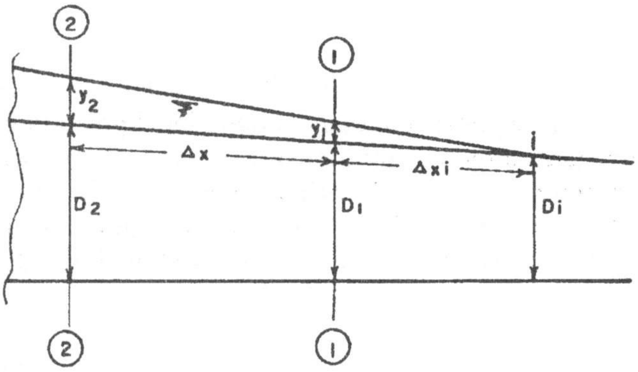

Figure 3. Initial calculation sections.

An initial y_2 value is obtained by applying Eq.17 with y_1 and y_2 known we have,

$$y_{av} = \frac{y_1 + y_2}{2} \qquad (19)$$

and

$$D_{av} = \frac{D_1 + D_2}{2} \qquad (20)$$

Seepage between sections 1 and 2 can be calculated from:

$$q_s = K\left(1 + \frac{y_{av} - h_o}{D_{av}}\right)\Delta x \qquad (21)$$

TABLE 2:Prelemenary Calculations of Example 1.

y_2(m) (1)	$\Delta y'_A$(m) (2)	Y_{av}(m) (3)	q_1(m^3/s/m) x 10 (4)	q_2(m^3/s/m) x 10 (5)	V_2(m/s) (6)	$[q_1+q_2]$ (m^3/s/m)x10 (7)	V_1+V_2 (m/s) (8)	Δq(m^3/s/m) x 10 (9)	ΔV(m/s) x 10 (10)	Δy_m(m) x 10^3 (11)	h_f(m) (12)	$\Delta y'_B$ (m) (13)
X_i = 12341.7m;			D_i = 7.5317m									
0.0000				0.0000								
X = 12331.7m;			ΔX = 10.0m;		D = 7.5337m							
0.0130				0.00340	0.0261							
X = 10831.7m;			ΔX = 1500.0m;		D = 7.8337m;		D_{av} = 7.6837m					
0.2646	0.5516	0.1388	0.5262	0.5296	0.2002	0.5330	0.2263	0.5262	1.7406	-2.150	0.3184	0.3163
0.2694	0.5563	0.1412	0.5264	0.5298	0.1967	0.5333	0.2228	0.5265	1.7059	-2.080	0.3002	0.2982
0.2155	0.5025	0.1143	0.5240	0.5274	0.2447	0.5308	0.2708	0.5240	2.1862	-3.230	0.6258	0.6226
0.2326	0.5196	0.1228	0.5247	0.5281	0.2271	0.5316	0.2532	0.5248	2.0096	-2.780	0.4866	0.4838
0.2287	0.5156	0.1209	0.5246	0.5280	0.2309	0.5314	0.2570	0.5246	2.0478	-2.880	0.5146	0.5118
0.2282	0.5152	0.1206	0.5245	0.5279	0.2314	0.5314	0.2575	0.5246	2.0525	-2.890	0.5182	0.5153
X = 9331.7m;			ΔX = 1500.0m;		D = 8.1337m;		D_{av} = 7.9837					
0.3675	0.4393	0.2978	0.5463	1.0743	0.2924	1.6024	0.5237	0.5464	0.6100	0.570	0.4384	0.4390
X = 7831.7m;			ΔX = 1500.0m;		D = 8.4337m;		D_{av} = 8.2837m					
0.4816	0.4141	0.4245	0.5623	1.6367	0.3399	2.7111	0.6322	0.5624	0.4749	0.985	0.4130	0.4140
X = 6331.7m;			ΔX = 1500.0m;		D = 8.7337m;		D_{av} = 8.5837m					
0.5827	0.4011	0.5321	0.5756	2.2124	0.3797	3.8492	0.7195	0.5757	0.3984	1.130	0.3999	0.4010
X = 4831.7m;			ΔX = 1500.0m;		D = 9.0337m;		D_{av} = 8.8837m					
0.6754	0.3927	0.6290	0.5872	2.7997	0.4145	5.01214	0.7942	0.5873	0.3484	1.190	0.3915	0.3927
X = 3331.7m;			ΔX = 1500.0m;		D = 9.3337m;		D_{av} = 9.1837m					
0.7621	0.3867	0.7187	0.5975	3.3972	0.4458	6.1970	0.8603	0.5976	0.3127	1.220	0.3854	0.3867
X = 1831.7m;			ΔX = 1500.0m;		D = 9.6337m;		D_{av} = 9.4837m					
0.8441	0.3820	0.8031	0.6068	4.0041	0.4744	7.4013	0.9202	0.6068	0.2856	1.230	0.3808	0.3820
X = 331.7m;			ΔX = 1500.0m;		D = 9.9337m;		D_{av} = 9.7837m					
0.9224	0.3787	0.8833	0.6151	4.6192	0.5008	8.6233	0.9751	0.6152	0.2642	1.240	0.3771	0.3783
X = -1168.3m;			ΔX = 1500.0m;		D = 10.2337m;		D_{av} = 10.0836m					
0.9976	0.3752	0.9600	0.6228	5.2421	0.5255	9.8613	1.0262	0.6228	0.2468	1.240	0.3740	0.3752

The discharge q_2 is then calculated from:

$$q_2 = q_1 + q_s \tag{22}$$

The second y_2 value is then calculated by applying Eq.17. Details of trails for section 1 and 2 are shown in Table 2. The first and second rows have been obtained with the above procedure. The y_2 value of the third and next rows may be obtained by interpolation such that the final y_2 value corresponds to $\Delta y'_A \approx \Delta y'_B$. After that the final results are given the subscript 1 and calculations start at a new section ΔX upstream from the last section and so on till the total channel reach is covered and the reference section is reached. In Table 2 the columns contain the following data:

Column 1: The assumed water depth, y_2;
Column 2: The difference between water surface elevations at section 1 and 2 and is given by:

$$\Delta y'_A = D_2 - D_1 + y_2 - y_1 \tag{23}$$

Column 3: The average depth given by Eq.20.
Column 4: Seepage between sections 1 and 2 given by Eq.20.
Column 5: Discharge at section 2 given by Eq.22.
Column 6: Velocity at section 2 given by:

$$V_2 = \frac{q_2}{y_2} \tag{24}$$

Column 7: $q_1 + q_2$.
Column 8: $V_1 + V_2$.
Column 9: $\Delta q = q_2 - q_1$.
Column 10: $\Delta V = V_2 - V_1$.
Column 11: This is the difference in the water surface due to the impact loss which is given by:L

$$\Delta y_m = \frac{\alpha q_2 (V_1 + V_2)\Delta V}{g(q_1 + q_2)}\left(1 - \frac{\Delta q}{2q_2}\right) \tag{25}$$

Column 12: This is the head loss due to friction and is computed from:

$$h_f = \left(\frac{n q_2}{y_2^{5/3}}\right)^2 \Delta x \tag{26}$$

Column 13: This is the difference between water surface elevations and is given by:

$$\Delta y'_B = \Delta y_m + h_f \tag{27}$$

In this example the increment ΔX is taken as 1500.0 m and hence ten steps are considered. In Table 2 it is evident that the impact loss Δy_m is considerably less than the friction loss, h_f. This is because water is lost at a small rate while friction action prevails along the channel reach.

In the last row of Table 2 if the value of y_2 = 0.9976 m is considered sufficiently close to 1.0 m, hence the new channel length L = the approximate channel length minus the last x value. A new calculation cycle is started with this new L value. Trials are then continued till the last row provides y_2 = 1.0 m with D_o = 10.0 m. On the other hand if the y_2 value of the last row of Table 2 is not close to 1.0 m, the increment of the last step Dx' is chosen to produce $y_2 \approx$ 1.0 m and the aforementioned procedure is followed with the new calculated channel length.

The final flow profile together with the approximate straight line profile are plotted in Figure (4). The calculated profile is 9.3% longer than the approximate linear one. The flow chart of the computer program used in carrying out calculations of the flow profile of Figure (4) is shown in Figure (5). In Figure (4), $\Delta x = \Delta x_i$ = 10.0 m.

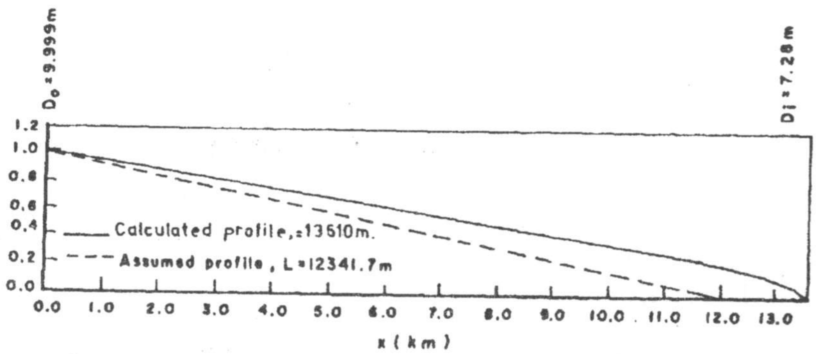

Figure 4. Calculated and assumed linear water-surface profiles.

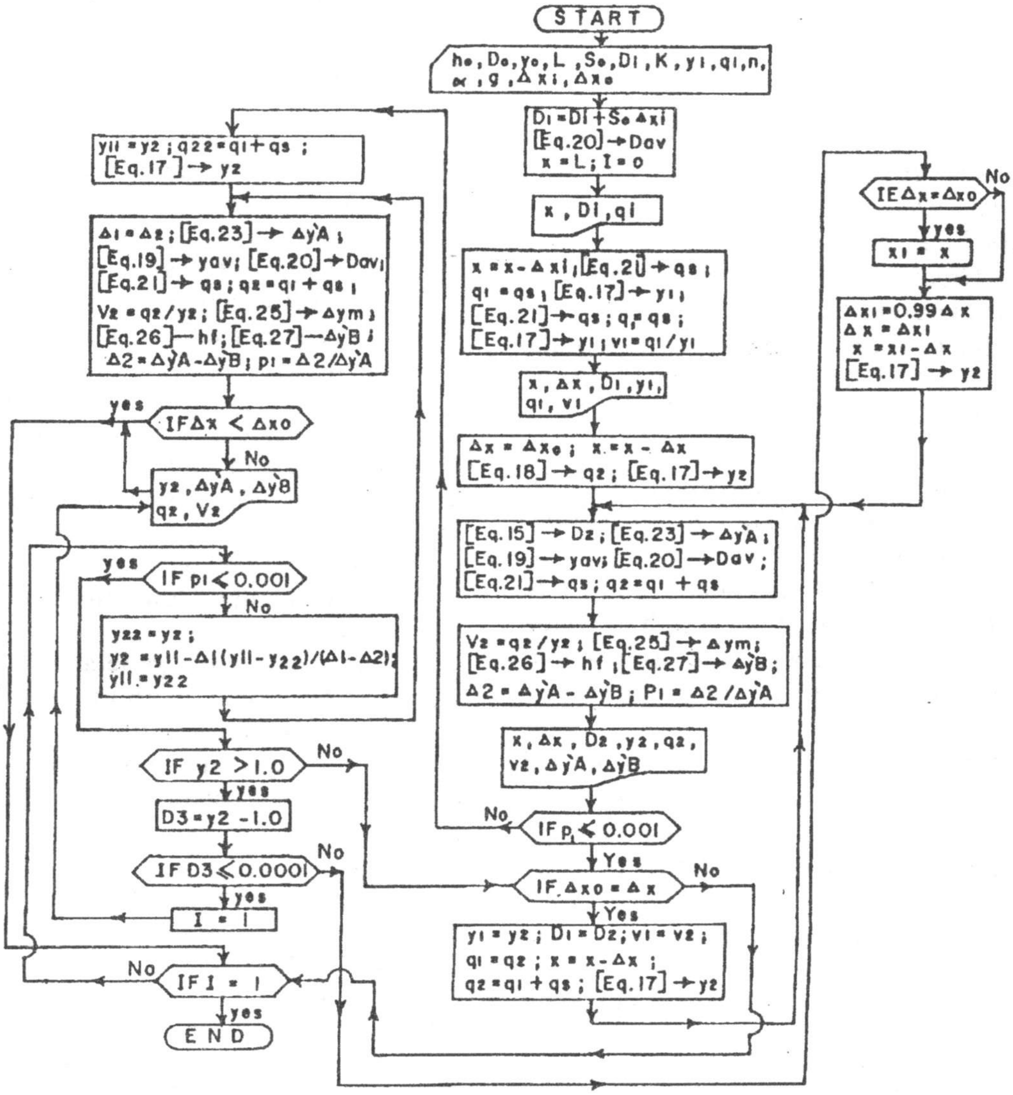

Figure 5. Flow chart of the computer program.

CONCLUSIONS

A new technique for estimating the length of a wide channel subject to seepage losses is presented. The technique provides the final channel length within two or three trial cycles only. It is proved that in seepage problems the impact losses can be ignored since they generally constitute about 0.3% of the total losses. The solution of a numerical example shows that the calculated channel length downstream from a reference section is about 9% longer than that of an approximate straight line profile in which losses are completely ignored.

APPENDIX I:

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APPENDIX II:

Notation

The following symbols are used in this paper:

A water area;
C Chezy coefficient;
D depth of top layer at a general section;
D_{av} average depth of top layer given by Eq.(20);
D_i depth of top layer at the downstream end of channel;
D_o depth of top layer at the reference section;
F Froude number;
g acceleration due to gravity;
h_f friction head loss;
h_o piezometric pressure head of the lower aquifer;
K hydraulic conductivity of top soil;
L channel length downstream from a reference section;
n Manning's coefficient;
Q channel discharge at a general section;
q discharge per unit channel width at a general section;
q_o discharge per unit channel width at the reference section;
q_s seepage per unit channel width between two sections;
q_s rate of seepage per unit channel width;
S_f friction slope;
S_o channel bed slope;
V velocity at a general channel section;
x distance between a general section and the reference section;
x_i channel length downstream from the reference section;
y water depth at a general channel section;
y_{av} average water depth as given by Eq.(19);
y_h hydraulic mean depth;
y_o water depth at the reference section;
α energy correction factor;
Δq $q_2 - q_1$;
ΔV $V_2 - V_1$;
Δx distance between two successive sections;
Δx_i distance between downstream channel end and the first upstream section;
Δy $y_2 - y_1$;
$\Delta y`_A$ change in water surface elevation given by Eq.(23);
$\Delta y`_B$ change in water surface elevation given by Eq.(27); and
Δy_m impact head loss given by Eq.(25).