# COMBINED CONFINED/UNCONFINED SEEPAGE BENEATH HYDRAULIC STRUCTURES

## Part I. Theoretical Study

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ABSTRACT

A theoretical approach, for the problem of combined confined/unconfined seepage beneath hydraulic structures, is presented. The effect of various factors, on the characteristics of combined seepage under the floor of a tail escape structure, is investigated. The floor is provided with one row sheet pile. The factors involving in the problem are; length of seepage face behind the floor S, variation of the upstream and downstream water levels $H_1$ and $H_2$, position of the sheet pile $\ell_1$ and its depth D, side slope angle $\theta$ and length of floor L. The depth of seepage flow, at the sheet pile $H_o$, is determined as a function of the above factors. Hence, the seepage discharge equation is predicted by applying Depuit's principles. Results are depicted in the form of curves and charts.

*Keywords: Seepage, Combined Seepage, Hydraulic structures.*

NOTATIONS

- D Depth of the sheet pile,
- $H_o$ Depth of seepage flow at the sheet pile above the impervious layer,
- $H_1$ Depth of flow at upstream side,
- $H_2$ Free board of the drain,
- h Potential just after the sheet pile,
- $K_s$ Coefficient of permeability,
- L Horizontal length of floor,
- $\ell_1$ Length of floor before the sheet pile,
- $\ell_2$ Horizontal distance between sheet pile and intersection of seepage line with the flow surface in the drain,
- Q Quantity of seepage per unit width,
- S Horizontal length of seepage face behind the floor,
- T Depth of pervious layer beneath the floor,
- t Thickness of the floor,
- Z Height of downstream water level above the impervious layer,
- $\theta$ The side slope angle.

## 1. INTRODUCTION

Seepage flow, beneath hydraulic structures, may either be confined or unconfined. Confined seepage occurs under gravity structures such as; regulators, weirs, power stations, dams, etc. Seepage flow through earth and rock-fill dams, and earthen embankment is considered unconfined.

Combination of both confined and unconfined seepage may also take place beneath hydraulic structures such as tail escape structures. Tail escape structures are characterized by discrepancy of the upstream and downstream seepage faces. Downstream seepage face is shaped to accommodate the lowered levels of the receiving drain. This often makes seepage flow separates from the downstream side of the sheet pile, creating free flow behind the sheet pile. Consequently, the sheet pile divides the flow domain, below the floor, into two adjacent zones; I and II as shown in Figure (1). The first zone, which is upstream the sheet pile, is equipped with confined seepage. In the other, which follows the sheet pile, the seepage line is lowered representing the free surface of unconfined seepage. Hereby, occurrence of confined and unconfined seepage at the same conditions presents a combined confined/unconfined seepage problem.

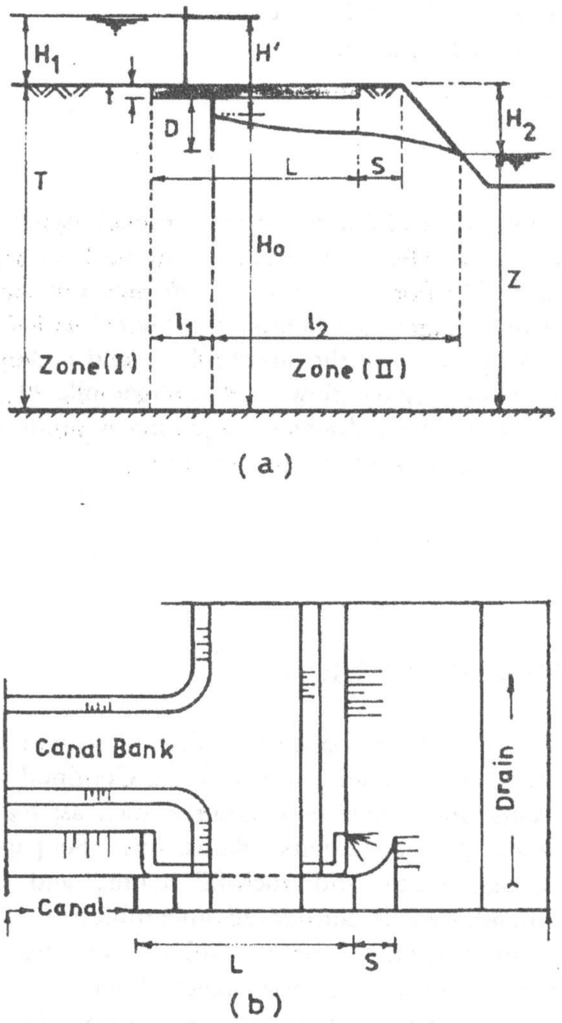


Figure 1. Definition sketch;
(a) longitudinal section,
(b) half plan.

Seepage characteristics for many seepage problems, concerned with confined or unconfined seepage beneath or through hydraulic structures, have been extensively investigated on different ways of approach [ 1,2,3,4,5,6]. In contrast to these, the problem of combined seepage beneath hydraulic structures has not nearly been dealt with. An attempt has been made to study experimentally the seepage mechanism beneath floor of a tail escape structure without sheepiling [7]. However, in practice, flat floors without sheet piles are not commonly used.

The present study is intended to investigate theoretically the factors affecting the characteristics of combined confined /unconfined seepage beneath a floor of tail escape structure provided with one row sheet pile. These factors are; distance S, depth of sheet pile D and its position $\ell_1$, depths $H_1$ and $H_2$, length of the floor L and it's thickness t, angle of the slopping seepage face $\theta$, and the depth of permeable soil foundation T. The distance S may be assendingly extended from the drain, creating an approaching distance for the canal flow to the drain. The value of floor thickness t can be neglected since it's value is very small compared to the depth T. The depth T is chosen to be a reference factor.

## 2. THEORETICAL APPROACH

In Figure (1-a), the seepage flow separates from both the floor and the sheet pile together. This is not only the probable case of separation, flow may separate from the floor only depending on values of $H_1$, $H_2$, and S. Separation increases as $H_2$ increases, while it decreases as $H_1$ and S increase. For a specific distance S, separation of flow from the floor only or from the floor and the sheet pile together depends on the higher effect of $H_1$ or $H_2$. These two cases are experimentally investigated. Results and complete discussion will be presented in part II (experimental study). The theoretical study in the present paper is only concerned with separation of flow from both the floor and the sheet pile, which is indicated in Figure (1-a).

The seepage discharge computed by Depuit's equation essentially depends on the flow depth at the sheet pile $H_o$. Hence, the theoretical study aims at predicting a general equation for the depth $H_o$. Then the seepage quantity can easily be evaluated applying Depuit's equation.

According to the solution of the complete elliptic integral of the first kind, concerned with confined seepage beneath hydraulic structures [3], the discharge equation in zone I, Figure (1-a), can be expressed as:

$$Q_{con.} = K_s . H' \frac{K'}{K},... \qquad (1)$$

where, $Q_{con}$ - is the discharge referred to confined seepage,

$$H' = T + H_1 - H_o,$$

K and K′ are the constants of complete elliptic integral of the first kind and
$K_s$ is the coefficient of permeability.

Substituting for H′ in Eq (1), yields,

$$Q_{con.} = K_s (T + H_1 - H_o) \frac{K'}{K}. \qquad (2)$$

Applying Depuit's equation for the unconfined seepage in zone II, Figure (1-a), the seepage discharge may be obtained as,

$$Q_{unc.} = K_s \left(\frac{H_o^2 - Z^2}{2\ell_2}\right),... \qquad (3)$$

where $Q_{unc.}$ - is the discharge referred to unconfined seepage, and

$$Z = T - H_2$$

Substituting for Z in Eq. (3), one get,

$$Q_{unc.} = K_s \left[\frac{H_o^2 - (T - H_2)^2}{2\ell_2}\right] \qquad (4)$$

Since, $Q_{con.} = Q_{unc.} = Q$, then equating Eqs (2), and (4), yields,

$$H_o^2 + 2\ell_2 . H_o \frac{K'}{K} = (T_2 - H_2)^2 + 2\ \ell_2 (T + H_1) \frac{K'}{K} \qquad (5)$$

Adding the term $(\ell_2 \frac{K'}{K})^2$ to each sides of Eq (5) and simplifying, we get,

$$H_o = \sqrt{(T - H_2)^2 + 2\ell_2 (T + H_1) \frac{K'}{K} + (\ell_2 \frac{K'}{K})^2} - \ell_2 (\frac{K'}{K}) \qquad (6)$$

Relating the variable involving in Eq (6) to the pervious layer depth T, Eq (6) becomes;

$$\frac{H_o}{T} = \sqrt{(1 - \frac{H_2}{T})^2 + 2(\frac{\ell_2}{T})(1 + \frac{H_1}{T})(\frac{K'}{K}) + (\frac{\ell_2}{T})^2 (\frac{K'}{K})^2} - (\frac{\ell_2}{T})(\frac{K'}{K}) \qquad (7)$$

where $1 - \frac{D}{T} < \frac{H_o}{T} < 1.0$

From the geometry of Figure (1),

$$\frac{\ell_2}{T} = \frac{L}{T} - \frac{\ell_1}{T} + \frac{S}{T} + \frac{H_2}{T} \cot \theta$$

The values of K and K'can be determined from special tables [8] according to the value of the modules m, where,

$$m = \cos\left(\frac{\pi}{2} . \frac{D}{T}\right) \sqrt{\tanh^2\left(\frac{\pi}{2} . \frac{\ell_1}{T}\right) + \tan^2\left(\frac{\pi}{2} . \frac{D}{T}\right)}. \qquad (8)$$

It is obvious from Eqs. (7) and (8) that the value of $H_o$ depends on $H_1$, $H_2$, S, L, $\ell_1$, D, and $\theta$. Hence, it is easy to calculate $H_o$ if values of these variables are known. Once the value of $H_o$ is determined from Eq (6), seepage discharge could be calculated using either Eq (2), or Eq. (3).

## 3. ANALYSIS AND DISCUSSION

As mentioned above, seven parameters are involved in the problem; $H_1$, $H_2$, S, L, $\ell_1$, D, and $\theta$. So, the effect of these parameters on, $H_o$, will be indicated graphically for specific conditions, as shown in Figures (2,3,4,... and 10). It is seem from the indicated Figures that, the effect of various parameters on $H_o$ is considerable. On one hand, the value of $H_o$ increases as well as values of $H_1$, S and L increase. On the other hand, $H_o$ values decrease whenever $H_2$, $\ell_1$, D and $\theta$ increase.

The value of $H_o$, in most cases, being minimum when $H_2/T$ ranges between 0.5 and 0.7. This may be referred to that, when $H_2/T \simeq 0.6$, the flow cross section area at the entrance to drain being more contracted. Such contraction causes backing effect to the flow which slightly rises the free surface again. When $H_2/T$=1.0, the values of $H_o$ sharply increase because that, the flow has no way to enter the drain through it's bed surface. Hence, the side seepage face of the drain becomes the only possible path for the flow to the drain.

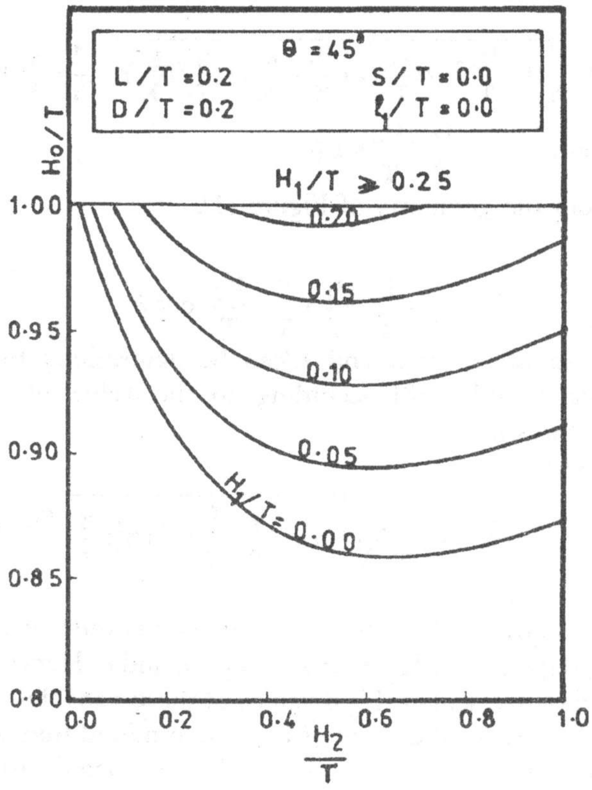


Figure 2. Effect of $H_2$ on $H_o$ for different values of $H_1$.

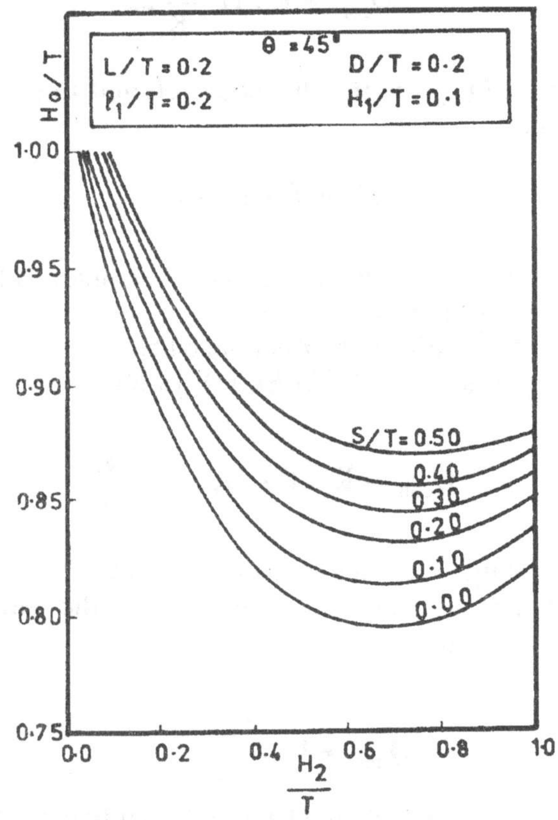


Figure 4. Effect of $H_2$ on $H_o$ for different values of S.

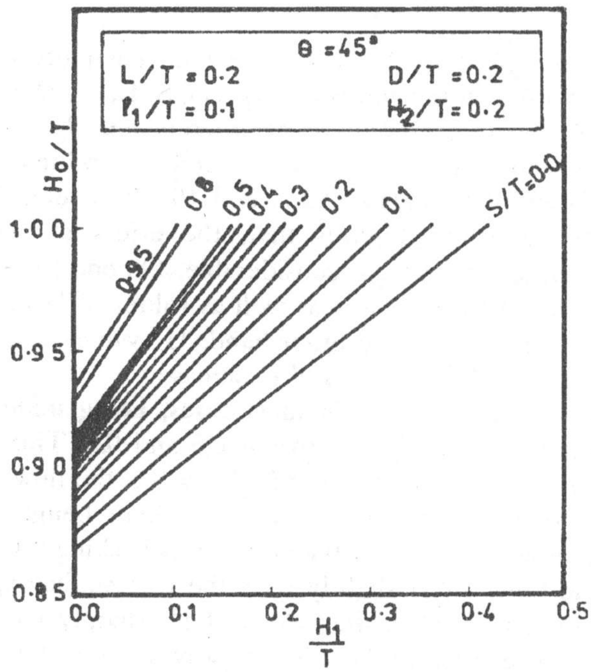


Figure 3. Effect of $H_1$ on $H_o$ for different values of S.

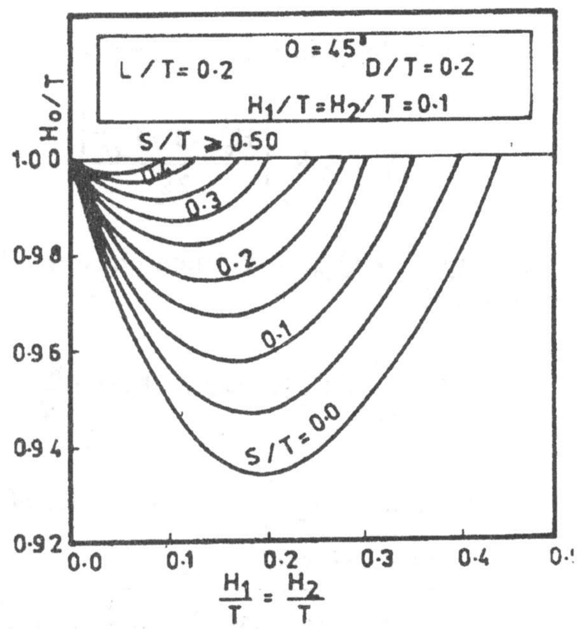


Figure 5. Effect of both $H_1$ and $H_2$ on $H_o$ for various values of S.

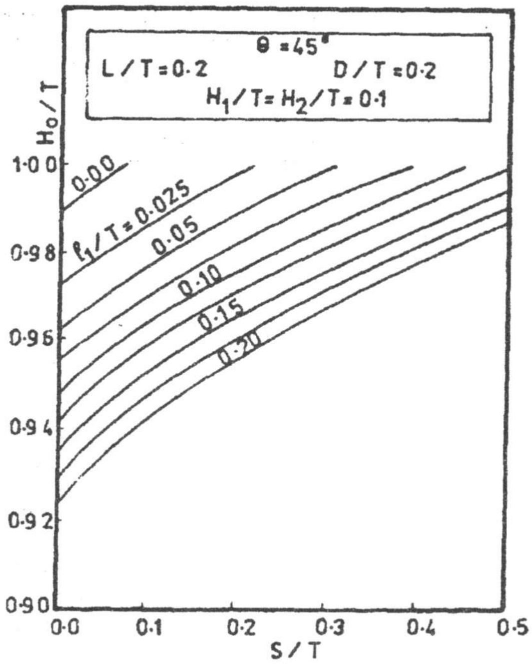


Figure 6. Effect of S on $H_o$ for different values of $\ell_1$.

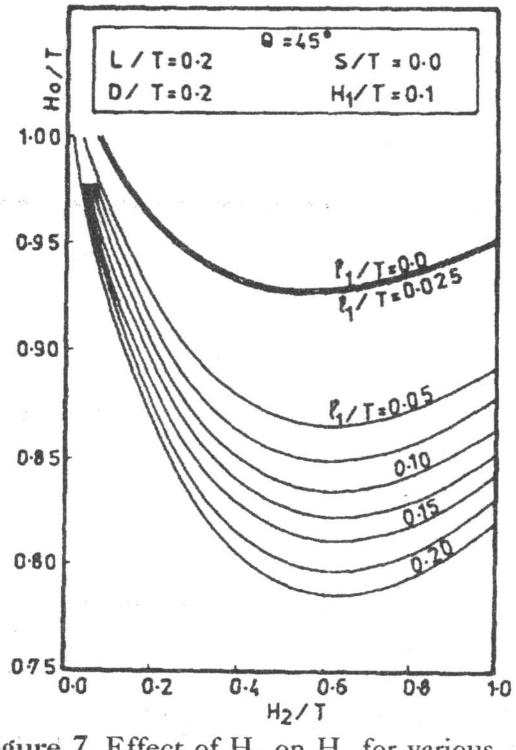


Figure 7. Effect of $H_2$ on $H_o$ for various values of $\ell_1$.

Referring to Figure (2) and (5), $H_o$ has a maximum value, equals T, for values of $H_1/T \geq 0.25$, and $S/T \geq 0.5$, respectively. For the same values of L, $\ell_1$, D, and $\theta$, in Figure (4) and (5), the head $H_1$ makes the minimum values of $H_o$ to occur at $H_2/T \sim 0.2$ in Figure (4) instead of $H_2/T \sim 0.7$ in Figure (5), when S/T=0.

Referring to Eq. (8), the term $\tan^2 (\frac{\pi}{2}.\frac{D}{T})$ is too small, compared with $\tanh^2 (\frac{\pi}{2}.\frac{\ell_1}{T})$. This makes the sheet pile depth D has a negligible effect on $H_o$, especially the depth D is not included in $H_o$ equation, Eq. (7),. Hence, the only possible effect of the sheet pile on $H_o$ values is obtained when $\ell_1/T$=0.0, as shown in Figure (8).

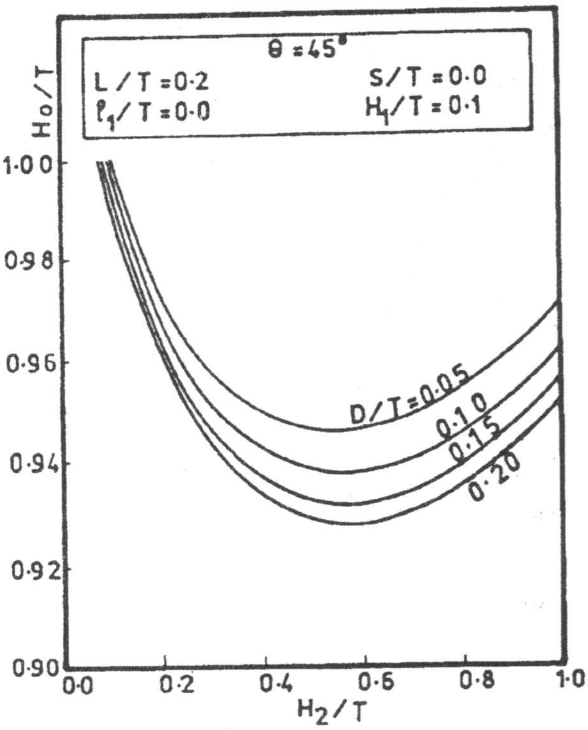


Figure 8. Effect of $H_2$ on $H_o$ for various values of D.

For the given values of L, D, $\ell_1$, $H_o$, and S in Figure (9), the maximum angle $\theta$, which guarantee continuous contact of the flow to the sheet pile, is 45°. That is, when D/T=0.2, $H_o/T$ should not be less than 0.8, otherwise flow leaves the sheet pile away.

With regard to the curve defined with L/T=0 in Figure (10), it presents a clear example, for combined seepage, in hydraulic structures practice. Such case, L/T=0, represents a single sheet pile without floor. A single sheet pile, retaining a head of water meets it's application if it is used as a cofferdam surrounding an excavated area prepared for construction.

It is seen from Figure (1-a) that the total potential on the floor H, is being the maximum when $H_2$=0. Substituting for $H_2$=0 in Eq. (7) results values of $H_o$ greater than T, i.e, $\frac{H_o}{T} > 1.0$. Abstracting the values of T from the resulted $H_o$ values gives the potential values h just behind the sheet pile, as shown in Figure (11).

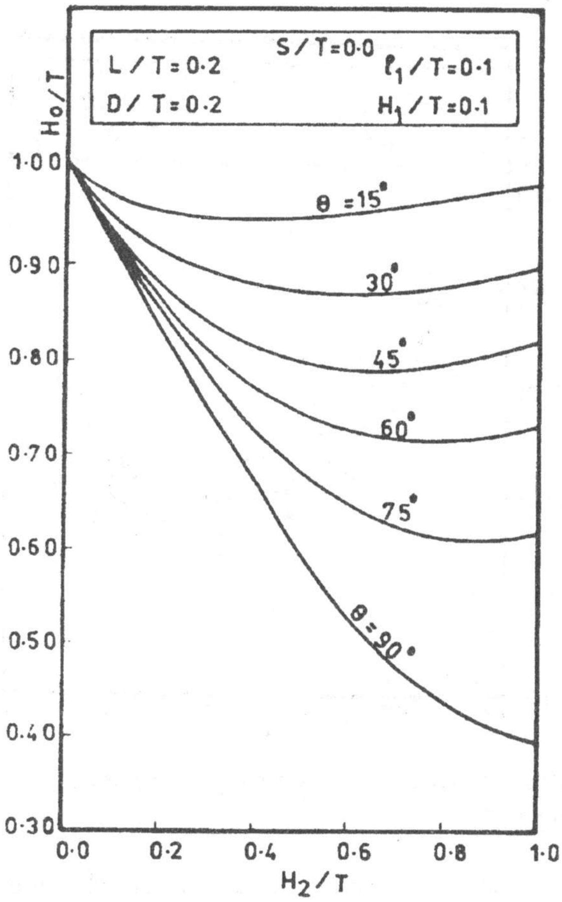


Figure 9. Effect of the angle $\theta$ on $H_o$.

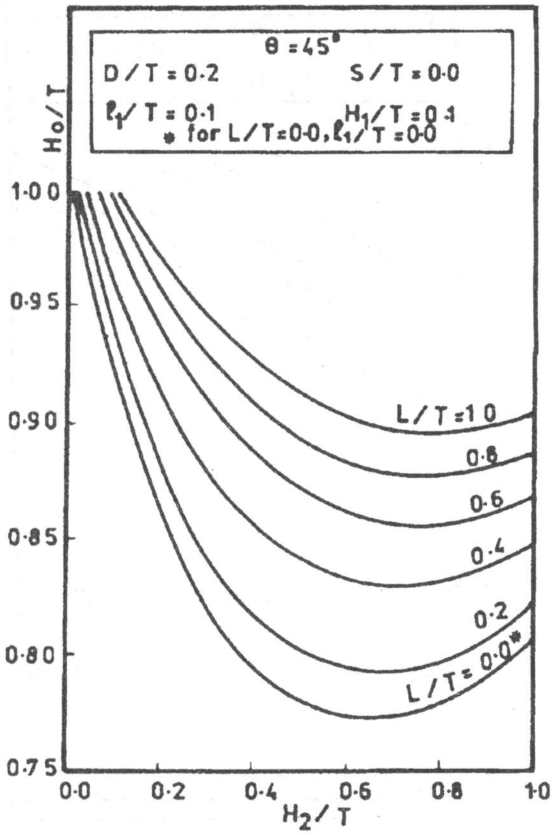


Figure 10. Effect of length of the floor L on $H_o$.

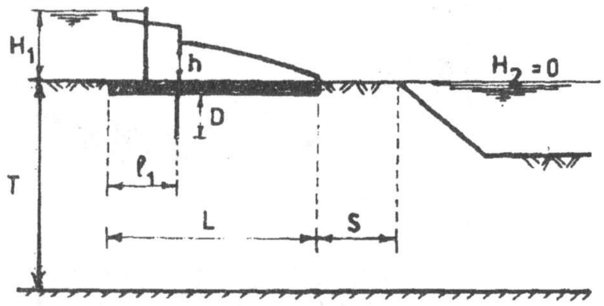


Figure 11. Definition sketch for potential diagram along the floor.

For $\ell_1/T$ = 0, i.e., the sheet pile is located at the upstream end of the floor, the variation of relative potential $h/H_1$ is plotted versus D/T for different values of S as shown in Figure (12). It is seen from Figure (12) that S gives higher effect, on h, than D. However, the effect of distance S diminishes as S increases. At S/T = 1.0 the effect of S becomes negligible.

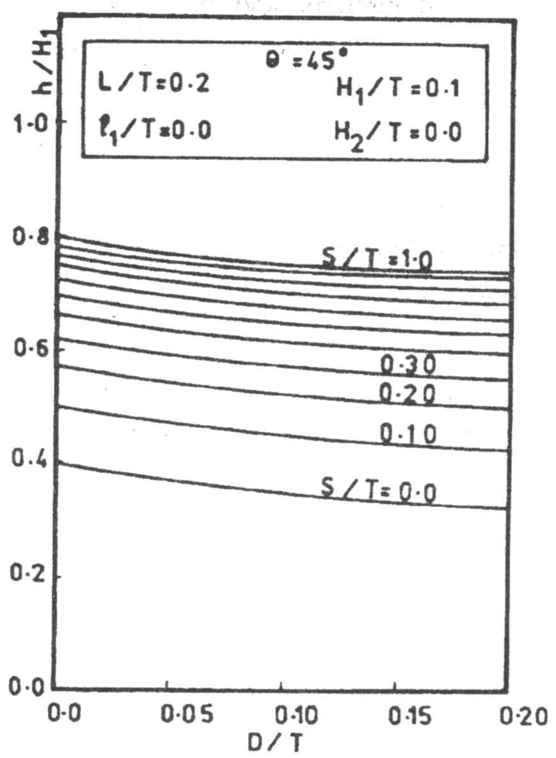


Figure 12. The remained potential head just behind sheetpile versus sheetpile depth, for various values of, S.

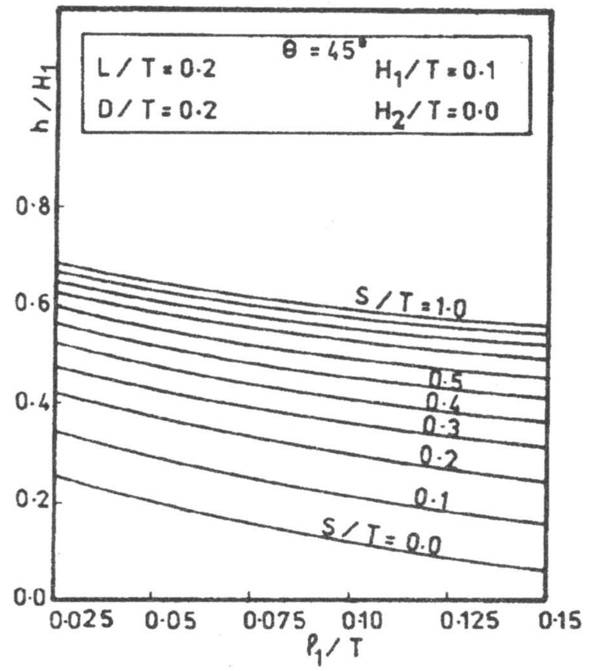


Figure 13. The remained potential head just behind sheetpile versus its position $\ell_1$ for different values of S.

The effect of sheet pile position ($\ell_1$) on h is shown in Figure (13). Nearly, the effect of distance S becomes negligible when S=T.

Finally, it should be noticed that, the predicted Equations Concerned with the depth $H_o$ and seepage discharge Q, in the present work, are checked experimentally. Results will be reported in part II (experimental study).

## 4. CONCLUSION

A theoretical solution is presented for the problem of combined confined unconfined/ seepage beneath hydraulic structures. The solution enables to determine the seepage quantity for wide range of different parameters. The solution can also be used for design of hydraulic structures subjected to combined confined/unconfined seepage. That is for dams, where the seepage quantity is more significant, it should be constructed as far as possible from the dropped area. For gravity structures; regulators and weirs, the uplift pressure and the exit gradient are considered important items in the design. Hence, they should be constructed as close as possible to the lowered area.

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