THEORETICAL AND EXPERIMENTAL ANALYSIS OF THE DYNAMIC RESPONSE OF A BEAM SUBJECTED TO MOVING LOADS

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ABSTRACT

The transverse vibration of a simply supported beam subjected to moving loads is analyzed. The analysis takes into consideration the shear deformation and the rotary inertia of the beam. An experimental set up is presented and the experimental results are discussed all together with the theoretical values obtained from a numerical Runge-Kutta solution. The results show that the dynamic response of the beam is affected clearly by the travelling velocity of the moving loads, specially for high travelling velocities. The experiments highlight the existence of a velocity generated signal which increases with the travelling velocity.

Keywords: Vibration of beams, Moving load problem, Experimental technique.

INTRODUCTION

The dynamic response of an elastic structure subjected to moving loads has been studied extensively over the past decade. These studies were initially directed at civil engineering applications such as railway tracks and bridges subjected to moving vehicle loads (1-7]. Recently, the moving load problem has been a topic of engineering interest because of its apparent in many modern machining operations such as high speed precision drilling and ballistic machining, beside the long truss-type space frame structures, [8-10]. The extreme positioning accuracy required in the modern machining operations requires minimizing of the induced vibrations to achieve the desired performance.

The earlier studies treated the problem using the elastic Bernoulli-Euler beam assumptions [11-13]. All of these studies solved the governing partial differential equation for an elastic structure excited by moving loads with constant velocity via an eigenfunction expansion. Other investigations, [14,15], developed some finite element models to simulate the response of the same type of structures (Bernoulli-Euler beam).

However, the response of a beam to a moving load with corrections for shear deformation and rotary inertia effects, which may be important for high speed moving loads, has not received much attention [16]. This study presents the steady state response of an elastic beam subjected to several moving harmonic loads. The analytical solution takes into account the shear deformation and the rotary inertia of the beam. An experimental set up is presented and comparison between the analytical and experimental results are discussed.

THEORETICAL ANALYSIS

Figure (1) shows a simply supported, initially undeformed, and at rest beam excited by a harmonic moving load ($P_{(x,t)} = P_o e^{i\omega t}$). The harmonic excitation force moves with constant velocity (v)

Considering Timoshenko beam by including shear deformation and rotary inertia, neglecting the viscous damping effect, the equation of motion governing the transverse vibration of the beam can be written as, [10, 17].

$$EL\frac{\partial^4 y}{\partial X^4} - (P_{(x,t)} - \bar{m}\frac{\partial^2 y}{\partial t^2}) - \bar{m}r^2\frac{\partial^4 y}{\partial x^2 \partial y^2} + \frac{EI\partial^2}{K'AG\partial x^2}(P_{(x,t)} - \bar{m}\frac{\partial^2 y}{\partial t^2}) - \frac{\bar{m}r^2\partial^2}{K'AG\partial t^2}(P_{(x,t)} - \bar{m}\frac{\partial^2 y}{\partial t^2}) = 0 \quad (1)$$

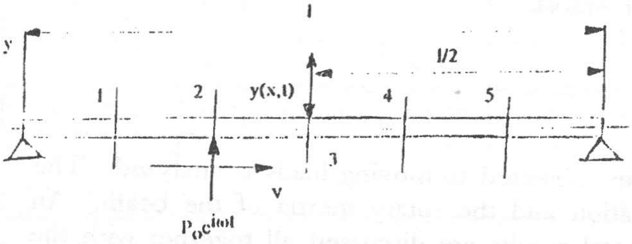

Figure 1. Model of a beam subjected to a moving harmonic load.

where; $y_{(x,t)}$ is the transverse deflection, $P_{(x,t)}$ is the applied force, is the mass per unit length, E is the modulus of elasticity, I is the second moment of area, A is the cross-sectional area, G is the shear modulus, l is the beam length, r is the radius of gyration and K′ is the shear coefficient (for rectangular cross-section).

The moving harmonic force $P_{(x,t)}$ may be written as;

$$P_{(x,t)} = P_o e^{i\omega t}\,\delta(x - vt) \quad (2)$$

where; v is the velocity of the moving force, ω is the circular frequency of the harmonic excitation and δ(x) is the Dirac delta function defined as;

$$\delta(x - vt) = 0$$
$$\int_{-\infty}^{\infty} \delta(x - vt)dx = 1 \quad (3)$$

In the case of simply supported beam the boundary conditions are:

$$y_{(0,t)} = (\frac{\partial^2 y}{\partial x^2})_{(0,t)} = 0 \text{ and } y_{(l,t)} = (\frac{\partial^2 y}{\partial t^2})_{(l,t)} = 0 \quad (4)$$

Zero initial conditions as assumed are;

$$y_{(x,0)} = (\frac{\partial y}{\partial t})_{(x,0)} = 0$$

Solving the equation of motion (Eq. 1) by using the modal expansion technique, one can assume the solution as;

$$y_{(x,t)} = \sum_{n=1}^{\infty} Y_n(t)\phi_n(x)dx$$

where, $\phi_n(x)$ are the modal shape functions and $Y_n(t)$ is the modal amplitudes which vary with time.

The orthogonal set of natural mode shape functions $\phi_n(x)$ are, [18] and [19];

$$\phi_n(t) = \sin(\frac{n\pi\ l}{x}) \quad (5)$$

Substitution for the time dependent modal amplitude Yn(t) leads to the modal equation of motion which could be written as;

$$\frac{d^4Y_n}{dt^4} + C_{nl}\frac{d^2Y_n}{dt^2} + C_{n2}Y_n = A_1\int_0^1 \phi_n(x)P_{(x,t)}dx - A_2\int_0^1 \phi_n(x)\frac{\partial^2}{\partial x^2}P_{(x,t)}dx + A_3\int_0^1 \phi_n(x)\frac{\partial^2}{\partial t^2}P_{(x,t)}dx \quad (6)$$

with;

$$C_{nl} = \frac{K'AG}{r^2\bar{m}l^2}\left[1^2 + r^2(n\pi)^2(1 + \frac{E}{K'G})\right]$$

$$C_{n2} = \frac{EIK'AG}{r^2\bar{m}^2}(\frac{n\pi}{1})^4$$

$$A_1 = \frac{2K'AG}{r^2\bar{m}^2 l} \qquad A_2 = -\frac{2EI}{r^2\bar{m}^2 l} \qquad A_3 = \frac{E}{\bar{m}l}$$

Substituting Eqs. (2) and (5) into (6) leads to;

$$\frac{d^4Y_n}{dt^4} + A_3\gamma\left[\sum_{n=1}^{\infty}\frac{d^4Y_n}{dt^4}\sin(\frac{n\pi}{1}vt)\right]\sin(\frac{n\pi}{1}vt) + C_{nl}\frac{d^2Y_n}{dt^2} + \gamma\left[A_1 + A_2(\frac{n\pi}{1})^2 - A_3(\frac{n\pi v}{1})\right]\left[\sum_{n=1}^{\infty}\frac{d^2Y_n}{dt^2}\sin(\frac{n\pi}{1}vt)\right]\sin(\frac{n\pi}{1}vt) + C_{n2}Y_n$$
$$= \left[-A_1 + A_2(\frac{n\pi}{1})^2 + A_3(\frac{n\pi v}{1})^2\right]\gamma g \sin(\frac{n\pi vt}{1})$$

In matrix form, Eq. (7) could be written as;

$$\begin{bmatrix} (1+\gamma M_1^2) & \gamma M_2 M_1 & \cdots & \gamma M_n M_1 \\ \gamma M_1 M_2 & (1+\gamma M_2^2) & \cdots & \gamma M_n M_2 \\ \gamma M_1 M_3 & \gamma M_2 M_3 & \cdots & \gamma M_n M_3 \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \gamma M_1 M_n & \gamma M_2 M_n & \cdots & (1+\gamma M_n^2) \end{bmatrix} \begin{bmatrix} \frac{d^4 Y_1}{dt^4} \\ \frac{d^4 Y_2}{dt^4} \\ \cdot \\ \cdot \\ \cdot \\ \frac{d^4 Y_n}{dt^4} \end{bmatrix}$$

$$+ \begin{bmatrix} (C_{11}+M_1^2 L_1) & M_2 M_1 L_1 & \cdots & (M_n M_1 L_1) \\ M_2 M_1 L_2 & (C_{21}+M_2^2 L_2) & \cdots & M_n M_2 L_2 \\ M_3 M_1 L_3 & M_2 M_3 L_3 & \cdots & M_n M_3 L_3 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ M_n M_1 L_n & M_2 M_n L_n & \cdots & (C_{n1}+M_{n1}^2 L_n) \end{bmatrix} \begin{bmatrix} \frac{d^2 Y_1}{dt^2} \\ \frac{d^2 Y_2}{dt^2} \\ \cdot \\ \cdot \\ \cdot \\ \frac{d^2 Y_n}{dt^2} \end{bmatrix}$$

$$+ \begin{bmatrix} C_{12} & 0 & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & C_{nn} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix} = \gamma \cdot g \begin{bmatrix} L_1 M_1 \\ L_2 M_2 \\ \cdot \\ \cdot \\ \cdot \\ L_n M_n \end{bmatrix}$$

where,

$$L_n = A_1 + A_2\left(\frac{n\pi}{l}\right)^2 + A_3\left(\frac{n\pi v}{l}\right)^2 \quad \text{for } n = 1, 2, \ldots,$$

$$M_n = \sin\left(\frac{n\pi vt}{l}\right) \quad \text{for } n = 1, 2, \ldots, \infty$$

$\gamma = \dfrac{P_o e^{i\omega t}}{g}$, and g with gravitational acceleration.

GENERALIZATION OF THE PROBLEM

Figure (2) shows the simply supported beam excited by several harmonic loads $P_1 e^{i\omega_1 t}$, $P_2 e^{i\omega_2 t}$ and $P_3 e^{i\omega_3 t}$ moving with constant speed v_1, v_2 and v_3.

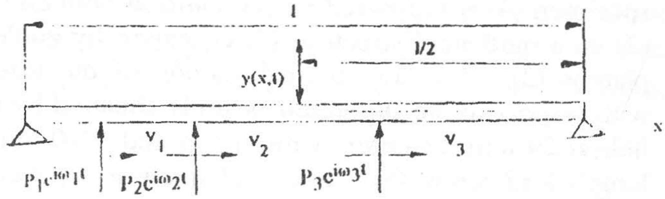

Figure 2. Model of a beam subjected to several harmonic moving loads.

The steady state response of the beam in such case could be obtained directly by linear superposition of the steady state response of the individual forces. In this case the general equation of motion will be in the form,

$$EI\frac{\partial^4 y}{\partial x^4} + \bar{m}\frac{\partial^2 y}{\partial t^2} - \bar{m}r^2\frac{\partial^4 y}{\partial x^2 dt^2} + \frac{\bar{m}}{K'AG}\left(\bar{m}r^2\frac{d^4 y}{dt^4} - EI\frac{d^4 y}{dx^2 dt^2}\right) \quad (9)$$
$$= P_1 e^{i\omega_1 t}\delta(x - v_1 t)P_2 e^{i\omega_2 t}\delta(x - v_2 t) + P_3 e^{i\omega_3 t}\delta(x - v_3 t)$$

The steady state response of the beam represented by Eq. (9) could be obtained directly by superimposed the steady state response of similar beams each one excited by only one of the excitations $P_j e^{i\bar{\omega}_j t}\,\delta(x-v_j t)$, providing that these forces are harmonic and regardless to their amplitudes, circular frequency or moving velocity [16]. Equation (9) is solved using the classical fourth-order Runge-Kutta method with fixed step size. The solutions run for a single harmonic force moving with a constant velocity and for two similar harmonic forces moving with the same linear velocity. The numerical values used are chosen to simulate the experimental set up. The values used are

$$\bar{m} = 0.863\text{Kg/m},\ A = 125\text{mm}^2,\ \lambda = 1100\text{mm},$$
$$I_x = 260.42\text{mm}^4,\ K' = \frac{5}{6},\ E = 200\text{GPa}$$

G = 80 GPa, r = 1.44, P_o = 50 N, ω = 125.6 rad/s
The results obtained will be discussed later with the experimental results.

EXPERIMENTAL SET UP AND PROCEDURE

Figure (3) shows the experimental set up in which 5x25 cross section and 1100 mm length steel beam specimen (4) is supported by pin joints at both ends (3) on a rigid steel structure (1) equipped by guide platens (2). The longitudinal motion of the load with respect to the supported beam is obtained by a helical 29 mm diameter, 4 mm pitch and 1200 mm length lead screw (5). The lead screw is screwed into the moving tables (6). The drive system consists of a 12 V DC motor coupled to a planetary gear box (7) and v-belt drive (8). The loads are applied through an extender (9) with adjustable length to ensure its contact with the beam. One end of the extender is equipped by a 6 mm inner diameter ball bearing to reduce the sliding friction and the other is screwed into the exciter head. A sine wave generator type 1027 B&K (10), a power amplifier type 2706 B&K (11) and two similar vibration exciters type 4809 B&K (12) are used to produce the harmonic loads acting on the beam. The response signal is picked by an 4332 B&K accelerometer (13) via a charge amplifier type 2635 B&K (14) to a high resolution Signal Analyzer type 2033 B&K (15). The freezed signals are recorded using 2308 B&K X-Y Recorder (16).

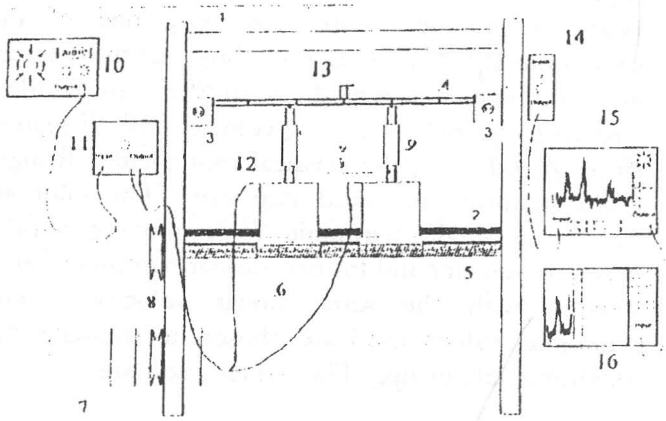

Figure 3. Experimental set up.

The beam is divided into parts by marking as shown in Figure (3) then mounted once for all series of experiments to cancel the effect of the interaction of the supports in the generated signal. Two sets of experiments were conducted, using one exciter only (single moving harmonic load) or by using two exciters distances 200 mm apart. Four differen longitudinal load speeds, namely 0, 0.20, 0.65, and 2.4 m/min were used in each set. The loading frequency was chosen as 20 Hz to be as near as practical to the first natural frequency of the system and the resulted signals at the beam mid-span fo each run was recorded. A summary of th experimental program is presented in Table (1) and some examples of the recorded response fo different loading conditions are shown in Figures (4), (5), (6).

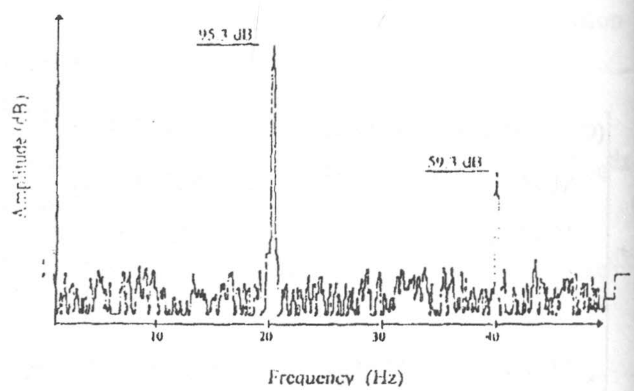

Figure 4. Response at mid. span to single harmon load acts at point 3.

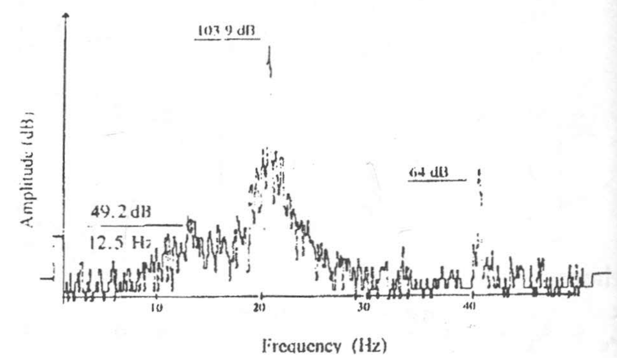

Figure 5. Response at mid. span to two harmon loads moving through point 1 and 2 with velocity (m/min.

RESULTS AND DISCUSSION

In the Runge-Kutta analysis the prescribed ax velocity of the moving load is defined by a no dimensional parameter given by;

$$\alpha = \frac{\pi v}{\lambda \omega_1} \tag{10}$$

where; v is the axial velocity of the moving load and ω is defined by;

$$\omega_1 = \left(\frac{\pi}{\lambda}\right)^4 \frac{EI}{\bar{m}} \tag{11}$$

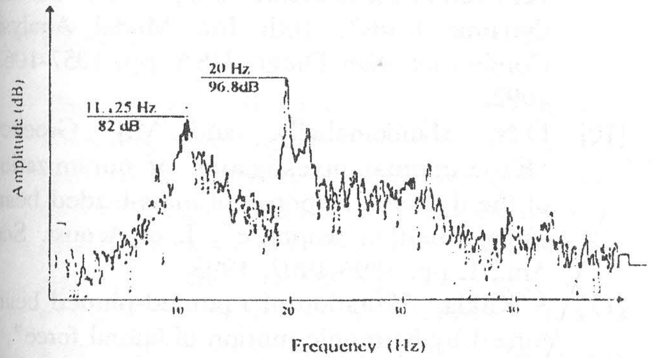

Figure 6. Response at mid. span to a single load moving through point 5 with velocity 2.4 m/min.

The value used in our analysis is w_1 = 528.2 rad/s. The mid-span deflection $y_{L/2}$ is normalized by the static deflection $y_{s\,L/2}$ when the load P_o is applied at the same point.

$$y_{sL/2} = \frac{P_o L^3}{48EI} \tag{12}$$

The results obtained for α = 0.12 and α = 0.5 are presented in Figures (7) and (8). These results were obtained using 5-term assumed functions (n=5). It is obvious that the deflection of the beam mid-span is affected by the travelling velocity of moving loads. Although the effect is small for small velocities, the deflection increase with about 60% for α = 0.5.

It is extremely difficult to achieve such linear velocities in the experimental approach. The velocities used in this approach were 0, 0.2 m/min, 0.65 m/min, and 2.4 m/min. These velocity values are very small compared with the clearly effected values obtained by the theoretical analysis. However, there are clear changes in the experimentally obtained dynamic behavior according to the travelling velocity (Figures (4), (5), (6)). The obtained response of the beam mid-span due to one load moving with the mentioned four speeds is presented in Figure (9) as a function of the load position. Figure (10) shows the response due to two loads moving simultaneously with the same speeds. Another remark which could not be obtained by the used numerical technique and highlighted clearly in the experimental graphs is the velocity generated response which obtained at a frequency ranges about 60% of the excitation frequency. This signal is clearly increased as a function of the load travelling speed.

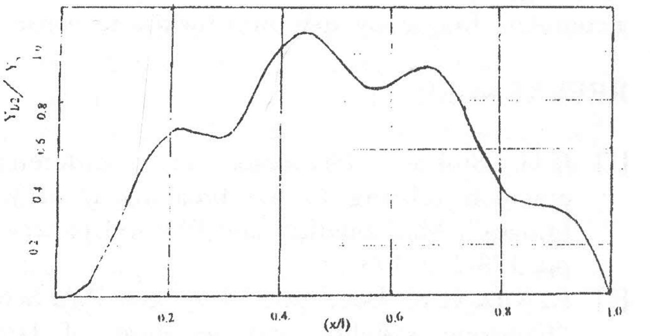

Figure 7. Normalized mid. span deflection as a function of load position, α=0.12.

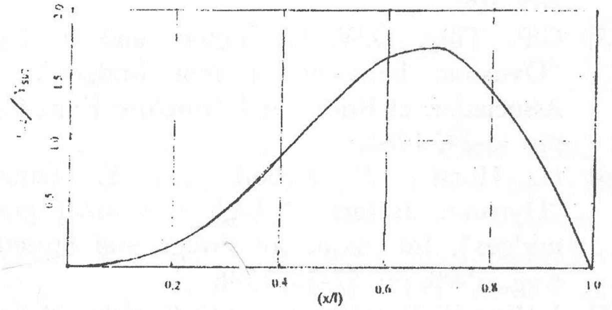

Figure 8. Normalized mid. span deflection as function of load position, α=0.5.

Some of the theoretical results correspond to experimentally obtained are normalized and calculated in db by using the well known transformation;

$$db = 20 \log_{10} \frac{Y_{L/2}}{Y_{sL/2}}$$

The comparison shows a very good agreement with respect to the experimental precision.

From the previous discussion one can conclude that:

The dynamic response of the beam is affected by the travelling velocity specially at high velocities.

A velocity generated response is clearly obtained in the experimental response. Such response could not be attained with the existing theoretical techniques.

The previous remark highlights the importance of designing an experimental set up which could be capable to achieve high travelling velocities and to detect its response with sufficient precision. The advantage of the experimental technique is to have a complete frequency spectrum for the response.

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