SEEPAGE UNDERNEATH DROP STRUCTURES

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ABSTRACT

A numerical model has been developed to simulate the groundwater seepage under drop structures with a wide range of geometries. The two dimensional steady state Laplace's equation is solved for the total potential head over a grid covering the area of the pervious soil foundation using a triangular finite element program. Design charts have been developed to present the obtained results for different structure geometries. These charts may be used to calculate the uplift pressure exerted along the base of the drop structure and the exit gradient at the exit face downstream for different geometries and effective head. The considered parameters are depth of sheet pile, upstream floor length, drop depth, total length of the structure and effective head. Their effects on the uplift pressure and maximum exit gradient are discussed.

Keywords: Hydraulic structures, drop structures, falls, seepage, uplift pressures, exit gradients, finite element.

Notation

The following symbo' .re used in this paper:

AB	inlet face
BCDEFG	subsurface contour of the floor;
d	drop depth;
FEM	finite element method
FEM2DV2	two dimensional finite element program;
g	gravitational acceleration;
GJ	exit face;
H	head difference between upstream and downstream water levels;
i	hydraulic gradient ;
Imax	exit gradient at point G;
k	hydraulic conductivity;
L	projection of the impervious floor on the horizontal;
L1	length of the upstream floor apron;
L2	length of the downstream floor apron;
Lds	Length of the exit face;
Lus	Length of the inlet face;
p	pressure head;
p/γH	relative pressure head
q	quantity of seepage per unit length of the structure;
ρ	water density;
S	depth of the end sheet pile;
T	pervious stratum depth;
U	uplift pressure at any point;
UC, UD, UE, and UF	uplift pressure at points C, D, E, and F, respectively;
z	elevation ;
[N]	shape function;
ψ	stream function; and
ϕ	velocity potential.

INTRODUCTION

The mean ground surface slope is about 10cm/km in lower Egypt and 12 cm /km. in upper Egypt. The water surface slope generally varies from 5 to 8 cm/km in main canals, and from 8 to 15 cm/km in branch canals [7]. An exception to this rule is the Fayum province, and the new reclaimed area in the western delta zone. For example the mean ground surface slope along Elhamam canal in the western Nuberia zone is about 200 cm/km. The longitudinal water surface slope of canals in such areas have large values. Such steep slopes must be reduced by introducing drop structures (falls) at appropriate places.

The hydraulic design of drop structure is given in detail in many publications[6,2]. The study of the seepage characteristics underneath the floors of drop structures has always been an important design consideration for irrigation engineers.

For steady state conditions, the head within a soil mass with known boundary conditions is governed by Laplace's equation. A wide variety of methods exists for its solution. These methods include analytical solutions using conformal mapping technique [1, 5, 16], analogue methods [1, 11], a method of fragments [1], and stochastic analysis [13]. Flow net is considered a powerful and versatile method in experienced hands, but they can be time consuming, and its accuracy is sometimes difficult to assess.

The work reported in the present paper covers the study of the characteristics of the seepage flow underneath the floors of single drop heading-up structures built in earthen canals and provided with end sheet piles. An example of such structures is the clear overfall weir provided with a single drop in its floor, shown in Figure (1). The studied seepage characteristics include the following:

1) The uplift pressures acting along the subsurface contour of the floor.
2) The maximum value of the exit gradients along the exit face which occurs at the downstream edge of the floor. Such a value is proportional to the force exerted on the soil grains [14], which tends to cause a piping failure.

The finite element method [10], is used to solve the problem. A computational model is designed to calculate the required seepage characteristics. The model is tested and verified. The accuracy of the results is assessed using an electrical analogue method [15]. The results are presented in the form of design charts. The effect of the variables on the seepage characteristics are discussed. To show the use of the obtained design charts, a numerical example is presented.

STATEMENT OF THE PROBLEM

The problem is shown schematically in Figure (2). The floor of the structure has a single drop of depth (d) located at distance L1 from its upstream edge. It has also a sheet pile located at the downstream edge and having a depth (S). The projection of the floor on the horizontal has a length equal to (L). The depressions of the floor thickness into the foundation soil are neglected. The structure is built in a homogeneous isotropic pervious soil. The effective head on the structure is (H).

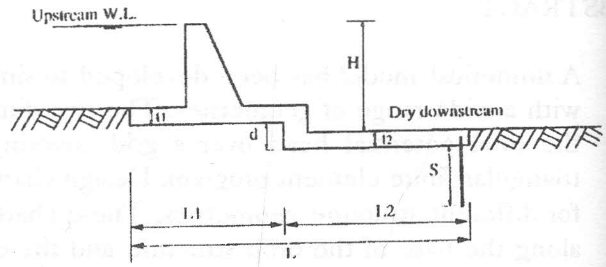


Figure 1. Definition sketch for a clear overfall weir provided with a single drop and a sheet pile at the downstream edge of the floor.

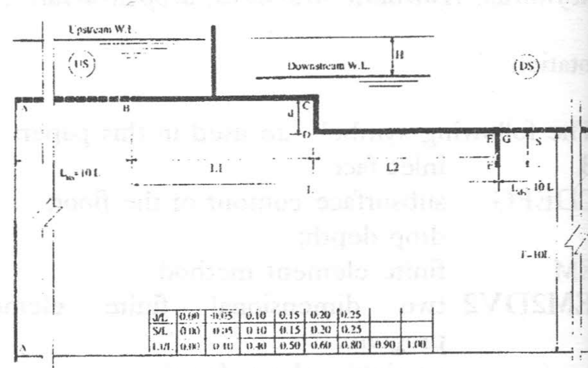


Figure 2. A schematic sketch for the problem.

In solving the seepage problems using the finite Element method, Muthukumaran and Kulandaiswamy, [9], recommended values for the length of both inlet and exit face such that Lus= Lds > 2.5L, and a value for the depth of the pervious layer T > 2.5L for short sheet piles (S < 0.5L) and T > 5S for long sheet piles(S>0.5L).

Preliminary studies are carried out to clarify the effect of change in Lus/L, Lds/L, and T/L on the studied seepage characteristics. Such studies show that for given values of Lus/L and Lds/L, the increase of T/L produce an increase in the uplift pressures at points E and F and a decrease in the uplift pressure at points C and D, but with a decreasing rate. For T/L>10.0, T/L has no effect on the uplift pressures at all the key points. For a certain values of T/L, the increase in Lus/L and Lds/L produce an increase in the uplift pressure at points C and D and a decrease in the uplift pressure at points E and F, but with a decreasing rate. For Lus/L=Lds/L>10.0, Lds and Lus has no effect on the uplift pressure at all the key points. Therefore we recommended values for the length of both inlet and exit face equal Lus=Lds=10L, and a value for the depth of the pervious layer equal T=10L. These dimensions are equivalent to infinite dimensions of porous media.

The objective of the present study is to determine the effect of the main parameters on the seepage characteristics, Such parameters can be written in dimensionless forms as follows: [L1/L, d/L, S/L, and H/L]

The considered seepage characteristics are also presented in dimensionless forms as follows:

1- The relative uplift pressures acting along the subsurface contour of the floor, U/H which control the stability of the structure.
2- The relative maximum exit gradient which occurs at the downstream e⁻ⅇ of the floor (point G), [Imax/(H/L)].

THE BOUNDARY CONDITIONS

The velocity potential, ϕ, and the stream function ψ, of the seepage flow satisfy the two-dimensional Laplace's equation;

$$\nabla^2 \phi = \nabla^2 \psi = 0.0 \qquad (1)$$

In the present study ϕ=-kh, in which h is the total head (p/ρg+z), and k denotes the coefficient of permeability.

where p pressure head, ρ is water density, and z is elevation.

Referring to Figure (2), the inlet face along the upstream bed (AB) and the exit face along the downstream bed (GJ) are both equipotential lines. Without losing the generality, the following relations can be written:

$$\left.\begin{array}{ll} \text{Along AB} & \phi_{AB} = -kH \\ \text{Along GJ} & \phi_{GJ} = 0.0 \end{array}\right\} \qquad (2)$$

This effectively assumes a datum at the downstream water surface.

The first streamline, ψ=0, coinsides with the subsurface contour of the floor (BCDEFG) and the last streamline, ψ=q coninmsides with the outer boundary (AAJJ), the flow is coming from the inlet face (AB) and draining out through the exit face (GJ). Thus

$$\left.\begin{array}{ll} \text{Along BCDEFG} & \frac{\partial \phi}{\partial n} = 0.0 \\ \text{Along AAJJ} & \frac{\partial \phi}{\partial n} = 0.0 \end{array}\right\} \qquad (3)$$

where n is the normal direction to the flow

METHOD OF ANALYSIS

The governing equation (1) has been solved numerically using the finite element method, for the given boundary conditions using the Galerkin formulation [3]. Adopting the shape function [N] as the weights, the weighted- residual of an element is given by

$$R^e = -\int [N]^T \left(k_x \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) dA \qquad (4)$$

Applying the Green's theorem and principle of finite element, the equilibrium equation for the seepage problem can be reduced to

$$\sum K^e \ \phi = \{ f \} \qquad (5)$$

where {f}= flow rate of point sources or sinks

$$k^e = -\int [B]^T [D][B] dA \qquad (6)$$

The [B] and [D] matricesare given by

$$[B]^T = \left[\frac{\partial [N]^T}{\partial x} + \frac{\partial [N]^T}{\partial y}\right] \qquad (7)$$

$$[D] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \qquad (8)$$

The confined flow problem under consideration can be solved by the direct application of equation (5).

THE DIMENSIONS OF THE FINITE ELEMENT MODEL.

The Finite Element Program FEM2DV2 [10] is used to calculate the required seepage characteristics.

To find out the effect of changing the various parameters on the seepage characteristics, two hundred and forty six runs of the computer program were carried out. Each run is characterised by certain dimensions for the floor and the thickness of the pervious foundation. The lengths of both the inlet and exit faces are taken equal (10 L) for all runs. The dimensions of the flow domain beneath the floor are discretized automatically using a specially designed subroutine.

The relative dimensions of the structure which are the relative drop depth (d/L), the relative end sheet pile depth (S/L) and the relative upstream floor length (L1/L) have had values varying from 0.0 to 0.25, from 0.0 to 0.25 and from 0.0 to 1.00; respectively.

To obtain design charts that cover all the possible cases which can meet the designer requirements in practice, the following cases are also considered;

1) The case of d/L=0, which means that the floor has no drop along its length.
2) The case of S/L=0, which means that the floor is constructed without sheet pile at its end.
3) The case of L1/L=0, although this case may be a hypothetical case, but it is useful to complete the construction of the design charts.

In order to carry out the finite element computations, the flow domain for each run is discretized using a triangular element mesh. Figure (3) shows the finite element mesh for just one case of the flow domains as an example.

EXPERIMENTAL VERIFICATION

In order to verify the calculated results from the numerical model, a twelve tests using electrical analogue method have been carried out, by the authors, with different boundary conditions and domain sizes along with random values of drop depth, d, upstream floor length, L1, downstream floor length, L2, and sheet pile depth, S, while the shape of drop structure remained unchanged, (Figure 2). A set of typical results is presented in Figure (4), where uplift pressure along the subsurface structure contour is plotted from both experimental and numerical results for the same dimensionless parameters. Upon observing the curves in these figures one can immediately see that a good agreement between experimental and theoretical results prevails. The maximum value of the percentage error between the experimental results and the numerical results is less than 5%. This effectively substantiates the use of the FEM. by virtue of its accuracy and versatility.

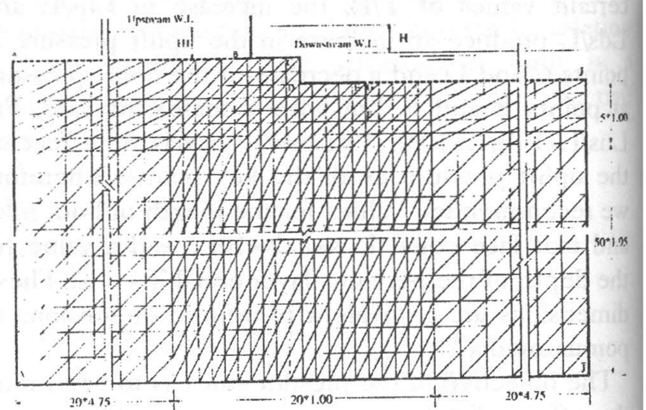
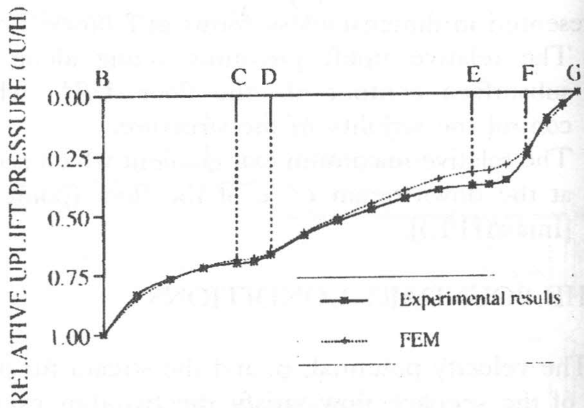
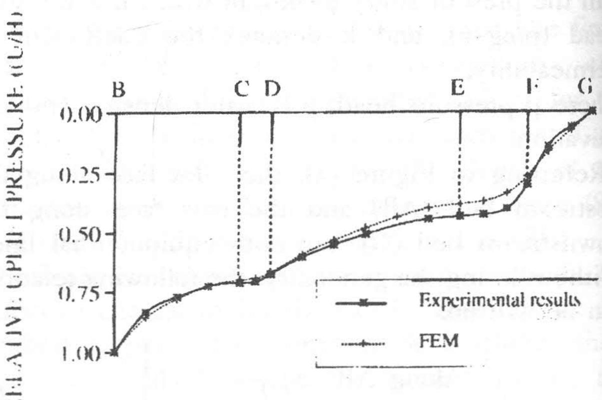


Figure 3. Finite element mesh for a general case of a single drop structure provided with an end sheet pile at the downstream floor edge.



(a) T/d=20. L1/d=4. L2/d=6, and S/d=1.62.



(b) T/d=20. L1/d=4. L2/d=6. and S/d= 2.14.

Figure 4. Relative uplift pressure distribution along the subsurface contour.

ANALYSIS OF RESULTS

The obtained results are used to:

1- Show the individual effect of each of the considered parameters on the studied seepage characteristics, Figures (5.a, b, and c).

2- Prepare design charts, which can be used directly by the designer to calculate the required seepage characteristics in practice, Figures (6) through (10).

EFFECT OF THE RELATIVE DROP DEPTH (D/L) ON THE SEEPAGE CHARACTERISTICS.

Figure (5.a) shows the effects of the relative drop depth (d/L) on the relative uplift pressures (U/H) at the key points, C, D, E, and F for Lus=Lds=T=10L, S/L=0.10 and L1/L=0.10 and 0.80. It is clear that the relative drop depth (d/L) has some effects on the uplift pressures along the subsurface contour with different extents.

The results also show that for all the tested values the relative uplift pressures at point (C) increases with increasing of drop depth (d/L). The increase of drop depth (d/L) causes a decrease in the relative uplift pressures at point D, (UD/H), for values of L1/L< 0.40, while it causes an increase in the values of (UD/H) for values of L1/L < 0.40. Also an increase in the drop depth creates an increase of the relative uplift pressures (UE/H) at point (E) as we can see that from figure (5.a) but this is much smaller than at point C and D and can be neglected for small values of L1/L. It is clear that the increase in drop depth produces a negligible increase on the relative uplift pressures (UF/H), at point (F). This is consistent with the relative position of points C, D, E, and F in relation to the position of the drop depth.

From figures (5.a), it is clear that the increase in relative drop depth produces a negligible decrease on the relative exit gradient at point (G) [Imax/(H/L)], for values of L1/L< 0.40, while it causes an increase in such values for L1/L > 0.40.

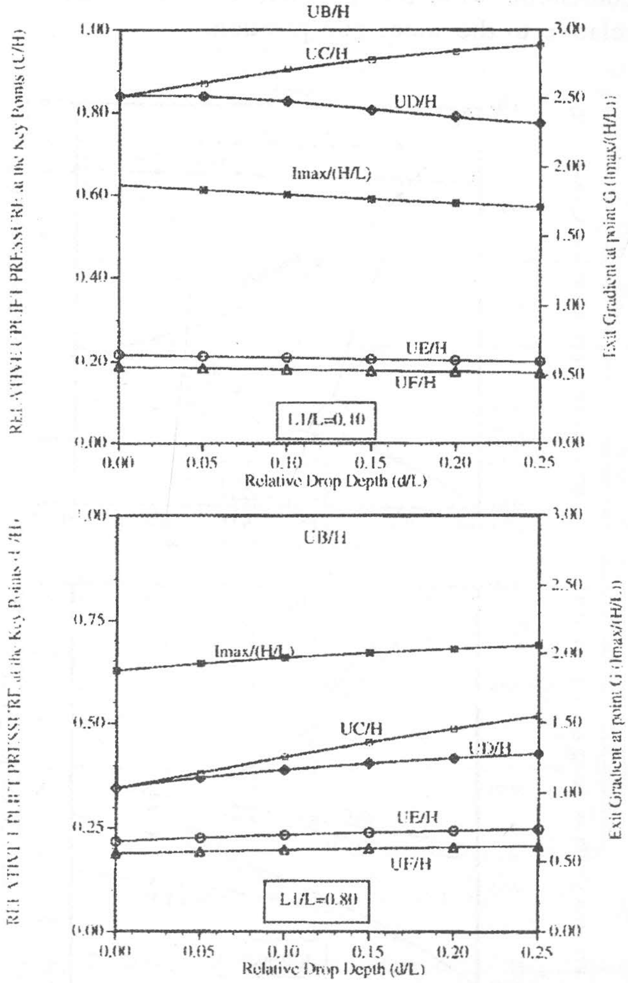


Figure 5-a. Effect of the relative drop depth (d/L) on the relative uplift pressure along the subsurface contour of the floor and the exit gradient at point G, for (S/L=0.10).

EFFECT OF THE RELATIVE SHEET PILE DEPTH (S/L) ON THE SEEPAGE CHARACTERISTICS.

Figure (5.b) shows the effects of the relative sheet pile depth (S/L) on the relative uplift pressures at the key points, for Lus=Lds=T=10L, d/L=0.10 and L1/L=0.10 and 0.80. It is clear that the relative sheet pile depth (S/L) has some effect on the relative uplift pressures along the subsurface contour. Figure (5.b) shows that the relative uplift pressures at all the selected key points (C), (D), (E), and (F) increase with the increase in the sheet pile depth (S/L), but with different extents. These results are consistent with the position of (C, D, E, & F) relative to the sheet pile position.

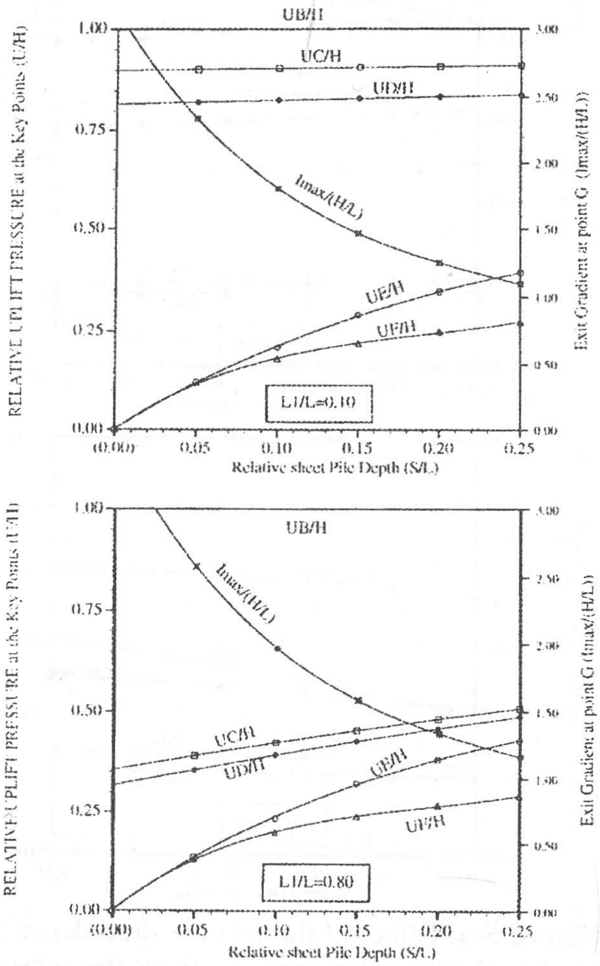


Figure 5-b. Effect of the relative sheet pile depth (S/L), on the relative uplift pressure along the subsurface contour of the floor and the exit gradient at the exit face for values d/L=0.10.

Figure (5.b) shows also that the increase in the sheet pile depth (S/L) causes a decrease with a great extent on the values of the maximum exit gradient at points G. This clarify the importance of using an end sheet pile to secure the stability of the structure against the piping failure.

EFFECT OF THE RELATIVE UPSTREAM FLOOR LENGTH (L1/L) ON THE SEEPAGE CHARACTERISTICS.

Figure (5.c) shows the effects of the relative upstream floor length (L1/L) on the relative uplift pressures at the key points, for Lus=Lds=T=10L, d/L=0.10 and S/L=0.10. It is clear that the relative upstream floor length (L1/L) has a great effect on the relative uplift pressures at points C and D. The relative uplift pressure at point (C) and (D) decrease rapidly with the increase in the upstream floor length (L1/L). An increase in the upstream floor length creates a negligible increase in the relative uplift pressures at points (E) and (F).

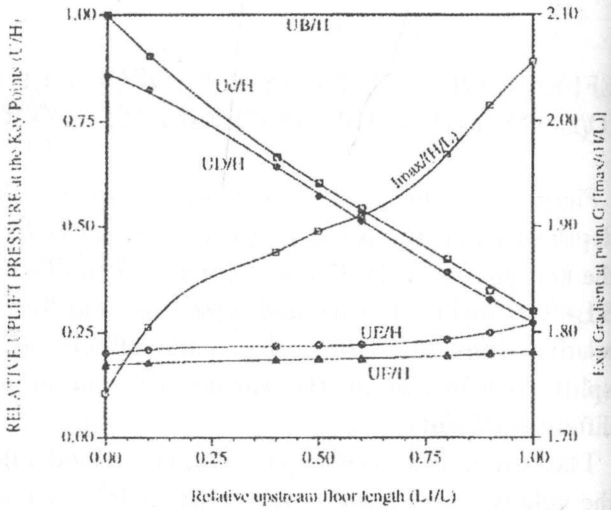


Figure 5-c. Effect of the relative upstream floor length (L1/L) on the relative uplift pressure along the subsurface contour of the floor and the exit gradient at point G, for values S/L=0.10 and d/L=0.10.

The relative upstream floor length (L1/L) has a great effect on the relative exit gradient at point G. The relative exit gradient [Imax/(H/L)] increases with a great extent as the relative upstream floor length (L1/L) increases.

DESIGN CHARTS

It is worthy to notice that representing each of the required seepage characteristics as a function of all the independent variables on the same graph is very helpful for design purposes. The graphical correlation method [8] and the computer statistical facilities are used to construct such graphs. The obtained design charts are presented in Figures (6) through (10).

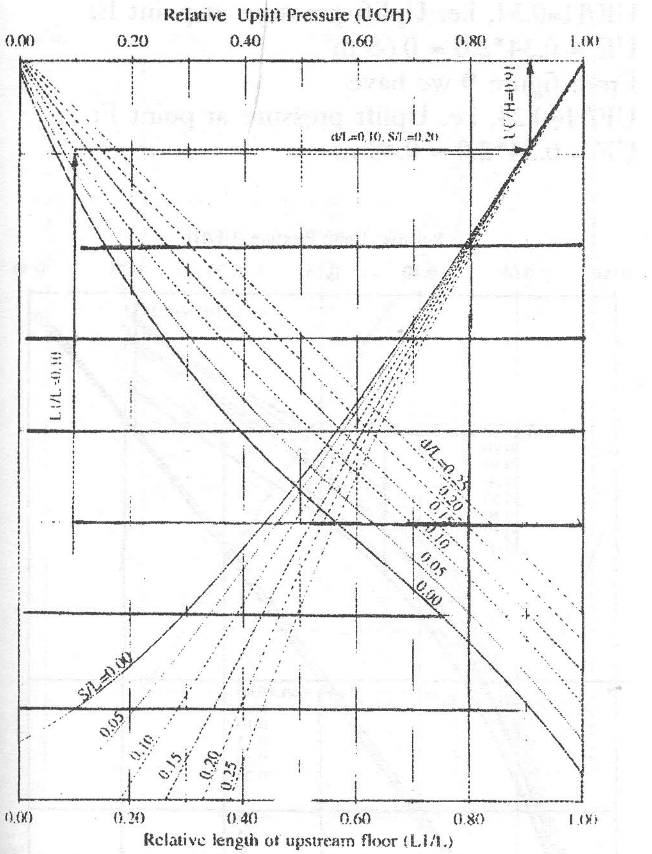


Figure 6. Relative between relative uplift pressure (UC/H) and upstream floor length (L1/L), sheet pile depth (S/L), and drop depth (d/L).

Figures (6,7,8 and 9) present in graphical form the calculated values of the relative uplift pressures at points (C), (D) ,(E) and (F); respectively as a function of the relative sheet pile depth (S/L), the relative drop structure depth (d/L), and the relative upstream floor length (L1/L).

Figure (10) presents in graphical form the calculated values of the relativeexit gradient at the downstream edge of the floor, point G, as a function of the drop structure depth (d/L), the relative sheet pile length (S/L), and the relative upstream floor length (L1/L).

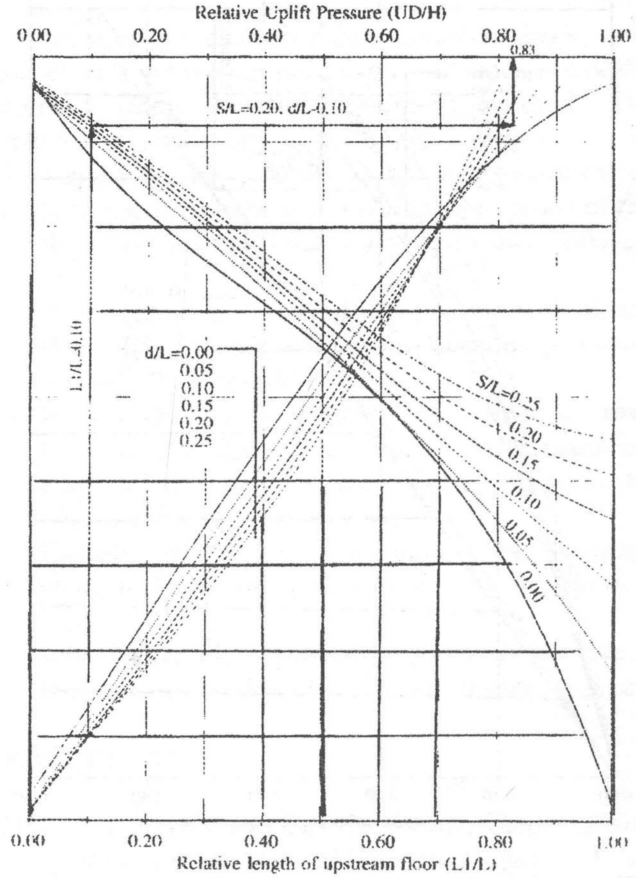


Figure 7. Relation between relative uplift pressure (UD/H) and upstream floor length (L1/L), sheet pile depth (S/L), and drop depth (d/L).

SOLVED EXAMPLE

A weir is to be constructed on a homogeneous isotropic soil of infinite extended. The weir has a drop in its floor . Figure (11) shows a longitudinal section of the weir, which covers the requirements of the hydraulic design. It is required to calculate the relative potential distribution and the exit gradient at the downstream edge of the floor.

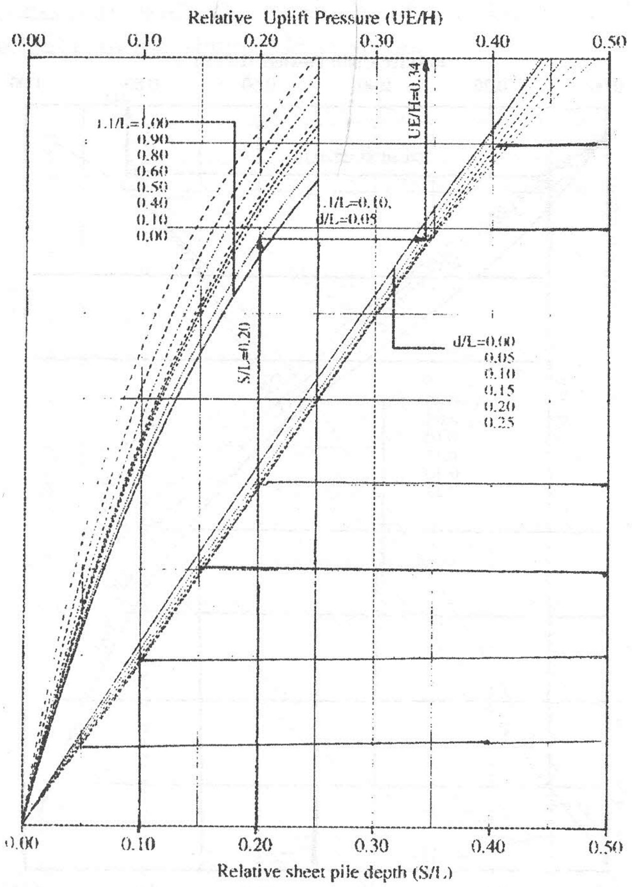


Figure 8. Relation between relative uplift pressure (UE/H) and sheet pile depth (S/L), upstream floor length (L1/L), and drop depth (d/L).

SOLUTION:

The extension of the floor behind the sheet pile as well as the floor thickness are small and can be neglected. The following approximate dimensions are used in calculation H=2.0 ms, L1=2.0 ms, L=20.0 ms, S=4.0 ms, and d=2.0 ms., therefore:

$$\frac{L1}{L}=\frac{2}{20.0}=0.10,\ \frac{S}{L}=\frac{4}{20.0}=0.20,\text{and}\ \frac{d}{L}=\frac{2}{20.0}=0.10$$

From figure 6. we have

UC/H=0.91, i.e. Uplift pressure at point C:
UC = 0.91*2.0 = 1.82 m
From figure 7 we have
UD/H=0.83, i.e. Uplift pressure at point D:
UD = 0.83*2.0 = 1.66 m

From figure 8. we have
UE/H=0.34, i.e. Uplift pressure at point E:
UE = 0.34*2.0 = 0.68 m
From figure 9 we have
UF/H=0.24, i.e. Uplift pressure at point F:
UF = 0.24*2.0 = 0.48 m

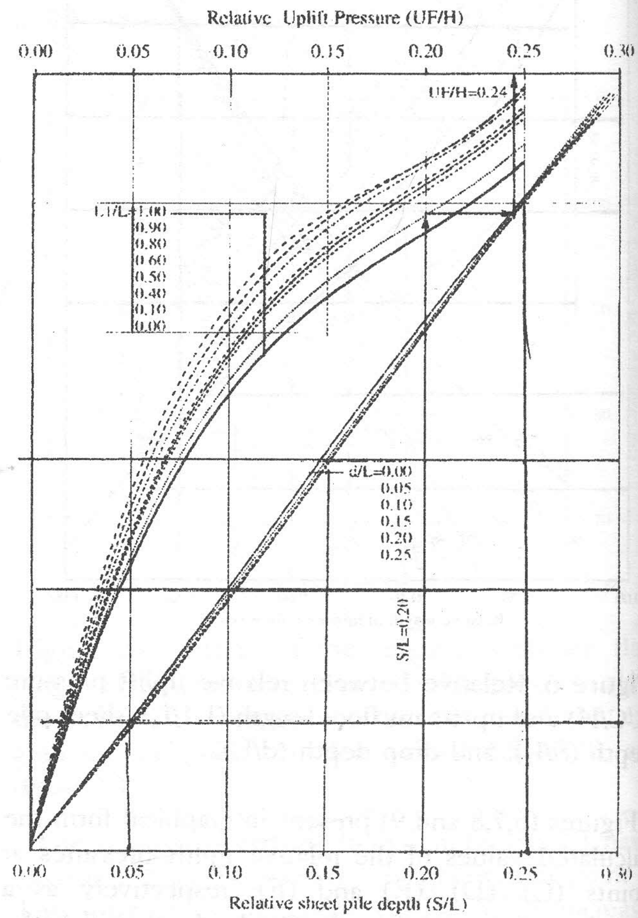


Figure 9. Relation between relative uplift pressure (UF/H) and sheet pile depth (S/L), upstream floor length (L1/L), and drop depth (d/L).

Figure (12) shows the uplift pressure distribution along the subsurface contour of the floor.
To calculate the maximum exit gradient at the exit face we should use Figure 10, therefore:

Imax/(H/L)=1.25
then Imax = 1.25*H/L = 1.25*2/20.0 = 0.125

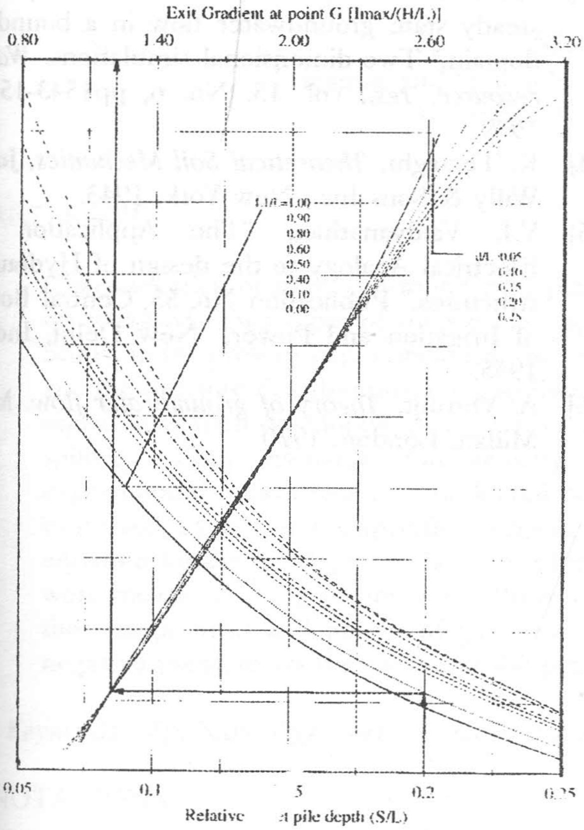


Figure 10. Relation between exit gradient [Imax(H/L)] at point G, and sheet pile depth (S/L), upstream floor length (L1/L), and drop depth (d/L).

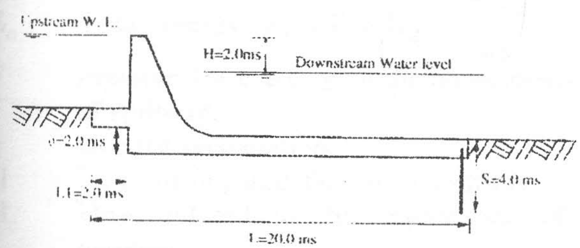


Figure 11. Longitudinal section of weir.

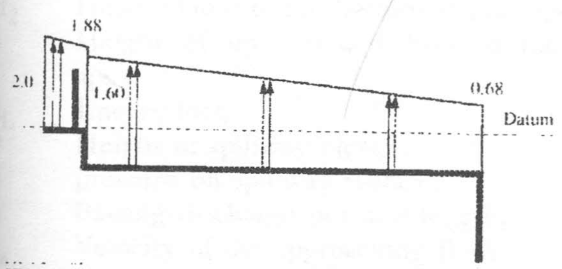


Figure 12. The uplift pressure diagram.

CONCLUSIONS

The seepage characteristics beneath single drop structure founded on permeable soil are numerically studied using the finite element method. The following conclusions may be considered:

1- Significant increase in the uplift pressures on both the upstream and downstream aprons of the floor may results if relatively deep sheet pile are used.
2- The maximum loss of uplift pressures along the floor drop occurs when the floor drop moves towards the upstream side.
3- In summery, this study strongly suggests that, keeping the floor drop closer to the upstream edge of the floor is the optimal location for design purposes.
4- Design charts to determine the seepage characteristics are given in figures (6) through (10).
5- solved example is presented to clarify the use of the obtained design charts in the design purposes.

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