SEEPAGE AROUND HEAD OR TAIL HYDRAULIC STRUCTURES
(Three dimensional problem)

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ABSTRACT

The problem of seepage around a head or a tail hydraulic structure, founded on a finite pervious stratum, is carried out experimentally by a three dimensional electric analogue model. The effect of the structure dimensions, position, and the thickness of the pervious stratum, on the seepage characteristics, are studied. A comparison between the results and some of two dimensional analytical solutions is presented. A simple relationship is expressed to determine the total quantity of seepage water around a head or a tail structure, taken all studied variables into consideration. The results are plotted on the form of curves and analysed.

Keywords: Seepage, Uplift, Groundwater, Head, Tail, Electrical-Analogue, Hydraulic-Structures.

NOTATION

A	digital Amme
B	water top width in channel-I;
C	discharge factor = q/q_c;
2-D	two dimensions
3-D	three dimensions
E	Structure width;
G	A/C Voltage Generator;
H	effective head on the structure;
H_1	upstream water head;
H_2	downstream water head;
I	electric current
k	coefficient of permeability;
L	Structure length;
L_c	approach length for the structure from the edge of the feeder canal or from the drain;
M	thickness of the pervious stratum of soil;
P	hydrostatic pressures;
q	quantity of seepage per unit width, from 3-dimensional studies,
q_c	quantity of seepage per unit width, from 2-dimensional studies.
Q_t	total quantity of seepage around the structure;
S	discharge factor depending on the method of solution
t	penetration depth of the structure on the soil;
U	uplift pressure;
v	discharge velocity through the soil;
V	digital Voltameter;
V_1, V_2	upstream and downstream voltage;
σ	Conductivity of the water.

INTRODUCTION

The seepage pattern beneath the floors of hydraulic structures, which is founded on finite or infinite pervious soil, can be determined analytically, for simple floor profiles [2,4,6 and 7]. The analytical solutions are based on Laplace's equation, for steady state flow of ground water motion in a 2-dimensional plane. For practical problems, the seepage flows under and around the hydraulic structures, therefore, can be solved as 3-dimensional problems. The analytical methods become cumbersome to solve these 3-dimensional problems and experiments are often used. The electrical analogue method was used for solving some of 2-dimensional problems [2, 8 and 11] and some of 3-dimensional problems [3 and 10]. Ujfoludi [10] studied the problem of seepage around a hydraulic structure, founded on two finite pervious layers. He studied the effect of the thickness and permeabilities of the two layers on both the quantity of seepage around the structure and the uplift pressures. In the present study, the problem of seepage around a head or a tail hydraulic structure, founded on a finite homogenous and isotropic permeable soil, is investigated. Figure (1) shows a schematic sketch of the problem. The total quantity of seepage around the structure, the uplift pressure distribution under the structure, the hydrostatic pressures on the structure sides, and the ground water variation around the structure are studied, for various values of the investigated variables. These variables are: the structure position from the feeder canal in head structures, or from the drain in tail structures (L_c), the thickness of the pervious layer (M), the width of the structure (E), the penetration depth in the soil (t), and the water top width of the channel (B). All variables are studied in relative forms to the structure length (L). The results of seepage discharge, which were obtained by Dachler [1]; Kovacs [5] and Pavlovsky [2,9] for the 2-dimensional problems, are compared with the present results.

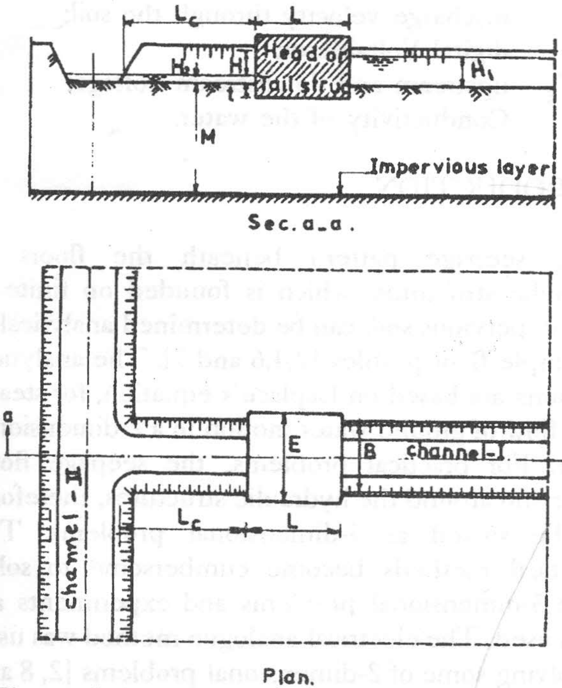

Figure 1. Schematic sketch of the physical problem.

It is worthy to note that the results of this paper may be used to solve some of the seepage problems for head pump stations or tail escapage control structures. The available solved problems of these structures are: evaluation of the quantity of lost seepage discharge, variation of ground water levels around the structure, and stability of suction or approach canal side slopes against exit gradients.

ELECTRIC ANALOGUE AND EXPERIMENTAL SET-UP

Laplace's equation, in addition to providing the governing differential equation for the steady state flow of ground water, is encountered in many other branches of engineering and physics. Among this is the problem of the steady flow of electricity. The reason for this correspondence becomes evident when one considers the nature of the governing law in this disciplines, that is, the counterparts of Darcy's law is Ohm's law for the current conduction. The correspondence between the steady state flow of water through porous media and the steady flow of electric current in a conductor is presented as follows: the total head (H), the coefficient of permeability (k), and the discharge velocity (v) are corresponded by the voltage (V), the conductivity (σ) and the current (I), respectively. However, the relation between Darcy's and Ohm's law is as follows:

$$\nabla 2\ H = \nabla\ 2V = 0 \tag{1}$$

Thus, to obtain the pattern of equipotential lines for a seepage problem, the flow domain can be transferred into an electrical conductor of similar geometrical form.

The experimental set-up is shown in Figure (2). It mainly consists of a prespex tank 120x80x15 cm (1); to represent the boundary of the flow domain around the structure, structure model, electrodes and electric circle. The investigated problem is symmetrical around the center line of the structure, therefore, a half of the problem dimensions is used for the study. The penetration part of the structure model through the flow domain is represented by a prespex plate (2) having dimensions and place according to the study requirements. The two channels (inflow and outflow faces) are represented by electrodes material 0.15 thick and 8, 20 cm width. The flow domain is represented by the water (4) and the depth of the water in tank (1) represents the finite depth of the permeable soil. The electric circle consists of a -16 channel A/C voltage Generator (G), a digital Ammeter (A) for measuring the current (Ampere) and a digital Voltameter (V) for measuring the volt at any point (Volt), by using a movable probe (5). The inflow and outflow potentials, V_1, and V_2, are controlled by the Generator (G) by using the movable probe (5). The values of V_1 and V_2 are fixed at 10 and 2 volts, respectively, for all experiments. i.e., the difference Voltage equals 8 volt. The conductivity of the water (σ) is measured for each run by using a conductivitor (siemens). The potentials are measured under and around the structure. The equipotential lines are plotted as a relative value to the difference voltage. The current is also recorded for each run and transformed into the seepage discharge.

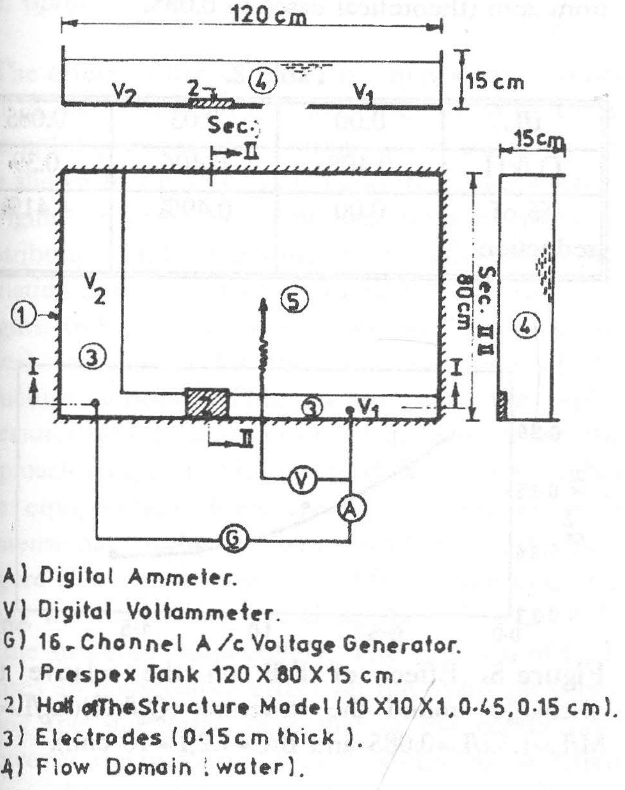

Figure 2. Experimental set-up.

ANALYSIS OF THE RESULTS

The investigated Hydraulic structure, in the present study, is assumed as a block having a length L and a width E. The structure foundation is penetrated by a depth t under the channel bed in the soil. The structure is founded on a homogeneous isotropic pervious soil layer of thickness M through channel I and far by a distance L_c from the edge of channel II. The dimensions of the two channels and the structure length (L) are kept constant for all experiments. The other dimensions of the structure (E & t) and its position (L_c&M) are varied and are put in relative forms as follows: E/L, t/L, L_c/L and M/L. For each variable, different values are chosen, while the other variables are kept constant. The effect of these variables are studied on the following:

1- The relative total quantity of seepage around the structure (Q_t/kH),
2- The relative uplift pressure distributions under the structure base (U/H),
3- The ground water levels and the hydrostatic pressures on the structure sides.

1. *Seepage Discharge*

Figure (3) shows the effect of L_c/L on the relative quantity of seepage discharge around the structure (Q_t/kH). The figure indicates that the value of Q_t/kH increases when the structure moves along the channel I to channel II. Q_t/kH is increased when the value of L_c/L is decreased. The value of Q_t/kH is doubled (0.25 to 0.50) when the structure is moved from L_c/L = 8.0 to the edge of channel -II (L_c=0.00). Therefore, the hydraulic structure should be built at a large distance L_c.

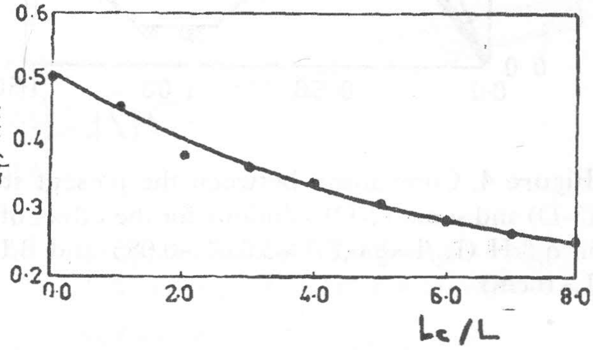

Figure 3. Effect of L_c/L on the relative total discharge around the structure (Q_t/kH) (E/L=2.0, M/L=1.5, t/L=0.085 and B/L =1.6, L=10 cm).

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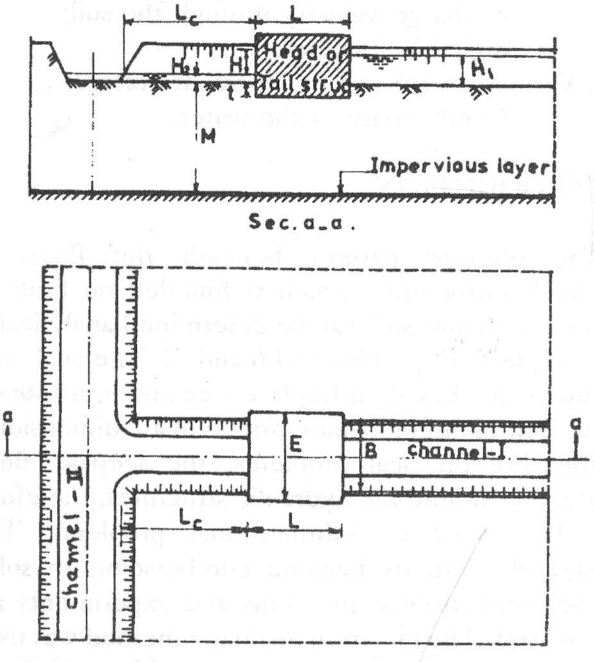

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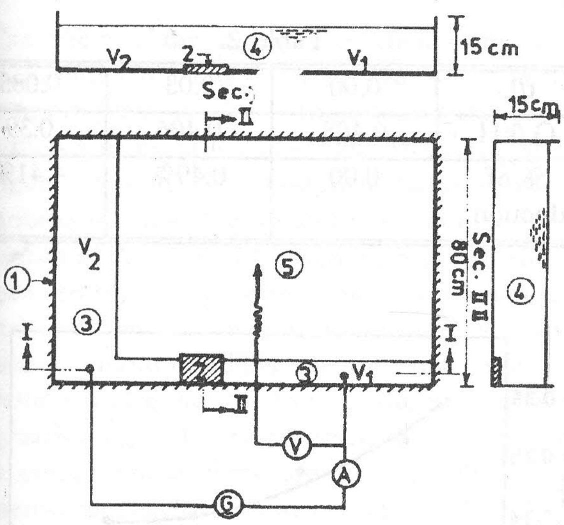

(A) Digital Ammeter.
(V) Digital Voltammeter.
(G) 16-Chonnel A/c Voltage Generator.
(1) Prespex Tank 120 X 80 X 15 cm.
(2) Half of The Structure Model (10 X 10 X 1, 0.45, 0.15 cm).
(3) Electrodes (0.15 cm thick).
(4) Flow Domain (water).
(5) Movable probe.

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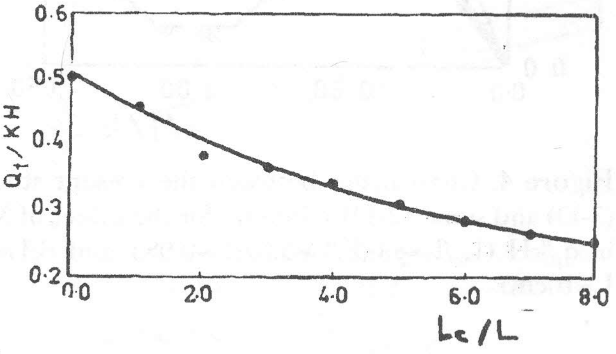

Figure 3. Effect of L_c/L on the relative total discharge around the structure (Q_t/kH) (E/L=2.0, M/L=1.5, t/L=0.085 and B/L =1.6, L=10 cm).

Figure (4) shows a comparison between the present results of $q = Q_t/B$ (3-dimension) and some of 2-dimensional results q_c, for varied values of M/L. From the figure, it is seen that the value of q/kH, for all values of M/L, is bigger than q_c/kH. This because, the lateral seepage around the structure is taken into consideration in the 3-dimensional study. Table (1) shows the values of the factor C ($C = q/q_c$) for various values of M/L and for 3-method of 2-dimensional solutions.

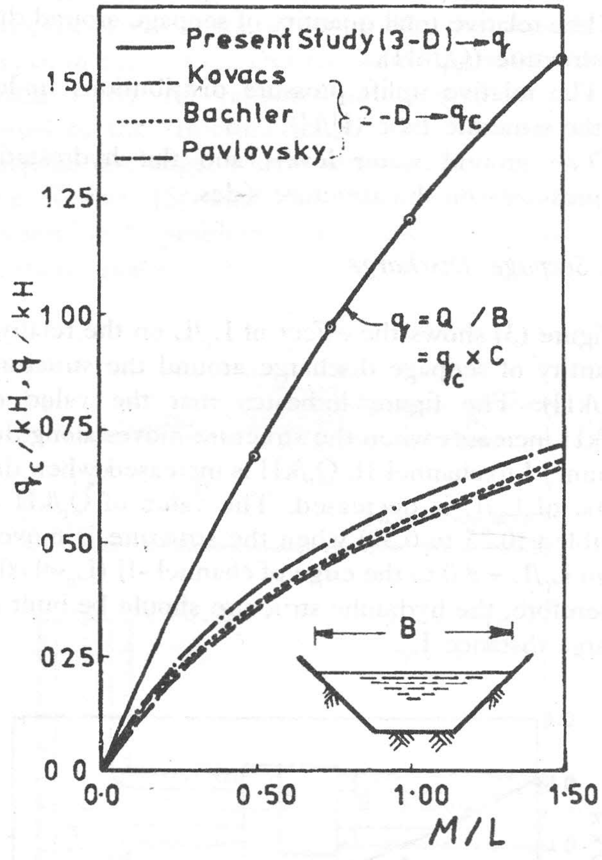

Figure 4. Comparison between the present study (3-D) and some (2-D) solutions for the effect of M/L in q_t/kH (L_c/L=8.0, E/L=2.0, t/L=0.085 and B/L=1.6 L=10cm).

Figure (5) shows that the value of Q_t/kH has a slight effect for the variation of the relative structure width E/B. The relative seepage discharge Q_t/kH is decreased by 3.70% when the value of the width E is enlarged from 0.50 B to 1.50 B.

Table 1.

2-D methods	C = q/q_c M/L 0.50	0.75	1.00	1.50
Pavlovsky	1.83	2.08	2.25	2.34
Kovacs	1.79	1.96	2.06	2.20
Dachler	1.90	2.07	2.17	2.26

Table (2) shows the effect of the relative penetration depth of the structure in the soil (t/L) on the value of Q_t/kH. The total seepage discharge is decreased by 4.41% for increasing the value of t/L from zero (theoretical case) to 0.085.

Table 2.

t/L	0.00	0.03	0.085
Q_t/kH	0.408	0.406	0.39
% of reduction	0.00	0.49%	4.41%

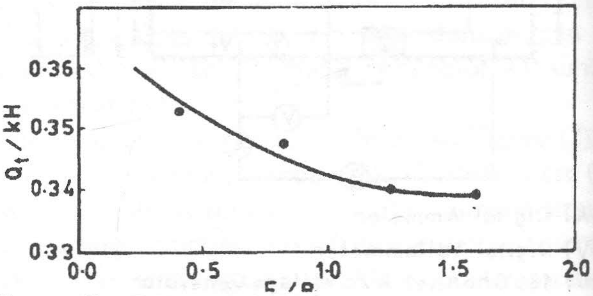

Figure 5. Effect of E/B on the relative total discharge around the structure (Q_t/kH)(L_c/L=3.0, M/L=1.5, t/L=0.085 and B/L=1.2, L=10 cm.).

From the upper analysis, the total seepage discharge has an appreciable effect by the variation of the structure position (L_c,M) while, it has a slight effect by the variation of the structure dimensions (E,t). For a wide range of results, a simple relation is expressed to calculate the total seepage discharge around a head or a tail hydraulic structure, as follows:

$$Q_t = \frac{11.15}{(M/L+3.0)} B * q_c * e^{(S-0.085L_c/L-0.55t/L)} \quad (2)$$

in which

S = 0.40 M/L - Kovacs solution
= 0.45 M/L - Dachler solution
= 0.47 M/L - Pavlovsky solution
B = Water top width in channel - I
q_c = Seepage discharge/unit width for 2-dimensional studies
= kH arsh (3.0 M/L) / π Kovacs solution (3)
= kH M/(L+0.8 M) Dachler solution (4)

and q_c for Pavlovsky solution can be obtained from Harr [2]

2. *Uplift Pressures*

The effect of the structure position along channel I (L_c/L and M/L) and structure dimensions (E/L) on the uplift pressure distribution under the structure are shown in Figures (6), (7) and (8), respectively.

Figures (6-a,b and c) show that the uplift pressure distribution under the structure are affected by the variation of the structure position, L_c/L. As shown in Figure (6-b), the pressure contours (equipotential lines) are moved for the same direction of the structure motion. This means that the uplift pressures under the structure are increased when the approach length (L_c) is increased. It is also seen that the equipotential lines are not straight as in 2-dimensional (pavlovsky) which is plotted in all cases. Figure (6-a and c) show the uplift pressure diagrams along the center line and along the side edge A-A′ of the structure, respectively. The variation of L_c/L causes an appreciable effect on the uplift pressures along the side edge than that of the center line. However, the maximum pressures value are occurred along the centerline of the structure. The 2-dimensional curve of pavlovsky, $L_c = \infty$) envelope all other curves. Therefore, the design of hydraulic structure by using 2-dimensional model results gives more safety.

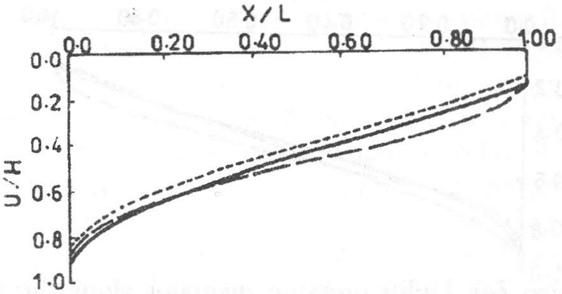

Figure 6-a. Uplift pressure diagrams along the ¢ of the structure.

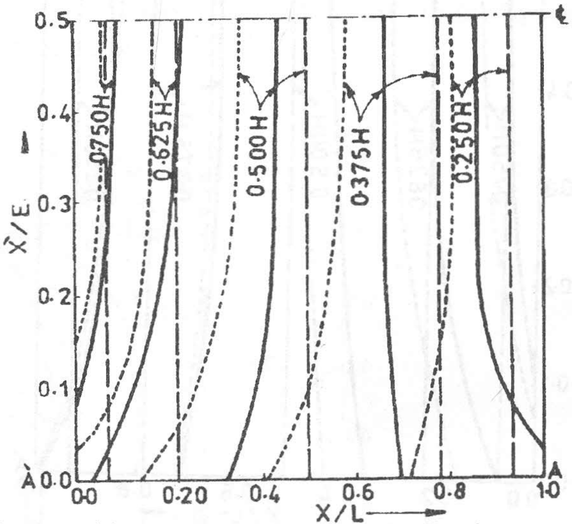

Figure 6-b. Uplift pressure distributions on the subsurface of the structure.

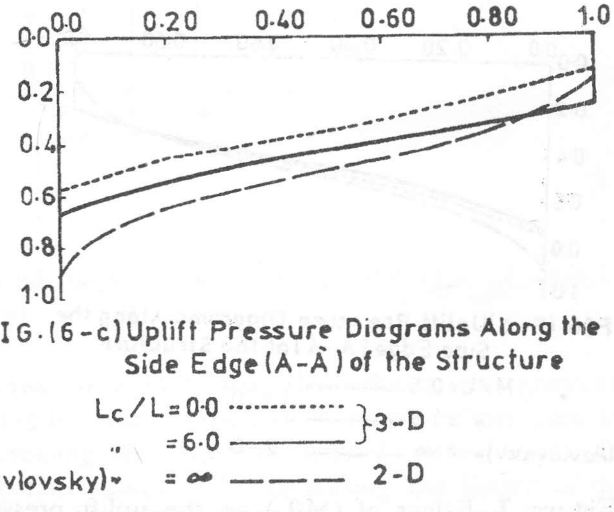

Figure 6-c. Effect of (L_c/L) on the uplift pressure distributions under the structure, (E/L=2.0, M/L=1.5, t/L=0.085 and B/L=1.6).

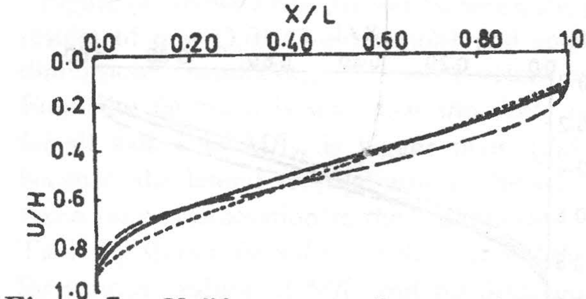

Figure 7-a. Uplift pressure diagrams along the ¢ of the structure.

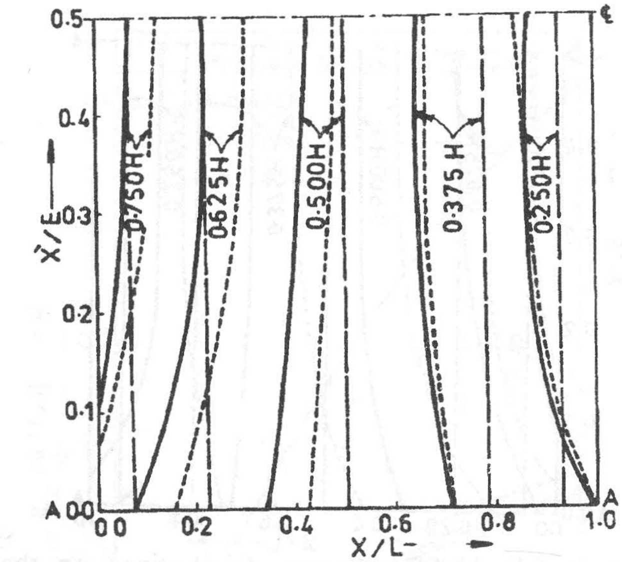

Figure 7-b. Uplift pressure distributions on the subsurface of the structure.

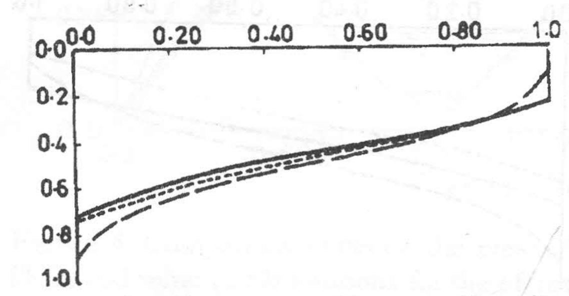

FIG.(7-c)Uplift Pressure Diagrams Along the Side Edge (A-A) of the Structure.

M/L=0.5 .... } 3-D

= 1.5 ———— } 3-D

(Pavlovsky) = ∞ — — — 2-D

Figure 7. Effect of (M/L) on the uplift pressure distributions under the structure, (L_c/L=3.0,E/L=2.0, t/L=0.085 and B/L =1.6.

Figure (7-a,b and c) illustrates the effect of the relative thickness of the permeable layer of soil (M/L) on the uplift pressure distributions under the structure base. From Figure (7-b), it is seen that the equipotential lines are parallel and they have unequal affect due to the variation of M/L. The equipotential lines are closed as we move to the downstream direction. The contour of 2-dimensional model results (pavlovsky) are moved to the opposite direction because the value of $L_c/L = \infty$. Figure (7-a and c) indicate that the, for the upstream 0.8L, the uplift pressures are decreased when the value of M/L is increased and vice versa for the remaining length of the structure. It is also noticed that the 2-dimensional model (pavlovky) covers all other curves. So that, the hydraulic structure can be designed as a 2-dimensional case.

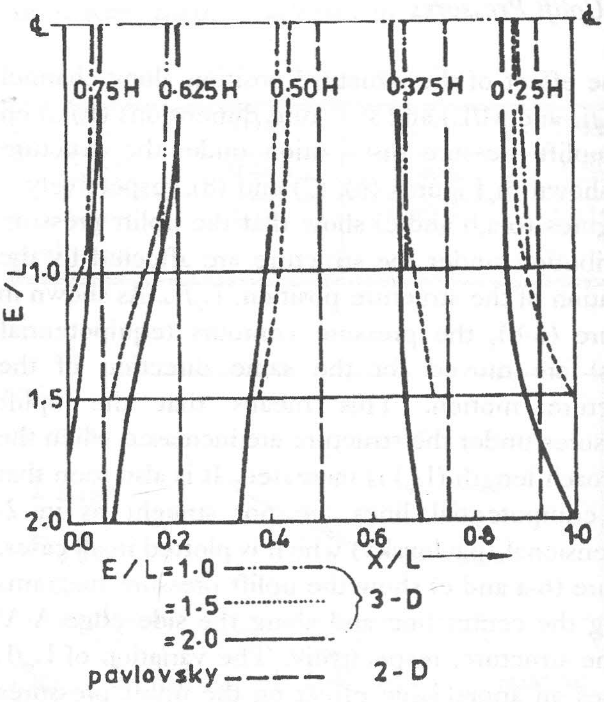

Figure 8. Effect of E/L on the uplift pressure distributions under the structure (L_c/L=3.0,M/L=1.5, t/L=0.085 and B/L =1.6.

Figure (8) shows the effect of the structure dimension ratio (E/L) on the uplift pressure distribution under its base. it is noticed that the equipotential lines are straight, for small widths of the structure, and they are closed to the 2-

dimensional model equipotential lines. Therefore the hydraulic structure is considered as a 2-dimension as the dimension ratio (E/L) is smaller. Practically, it is assumed as a 2-dimension model for the ratio E/L ≤ 1.0.

3. Ground Water Levels

Figure (9) shows the effect of L_c/L on the location of equipotential lines around the structure, for constant values of all other variables. Each equipotential line represents the contour of equal water level measured from a datum. The water level in channel -II is taken as a datum and the water level in channel-I equals H. From the figure, it is seen that the ground water levels around the structure are raised when the relative approach length for the structure from channel II (L_c/L) is decreased. The exit velocity gradients along the sides of channel -II and the approach length of channel II (L_c) are increased when the value of L_c/L is decreased. So that, the variation of the ground water levels and exit gradients should be taken into consideration, for the stability of the side slopes and for the design of lining canals. Figures (9) and (11) also indicates that the hydrostatic pressures along the sides of the structure are increased when the value of L_c/L is increase. Increasing the value of L_c/L from zero to 6.0 causes an increase in the hydrostatic pressures by 5%-25% .

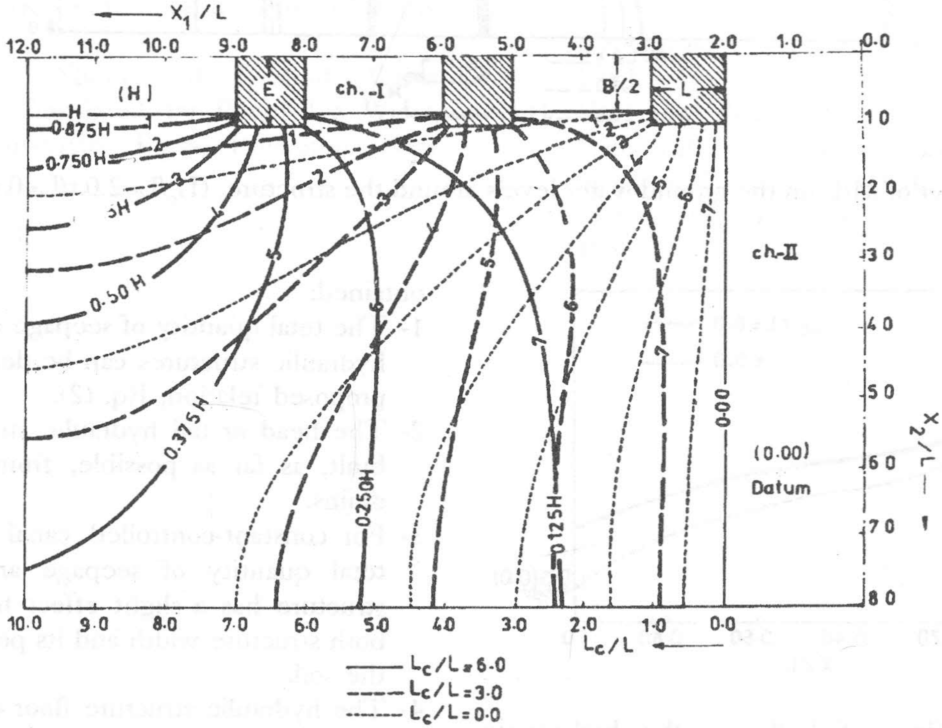

Figure 9. Effect of L_c/L on the ground water levels around the structure, (E/L=2.0,M/L=1.5, t/L=0.085 and B/L =1.6).

Figure (10) shows the effect of M/L on the location of the equipotential lines around the structure. The figure indicates that the ground water levels around the structure are affected by the variation of M/L. The ground water levels are increased, for the area downstream the structure, when the value of M/L is increased and vice versa for the remained area. Hence, it is clear that the velocity exit gradients along the sides of the two channels are increased for increasing the value of M/L. In other words, for constant values of M, increasing the length of the structure decreases is the exit gradients along the sides of the two channels.

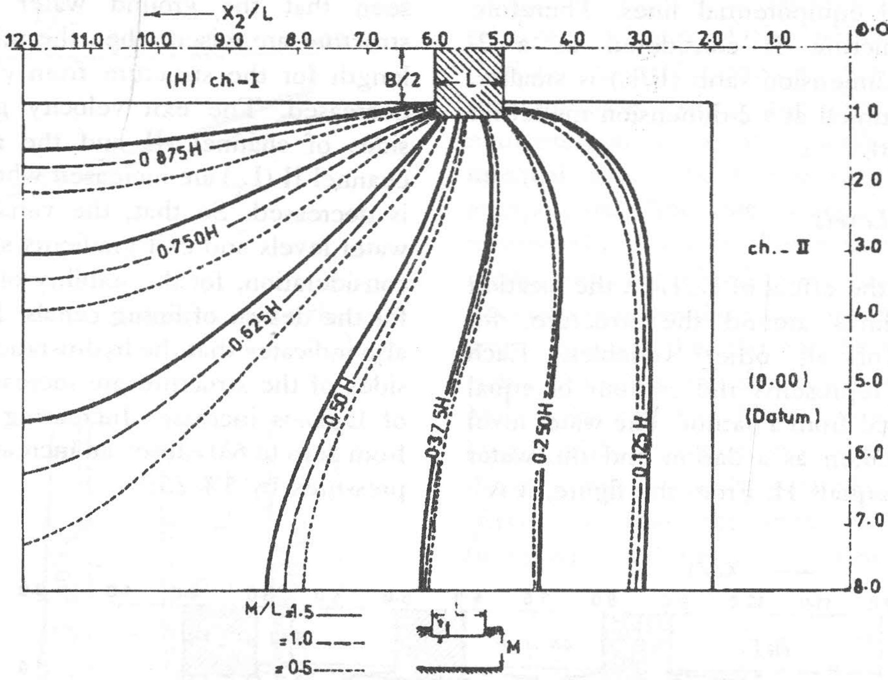

Figure 10. Effect of M/L on the ground water levels around the structure, (L_c/L=2.0,t/L=0.085 and B/L =1.6).

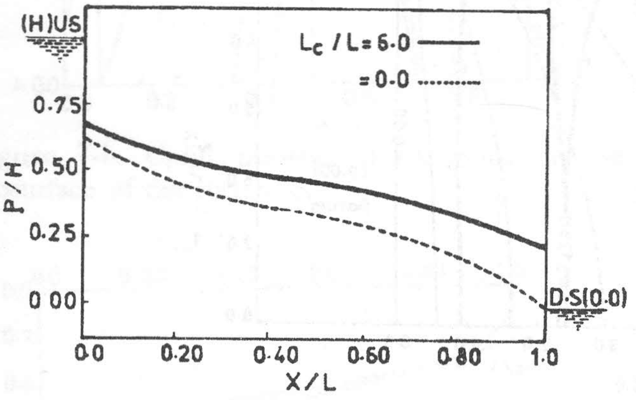

Figure 11. Effect of L_c/L on the hydrostatic pressures along the side walls of the structure, P/H (E/L=2.0,M/L=1.5, t/L=0.085 and B/L =0.6).

CONCLUSIONS

The effects of dimensions and position of head or tail hydraulic structures on the seepage characteristics have been investigated, using a 3-dimensional electric analogue model. From the analysis of results, the following conclusions are obtained:

1- The total quantity of seepage around head or tail hydraulic structures can be determined from the proposed relation, Eq. (2).
2- The head or tail hydraulic structures should be built, as far as possible, from feeder canals or drains.
3- For constant-controlled canal cross section, the total quantity of seepage around the control structure has a slight effect for the variation of both structure width and its penetration depth in the soil.
4- The hydraulic structure floor can be assumed as 2-dimensional when the dimensions ratio E/L ≤ 1.0.
5- The design of hydraulic structures, using 2-dimensional model analytical solutions, is recommended and it gives more safety.
6- For a constant depth of the pervious stratum, the exit velocity gradients along the sides of the investigated channels are decreased for increasing the structure length.

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