OPTIMUM CONTROLLER DESIGN FOR AIRCRAFT ENGINE

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ABSTRACT

This paper applies the optimization techniques in the design of a multivariable optimum controller for systems with disturbance. The controller uses a number of output variables less than the state variables representing the system.The optimum controller guarantees a fast response of the system, minimum overshooting of the outputs , minimum steady state errors and minimizing the settling times. The method is applied for the design of the controller of an aircraft gas turbine with known linear state variable model. The optimum PID and PI controllers are obtained. The speed response of the low and high pressure spools of the gas turbine with each controller are calculated and compared.

Keywords: Gas-turbine control, Optimal controllers, Multivariable control, Aircraft gas turbine.

NOMENCLATURE

- A Transfer matrix of the state variables .
- B Transfer matrix of the control signal inputs.
- C Transfer matrix between outputs and the state variables.
- d Disturbance vector matrix
- E Transfer matrix of the disturbance.
- X State vector matrix.
- P Matrix of the proportional elements of the controller
- Q Matrix of the integral elements of the controller.
- U Open loop input vector matrix
- R Matrix of the derivative elements of the controller.
- Y Output vector matrix.
- J Objective function.
- H Time interval.
- n_i The rise time of the output y_i divided by H.
- U_1,U_2 Open loop inputs.
- x_1 High pressure spool speed.
- x_2 Low pressure spool speed .
- x_3 Jet pipe nozzle area.
- x_4 Fuel flow rate.
- y_1 First output equals to x_1
- y_2 Second output equals to x_2
- y_{iss} Steady state response of the output number i, [i=1,2].
- V_1 Required high pressure spool speed.
- V_2 Required low pressure spool speed.
- t_{s1} Settling time of the output y_1 based on 2%.
- t_{s2} Settling time of the output y_2 based on 2%.
- t_{ri} The rise time of the output number i,[i=1,2].
- T_{max} The period of time for calculating the response.

INTRODUCTION

The problem of designation of a controller for multivariables system is treated from different point of views.Seraji and Tarokh [1] use a cyclic augmented system to replace the state variable system and found the controller that introduces certain poles for the closed loop of that system to assure the stability . Gray and Taylor[2] apply the frequency domain methods in the design of nonlinear feedback loops. Mahmoud and McLean[3] applied a single control law in regulating the dynamic responses of a linear and non-linear model for jet engine as application for multivariable system.Al-Bahi and Abdelrahman[4] tried to find a

single control law suitable for the different operating conditions of an aircraft engine .
This work aims to find the optimal PID and PI controllers that minimize the steady state errors,the overshootings and the settling times for the outputs of a multivariable system with disturbance .The method is applied to control an aircraft engine.

PROBLEM FORMULATION

Considering the multivariable system with disturbance shown in Figure (1) and can be represented by the following state equations:

$$\dot{X} = A X + B U + E d$$
$$Y = C X \quad (1)$$

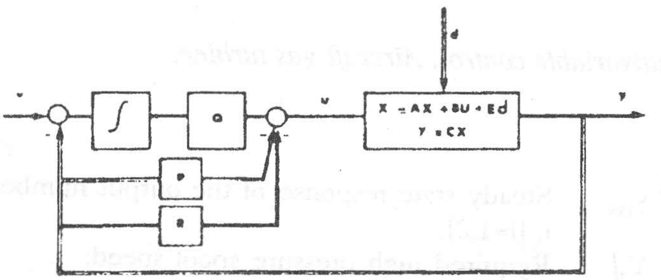


Figure 1. Multivariable PID controller and system model.

The problem is to design multivariable controllers of the type PID and PI that use only the available number of output variables which is less than the number of the state variables. These controllers satisfy the following requirements:

1- Minimizing the steady state errors.
2- Minimizing the settling time for the outputs i.e. assures rapid response of the system .
3- Minimizing the overshooting in the state variables.
4- For disturbances of any time function the outputs exhibit transient responses but will not be affected in the steady state.
5- Each constant of the closed loop controller must be between pre-defined values.

The state model of the closed loop with the suggested PID controller is :

$$U = Q \int_0^t (V - Y) \, dt - PY - R \frac{dY}{dt} \quad (2)$$

It is required to find the controller constants of the matrices Q,P and R that minimize the following objective function:

$$J = \sum_{i=1}^{i=2} t_{s_i} \int_{t_{r_i}}^{\infty} |y_i - y_{i_{ss}}| \, dt + \text{Penalty function} \quad (3)$$

The penalty function must increase the objective function rapidly whenever a parameter of the Q, P and R matrices come near the maximum or minimum limits of its value during the optimization process.This guarantees that the obtained parameters of the controller are in the permissible range of manufacturing.

The optimization technique of Awad[5]based on the gradient method coupled with random search optimization for the first steps of minimizing the objective function is applied.

Design of optimal controller for aircraft gas turbine

The twin spool aircraft gas turbine shown in Figure (2) have low and high pressure compressors, low and high pressure turbines and a nozzle. The flow compatibility must be satisfied between the compressors, the turbines and finally between the low pressure turbine and the nozzle.This makes the equations representing the aircraft gas turbine of that type strongly non-linear[6]. In the literature many linear models are developed to represent it in different working conditions . Some models are suitable for the take-off conditions and others are for subsonic cruise[7].

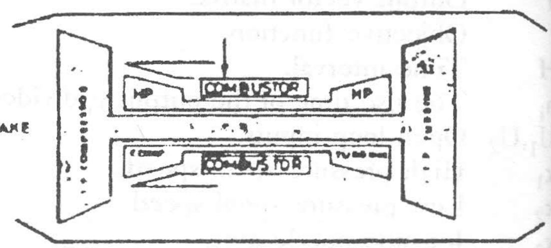


Figure 2. Twin spool Gas turbine.

The method is applied to a linearised perturbation model introduced by Mueller[8]. This model includes the altitude in the disturbance d matrix.

$$\dot{x}=\begin{bmatrix}-1.268 & -0.04528 & 1.498 & 951.5\\ 1.002 & -1.957 & 8.52 & 1240\\ 0 & 0 & -10 & 0\\ 0 & 0 & 0 & -100\end{bmatrix}x+\begin{bmatrix}0 & 0\\ 0 & 0\\ 10 & 0\\ 0 & 100\end{bmatrix}U+\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix}d \quad (4)$$

It is required to find the following twelve constants of the PID controller that minimize the objective function J.

$$P=\begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}, Q=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{21} & Q_{22}\end{bmatrix}, R=\begin{bmatrix}R_{11} & R_{12}\\ R_{21} & R_{22}\end{bmatrix} \quad (5)$$

$$J=\sum_{k=1}^{k=3} C_k \sum_{i=1}^{i=2} t_{s_i} \sum_{j=n_i}^{j=2000} |y_{ij}-y_{i_{ss}}| + \text{penalty function} \quad (6)$$

where,

$$C_1=V_1 \qquad C_2=V_2 \qquad C_3=d \quad (7)$$

and the constraints are :

$$P_{min} < P_{ij} < P_{max}$$

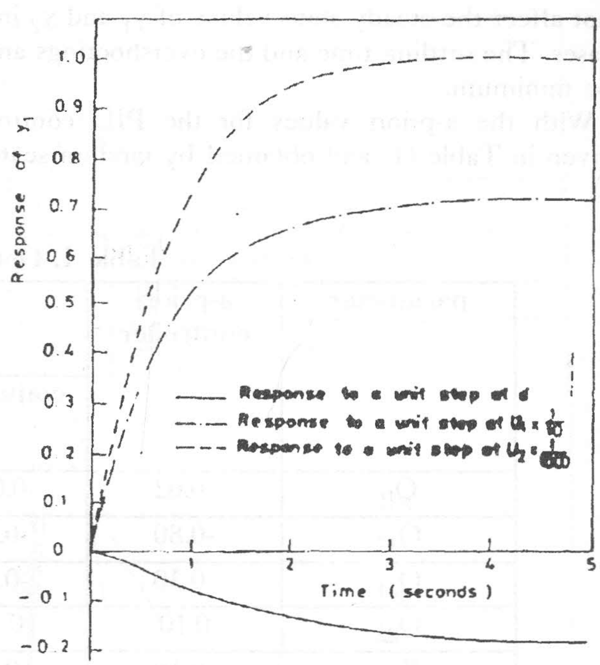
$$Q_{min} < Q_{ij} < Q_{max} \quad (8)$$

$$R_{min} < R_{ij} < R_{max}$$

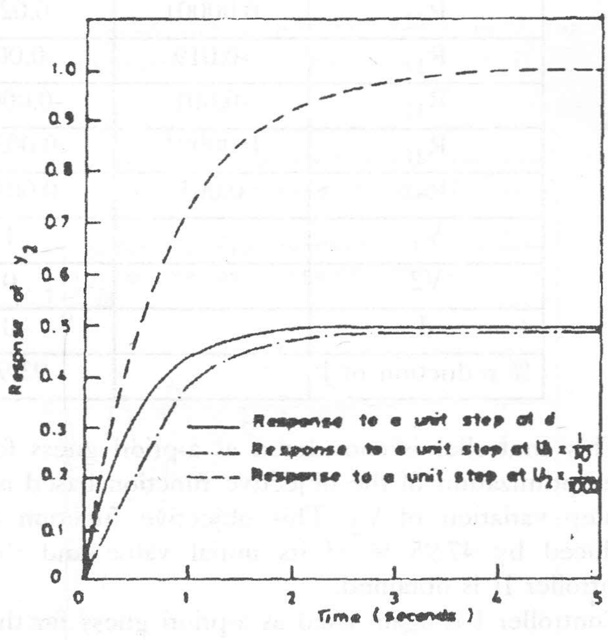
Optimal PID controller

Starting with the linear model of equation 4 and testing the open loop model of the gas turbine for step changes in U_1, U_2 and d respectively. The numerical solution of y_1 and y_2 is obtained by applying the Runga-Kutta [9] method of the fourth order using time interval H=0.005 second and the period of time T_{max} = 10 seconds. i.e for each time response 2000 points are calculated to assure good accuracy of the solution. The obtained responses are shown in Figure (3a,b). They are slow, altered by step change in U_1 or U_2 or d, having large steady state errors and indicating interaction.

It is required to get the optimum PID controller such that for a unit step change in V_1 the corresponding output y_1 exhibits a fast transient response lasting less than 5 seconds and changes by unity in the steady state. The steady state value of y_2 remains constant. This also stated for the change of V_2.



a Speed response (y_1) of the high pressure spool



b-Speed response (y_2) of the low pressure spool.

Figure 3. Open loop speed responses of the Gas turbine spools.

Furthermore a step change in the altitude d does not affect the steady state values of y_1 and y_2 in all cases. The settling time and the overshootings are to be minimum.

With the a-priori values for the PID controller given in Table (1) and obtained by random search optimization . The twelve constants of the controller are to be constrained by ± 10. The optimization technique is applied to minimize the objective function based on a step variation of V_1. The objective function is reduced by 93.9% of its initial value and the optimal controller I is obtained.

Table 1. Optimized PID Controllers.

parameter	a-priori controller	Optimized controllers			
		controller I	controller II	controller III	controller IV
$Q_{!!}$	-0.02	-0.02129	-0.3794	-0.3038	-0.6351
Q_{12}	-0.80	-0.1020	-0.1404	-0.1259	-0.1763
Q_{21}	-0.30	-0.0603	-0.0569	-0.0270	0.0097
Q_{22}	0.10	0.1993	0.1864	0.2097	0.2480
P_{11}	-0.16	-0.1293	-0.1305	-0.1301	-0.0501
P_{12}	0.06	0.0606	-0.0526	0.0595	-0.0821
P_{21}	0.012	-0.0289	0.0172	-0.0252	-0.0427
P_{22}	0.00001	0.0279	0.0504	0.0284	-0.0475
R_{11}	-0.019	-0.0055	0.0291	-0.0090	-0.0497
R_{12}	-0.001	-0.00005	-0.0198	-0.0019	-0.0496
R_{21}	0.00001	0.00523	0.0034	-0.0003	-0.0058
R_{22}	0.001	0.00116	0.0011	0.0047	0.0059
V_1		1	0	0	1
V2		0	1	0	1
d		0	0	1	1
% reduction of J		93.9%	47.9%	39.8%	57.02%

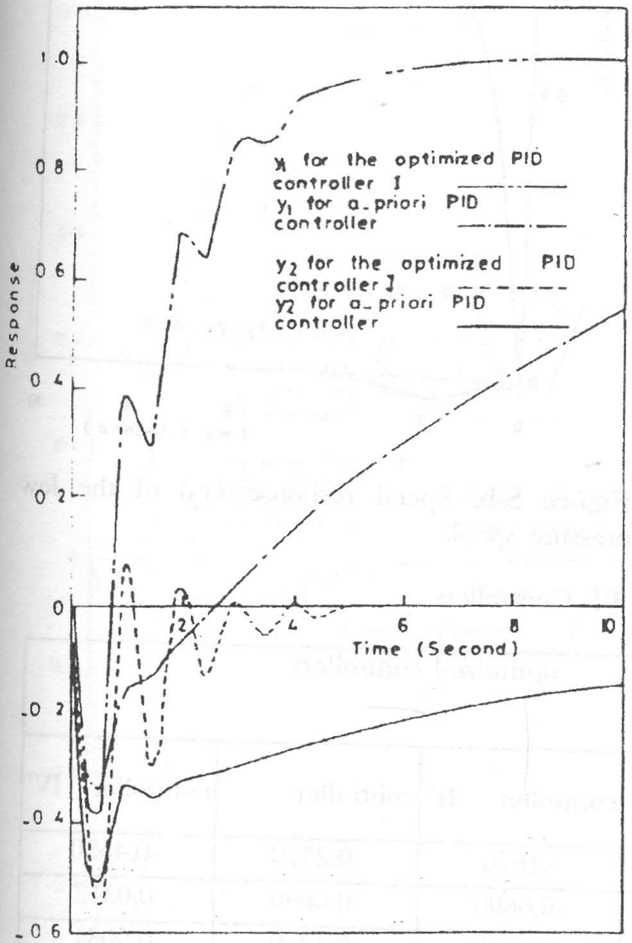
This controller is considered as a-priori guess for the optimization of the objective function based on a step variation of V_2. This objective function is reduced by 47.95 % of its initial value and the controller II is obtained.

Controller I is again used as a-priori guess for the minimization of the objective function based on a step change in the altitude d . This is reduced by 39.7% of its initial value.

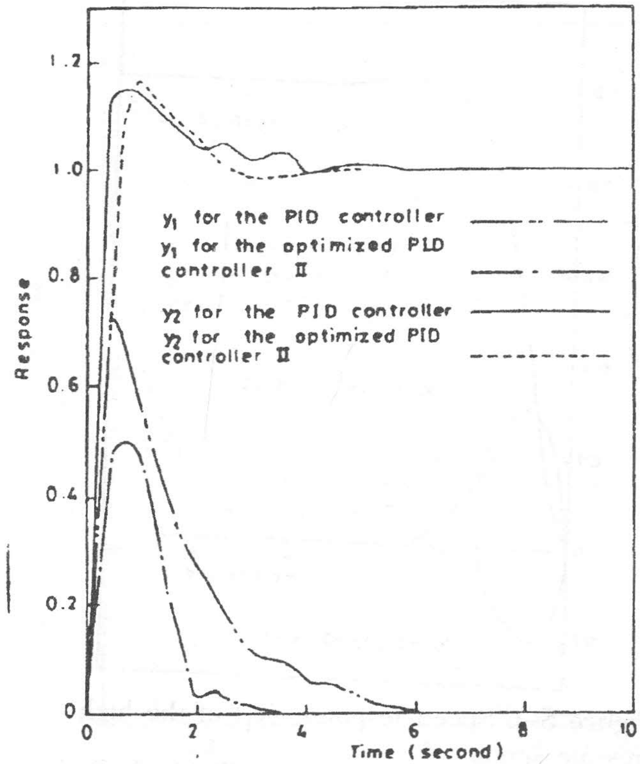
Optimizing the controller to minimize the objective function based on step changes in V_1, V_2 and d. The optimal controller IV is obtained . The responses of the gas turbine with the a-priori controller and with controllers I,II,III,IV are shown in Figures (4a,b,c)and (5a,b). The values of the controller constants are given in Table (1).

Optimal PI controller

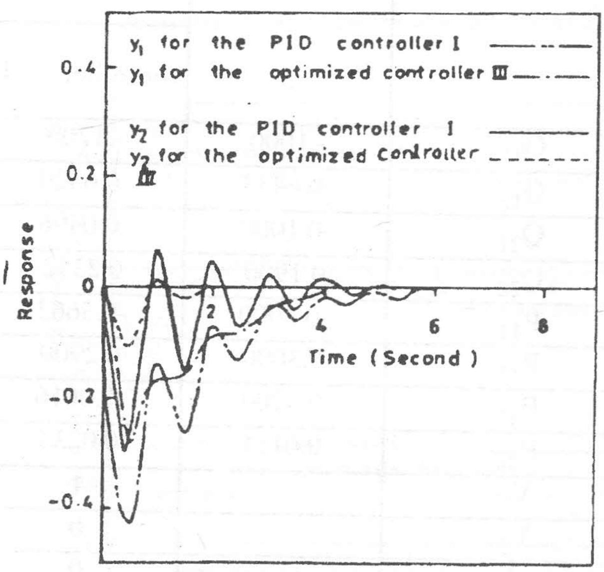
Investigation of table 1 shows that the obtained values of the derivative matrix **R** are relatively small to that of **P** and **Q** matrices. This gives the idea to optimize a PI controller suitable for the gas turbine.



a-Speeds due to unit step change in V_1 for apriori and optimized model I



b-Speeds due to unit step change in V_2 for controller I and optimized controller II



c-Speeds due to unit change in d for controller I and optimized controller III

Figure 4. Closed loop speed response of the Gas turbine spools with optimized PID controllers.

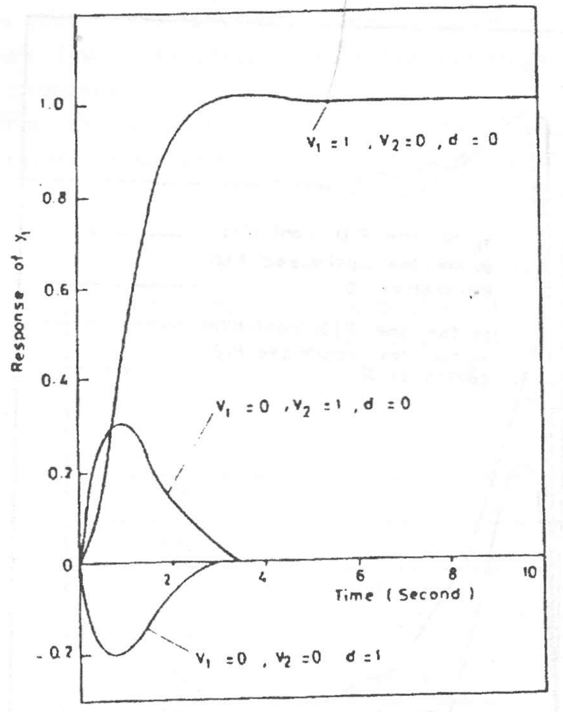


Figure 5-a. Speed response (y_1) of the high pressure spool.

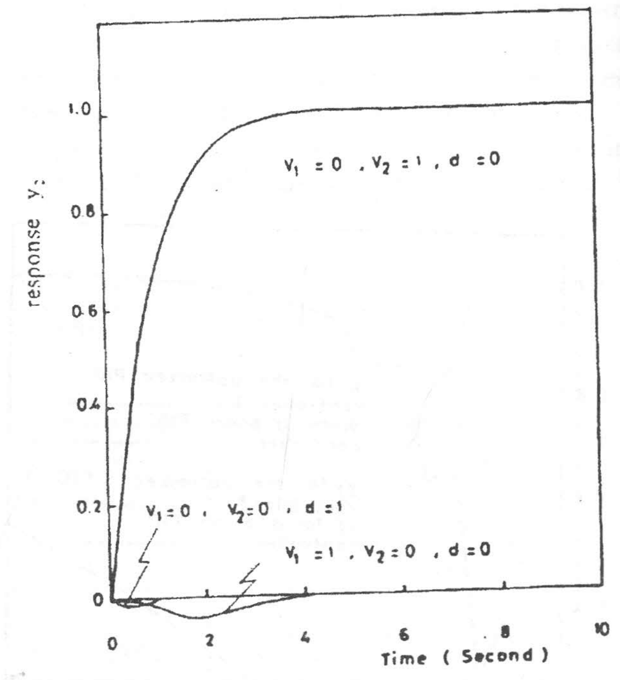
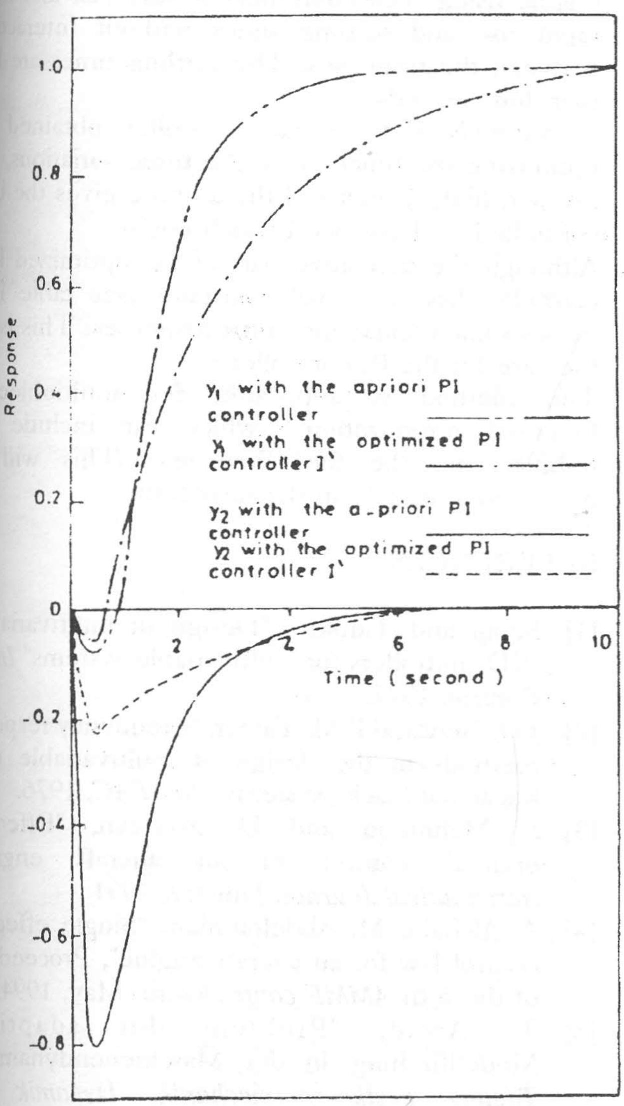


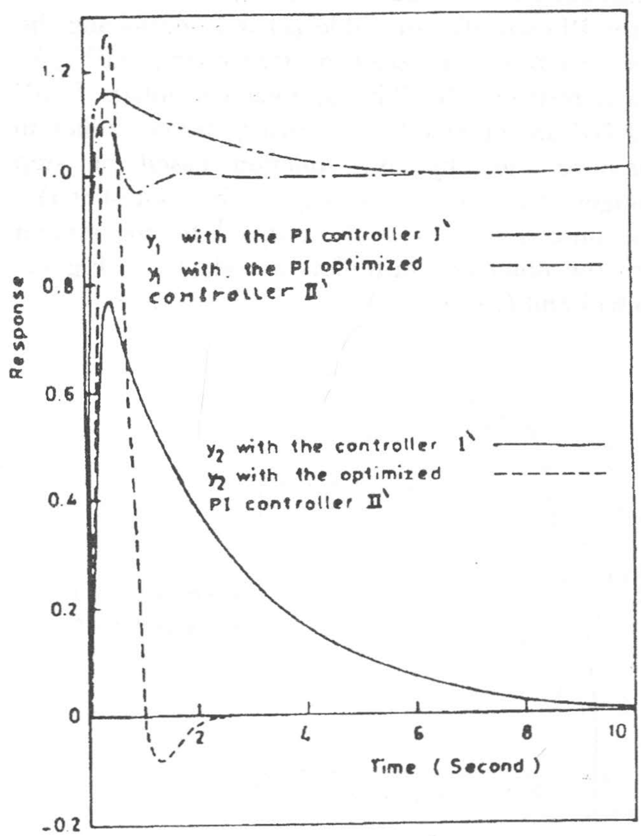
Figure 5-b. Speed response (y_2) of the low pressure spool.

Table 2. Optimized PI Controllers

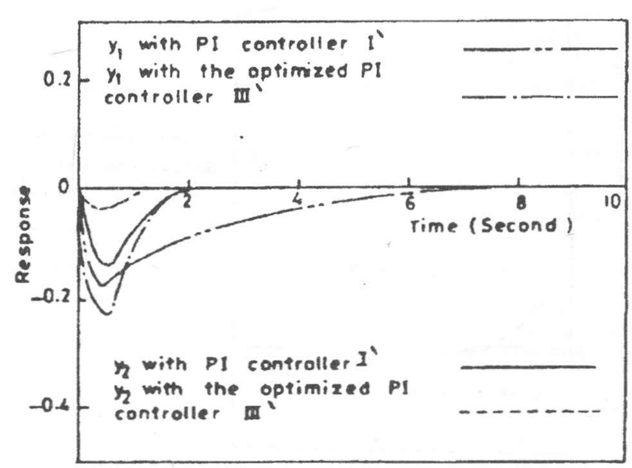
parameter	a-priori controller	optimized controllers			
		controller I'	controller II'	controller	controller IV'
Q_{11}	-.1000	-.1700	-.1670	-0.2310	-0.4630
Q_{12}	-0.6433	-0.6520	-0.6690	-0.6690	0.0312
Q_{21}	-0.1000	-0.0496	-0.1210	-0.1260	0.0081
Q_{22}	0.1900	0.2312	0.1820	0.1971	0.2612
P_{11}	-0.6000	-0.5662	-0.5560	-0.5620	-0.0085
P_{12}	0.3000	0.2900	0.2890	0.2970	-0.0085
P_{21}	0.0200	-0.0016	0.0240	-0.0085	0.0280
P_{22}	0.0111	0.0233	-0.0124	0.0314	0.2280
V_1		1	0	0	1
V_2		0	1	0	1
d		0	0	1	1
% reduction of J		49.6%	60.8%	35.3%	96.9%



a- Speeds due to unit step change in V_1 for apriori and optimized controller I`.



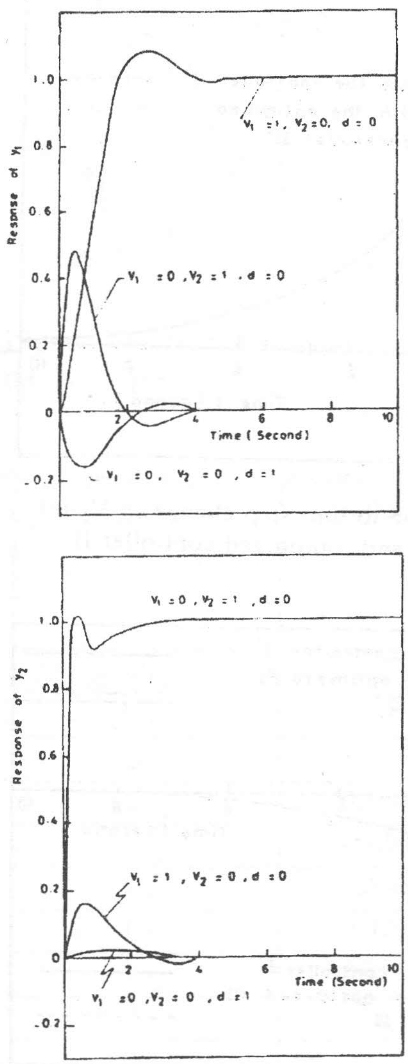
b- Speeds due to unit step change in V_2 for controller I` and optimized controller II`



c-Speeds due to unit step change in d for controller I` and optimized controller III`.

Figure 6. Closed loop speed response of the Gas turbine spools with optimized PI controllers.

The above steps applied to obtain the PID controller are followed starting with the initial guess of the PI controller of Table (2) and minimizing the objective functions based on step change in V_1, V_2 and d respectively. The optimal controllers I' ,II' and III' are obtained. Optimizing the controller to minimize the objective function based on step changes in the three inputs gives the controller IV'. The time responses of the outputs for the system with the obtained controllers are shown in Figures (6a,b,c) and (7a,b).



a- Speed response (y_1) of the high pressure spool.

b- Speed response (y_2) of the low pressure spool.

Figure 7. Closed loop speed responses due too step changes in V_1, V_2 and d using the optimum controller IV.

CONCLUSIONS

The PI and PID controllers obtained by the above method gives very good responses that fit the demands of aircraft engines. The performance of the engine using such controllers is very fast and have rapid rise and settling times without interaction between the responses. The settling times are less than four seconds.

Naturally the optimal controller obtained by optimizing the function for the three variations, the low and high speeds and the altitude gives the best controlled performance of such engine.

Although the derivative part of the optimized PID controller has very small constants (see table 1) ,it causes some oscillations in the responses. This is not the case for the PI controller.

The method is applicable for multiobjective functions optimization which can include the stability and the fuel flow rate. This will be published separetly in the near future.

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